Are Economics-Based and Psychology-Based Measures of Ability the Same?

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ABSTRACT

Economists rely on psychology-based IQ and achievement test scores to assess ability. Yet human capital models of lifetime earnings propagation entail human capital production function parameters that incorporate ability parameters. This paper makes use of human capital theory to derive a highly nonlinear, but empirically tractable, earnings function which when estimated yields parameters representing cognitive ability. Given that the National Longitudinal Survey (NLS-Y) now has up to 22 years of data on each individual respondent, we estimate these earnings functions for each individual to extract individual-specific estimates of ability. We then compare our estimated ability parameters with independently obtained intelligence test scores for these same individuals in the NLS-Y data. We find a significant positive correlation between our measures and the independently obtained psychologically-based cognitive test scores. These measures adhere to predicted relationships regarding types of ability and schooling. We find a positive correlation between intellectual prowess (the ability to create new human capital from old) and years of school, and a negative relation between years of school and stocks of knowledge, as predicted by human capital theory. However, unlike the psychology-based measures, our ability estimates yield a weaker relationship with race, thereby implying the possibility of greater racial biases in the psychologically-based measures than our economics-based measures.

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1. Introduction

Cognitive ability reflects an individual’s capacity to perform proficiently in intellectual pursuits. For our purposes, we characterize cognitive ability to comprise intelligence as well as knowledge based skills. We view intelligence as the ability to think and solve problems, and knowledge as the ability to recall and use prior learned materials. Two strands of research measure ability. The first, and most common, is based in psychology. This strand originated in 1904 when the French government commissioned Alfred Binet to develop tests, now known as the IQ (intelligence quotient), to measure various aspects of cognitive skills. Nowadays there is a myriad of such tests. They examine reading comprehension, arithmetic and algebraic skills, spatial relations, vocabulary, and other components of cognitive ability. The second research strand is based in economics, but is less well known. This latter approach uses life-cycle human capital theory to derive an estimable nonlinear earnings function that according to the literature (beginning with Ben-Porath, 1967) contain human capital production function parameters denoting intelligence as well as a parameter denoting knowledge.

Currently practitioners, including social science research scholars as well as economists, generally ignore the economics-based approach when they measure ability, mostly because the economics-based technique is difficult to implement. In order to identify ability parameters for particular individuals, the economics-based technique requires a highly nonlinear specification incorporating sufficiently long panel data, which at least in the past were not readily available. Thus, instead of doing their own estimation, economists usually rely on psychologists’ IQ and achievement tests to identify ability. Typically, to account for ability, economists use these measures as independent variables.

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1 The whole notion of testing dates back at least to 2200BC when the Chinese emperor had his officials tested every three years to determine if they were fit. Written tests covering civil law, military affairs, agriculture, revenue and geography were introduced by the Han Dynasty (202BC to 200 AD), but these were designed to measure knowledge more so than intelligence (Gregory, 2011). Currently, tests begin at birth with the Apgar test, and continue throughout life in school, encompassing tests for learning disabilities, giftedness, vocational interest, college admission, driver’s licenses, marital compatibility, and more. The mainstay of psychological tests entails measuring aspects of personality and intelligence. Like in empirical economics, psychological tests represent a limited sample of behavior from which the examiner draws inferences about the total domain of relevant behavior. For example, the number of words defined in the vocabulary subset of the Wechsler test signals the examinee’s general knowledge of vocabulary. Making such inferences is known as assessment in the psychology literature.

2 We concentrate mostly on the AFQT because it is used widely by economists to control for ability in earnings function and other statistical analyses, and because it is contained in the National Longitudinal Survey of Youth (NLS-Y). However, as a robustness check, we present some additional results for fifteen other IQ type tests.
in simple earnings function regressions. When panel data are available, they often rely on fixed-effects techniques to hold individual-level ability constant.

Earnings functions that incorporate intelligence tests as independent variables estimate the effects of ability, rather than measure ability itself. The same can be said for fixed-effects regressions which are designed to net out individual specific heterogeneity, including ability. However, with the advent of speedier computers, better optimization programs, and longer panels, measures of ability based on human capital theory can now be retrieved by estimating parameters of a nonlinear earnings function. With sufficiently long panels one can estimate such ability parameters for specific individuals. From these estimates, one can aggregate the data to estimate average ability for selected groups, such as all those employees with a given level of schooling or all those employees of a particular racial group.\(^3\) The advantage of this economics-based approach is one can obtain measures of ability founded on the economics principle of optimization which results when individuals accumulate human capital to maximize the present value of lifetime earnings, rather than be rooted in the psychology principles underlying intelligence testing of which there is a large literature (Hogan, 2007).

Obtaining ability measures using economics-based models is important for a number of reasons. First, the economics-based approach yields several distinct types of ability which enables one to test theories emanating from human capital theory. For example, theory predicts that intellectual prowess (what we will define as the ability to create new human capital from old) causes one to get more schooling whereas a greater innate stock of knowledge causes one to go to school less. Additionally, theory predicts higher discount rates (which our model also estimates) are associated with less years of school. Checking the strength of these predictions serves to test the validity of the life-cycle human capital models.

Second, obtaining ability measures using economics-based models is important because it enables one to compare these new economics based ability measures to previously obtained psychology based measures. If they differ, one can question the

\(^3\) As will be discussed later, the economics-based approach is limited to those who have a work history. Whereas, this limitation could (but probably does not) bias inferences about the non-working population, it does not bias conclusions comparing the economics-based and psychology-based approaches for observationally equivalent workers, which is the main point of this paper.
validity of one or the other. Though designed for different purposes, credence is enhanced regarding the reliability of each if both correlate with each other.

Third, the ability measures based on the economics approach can be used in a Mincer-type earnings function along with psychology based measures to see which better explains earnings variation. One might be reluctant to use psychology-based test scores rather than economics-based measures should the explanatory power of the economics-based measures appear to have more explanatory power than the standard AFQT ability measures when accounting for earnings variation.

Fourth, and perhaps more controversial but even more important, one can test whether traditional IQ type tests are racially biased, as many allege is the case. This is a real possibility given that the different cultures of minority groups are typically not taken into account when formulating questions for psychologically-based intelligence tests. As will be shown, the economics-based ability parameters we estimate are obtained from a model conceptually independent of race. As such, notions of race do not enter the structural definition of our human capital based ability measure, though they do enter the estimation process if racial discrimination affects earnings. Thus at least from a theoretical standpoint, our ability measures are conceptually race neutral. From both pedagogical and policy perspectives this race neutrality is important because finding no racial differences in economics-based ability measures would be strong evidence that IQ type measures are indeed racially biased, since those measures do differ by race, whereas our economics-based measures would not. On the other hand, should our economics-based measures differ by race in the same way IQ measures differ by race, one can make either the case that neither measure is racially biased, or alternatively one can make the case that wage discrimination is racially biased in exactly the same way that cultural

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4 A long history of intelligence test cultural biases and their implications begins when Henry Goddard, a prominent American psychologist, lobbied for restrictive immigration laws based on his “finding” that over 80% of the Jewish, Hungarian, Italian, and Russian immigrants he tested using “translated” Binet-Simon IQ tests were “feeble-minded” (Goddard, 1912). He somewhat recanted in 1928, but nonetheless Goddard’s intelligence testing opened up the question of cultural biases in implementing such tests. The same can be said for achievement and numerous other type tests (Gregory, 2011).
biases have inculcated the psychologically-based test score measures. In either case, the academic profession will be better informed how ability differs by race.

In addition, there are other related problems with standardized ability tests that get less attention. For example, tests can be inaccurate if examinees have undiagnosed disabilities. This is especially true with children who are often poor informants of their capabilities. Vernon and Brown (1964) report the case of a young girl admitted to a hospital for the mentally retarded for five years because she scored a 29 on the Stanford-Binet test. The child was released when hospital attendants finally realized she was deaf and had an IQ of 113 when measured on a performance-based instead of oral test. Children who blink, squint, or lose their place could be visually impaired which also could affect results on these tests. Similarly there is a whole literature on how race, experience, ethnicity and other characteristics of the examiner can affect test results (e.g., Terrell, et al., 1984). The same can be said for examinee background and motivation including problems regarding test anxiety. These imperfections in psychological tests and the interpretation of their results could imply a need for a completely different methodology for assessing ability. The economics-based approach is one possibility (though not applicable to children since one needs data on labor market experience).

Finally, fifth, one other important aspect of this paper is how it emphasizes individual heterogeneity. Currently literally hundreds of articles “account” for unobserved individual heterogeneity by “netting out” person-specific effects. Studies that assume person-specific parameters are of three genres. First, random coefficient models and extensions including the correlated random coefficients model (Heckman, et al. 2009) assume individual parameters vary across individuals in accord with a particular distribution but do not identify each individual’s actual parameter values. Nor are they usually certain of the underlying parameter distribution, which one usually assumes to be normal. Second, other panel data models adjust for person-specific slope parameters (Polachek and Kim, 1994; and Pesaram, 2006), but these are often limited to one parameter besides the intercept. Finally non-parametric approaches get at heterogeneity essentially by grouping individuals according to related (neighboring) measured...
characteristics within optimal “band widths” (Racine and Li, 2004). Rather than individual-specific parameters, they obtain “group-specific parameters.” Our approach makes use of long enough panels to obtain individual specific measures for each coefficient in the earnings function model we employ, including parameters specifying ability.

Of course a number of assumptions underlie the economics-based approach. First, the approach assumes individuals plan their human capital investment strategy based on expectations that they seek to work each year of their working life. This means individuals do not leave the labor force for family reasons and that they do not alter their intended human capital investment plans based on unexpected spells of unemployment. It is why we concentrate on males who generally have continuous work histories.5 Second, the approach assumes a relatively simple human capital production function. We assume individuals use their time and existing human capital to create new human capital, but we ignore other inputs such as books and computers as well as parental, teacher, and school quality inputs which can also be used to create additional human capital. In our model high ability people can create a given amount of human capital with smaller time inputs. Third, we ignore non-cognitive skills, but these could be incorporated by modifying the human capital production function. Fourth, the approach assumes labor markets reward individuals based on their existing stock of human capital, and that neither incomplete information nor incentive pay governs worker earnings. Fifth, we assume all human capital production function parameters remain constant throughout each person's life. In the context of our model, this means we assume that ability does not change over one’s lifetime though modifications can be made to parameterize changes in measured ability as environmental factors such as job, industry, or location change. Finally sixth, it relies on individuals with a significant work history. Obviously, those with a work-history constitute a select sample of the population. However, in our case, this selectivity does not preclude inferences obtained when comparing the economics-based and psychology

5 One potential bias we face in comparing whites and blacks is that blacks may select different types of jobs if they expect more spells of unemployment than whites. If blacks insure against risk they might seek jobs with more human capital investment thereby mitigating observed differences in our ability measures. We adjust for this somewhat by comparing blacks fully employed in every year to those with unemployment spells.
based methods because precisely the same individuals are used in evaluating the two approaches. On the other hand, selectivity biases could come about when making inferences about racial differences in ability if white workers are different in ability than black workers, for example, if black workers are relatively more able than white workers compared to black and white non-workers. However, as will be shown later, we find that the ability advantage of workers to non-workers is similar for both blacks and whites, so that this bias is at worst very small.

The remainder of our paper is organized as follows: Section 2 derives a theoretical model from which we are able to identify three measures of cognitive ability. Section 3 describes the data. Section 4 presents the estimation procedure and empirical results. In this section, we illustrate how our parameters are consistent with population-wide estimates from past studies, and how our ability parameters are consistent with ability measures from psychology-based tests. Further, in this section, we present evidence that intellectual ability is positively associated with school level whereas knowledge-based skills are not, as predicted by human capital theory. Finally, in that section we discuss issues regarding selectivity and robustness. Finally, Section 5 concludes.

2. Using the Life-Cycle Human Capital Model to Estimate Ability

Most empirical studies adopt single equation log-linear Mincer earnings functions to parameterize earnings. The beauty of estimating simple Mincer earnings functions is computational ease. Assuming schooling and experience are exogenous, Mincer earnings functions are easily estimated by OLS. Although more recent analyses question whether such OLS estimation procedures are free of econometric biases (Heckman, Lochner and Todd, 2006), in reality the Mincer model is a simplification based on Taylor approximations of a more complex function. Underlying the Mincer model is a life-cycle

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6 There are also some conceptual biases regarding how to interpret such parameters as the schooling and experience coefficients which many take to measure rates of return to school and experience.
earnings generating process that yields a highly nonlinear earnings function. From this nonlinear function one is able to identify several cognitive ability parameters based on the production function of human capital. We define two of these parameters to depict intelligence because they measure the ease which an individual can create new human capital from old human capital, a process which entails innovative thinking, but not necessarily rote recall of facts. The first of these parameters is the individual’s human capital production function output elasticity, and the second is the individual’s human capital production function total factor productivity parameter. These two intelligence measures of cognitive ability are distinct from one’s knowledge base, which in our framework is defined more as rote knowledge and depicted as the individual’s earnings power devoid of human capital investments. In our framework, this is one’s human capital at the time one begins school. We also provide estimates of a combination of intelligence and knowledge which is our estimate of one’s human capital stock measured at the time one graduates from school and enters the labor market.

The derivation of the earnings function containing these parameters entails the typical economics-based maximization paradigm. It assumes an individual invests in human capital to maximize the present value of expected lifetime earnings. Based on this optimization process, one can derive optimal human capital investment, optimal human capital stock, and optimal earnings over a person’s lifetime. In the model one’s earnings over the life-cycle are directly proportional to one’s human capital stock. Each year one’s human capital stock is augmented by the amount of new human capital one creates through schooling and on-the-job training, and one’s human capital stock is diminished by the amount human capital depreciates. Creating new human capital entails using time and existing human capital to produce new human capital, given one’s ability. The greater one’s ability the more human capital one can produce, and the more rapidly one can increase earnings power from year-to-year (Ben-Porath, 1967). The result is a nonlinear earnings function with three parameters reflecting different kinds of ability.

Mincer’s log-linear specification gets around these nonlinearities by assuming time-equivalent human capital declines linearly with age. In reality, the time path of human capital acquisition is more complicated. Taking this into account yields a highly nonlinear earnings function.
Whereas not everyone believes in the human capital approach as the basis for one’s earnings, the model is surprisingly robust compared to other models in explaining life-cycle earnings patterns. For example, screening models explain why education enhances earnings; occupational segregation models explain why women earn less; efficiency wage models explain certain wage premiums; and productivity enhancing contract models explain upward sloping (though not necessarily concave) earnings profiles; but none of these theories simultaneously explain all of these issues as does the human capital model. But more important, these other models allow not one to identify ability from estimated parameters. For this reason we adopt the human capital model to approach the problem of measuring ability.8

2.1. The Ben Porath Model

The Ben-Porath (1967) model assumes individuals invest in themselves to maximize expected lifetime earnings.9 Investment is governed by a production function in which one combines own time and ability along with past human capital investments to create new human capital. At the margin, one equates the marginal cost and marginal gains of creating new human capital. The marginal cost of each unit of investment is essentially the foregone earnings of the time needed to produce a marginal unit of human capital.10 The marginal gain is the present value of each additional unit of human capital. Ben-Porath’s innovation was to realize that the finite life constraint implies the marginal gain declines monotonically over the life-cycle (at least for individuals that work continuously throughout their lives).11 The equilibrium implies a human capital stock that rises over the life-cycle at a diminishing rate. This yields the commonly observed concave earnings profile.

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8 Some have criticized the Mincer’s derivation of the earnings function because it does not explain why people choose a particular amount of education since equilibrium in his original model (1958) entails individuals who differ in schooling but have no difference in lifetime earnings. However, Johnson (1978) shows how schooling levels differ across individuals based on the ability parameters of the human capital production function, which we estimate individual-by-individual in this paper.
9 Incorporating labor supply enables one to maximize utility potentially enabling one to identify specific taste parameters, but doing so requires a number of additional assumptions to identify key earnings function parameters.
10 In more complicated models this cost also includes expenses for goods such as tuition, books, computers, and other material inputs to create human capital. as does Mincer we assume the goods components are offset by earnings during the investment process.
11 See Polachek (1975) for the case of discontinuous labor force participation.
The closed-form solution to Ben-Porath’s earnings function is highly nonlinear. At the time of its discovery in 1967 few computers were fast enough to easily estimate the parameters. However, shortly thereafter, Haley (1976) was able to estimate a version, but he simplified the estimation because not all parameters were readily identifiable. Given these computational difficulties, most scholars relied on a linearization. This linear-in-the-parameters specification has become known as the Mincer log-linear earnings function, or simply the Mincer earnings function. One problem is that Mincer’s simplification does not allow one to identify ability.

Given the advent of faster computers and longer panels of individual data, we feel now is a good time to reexamine Haley’s approach. Further, as mentioned above, given sufficiently long panels for particular individuals, the approach enables one to compute ability parameters person-by-person. Obtaining person-specific ability measures addresses one aspect of unobserved heterogeneity, a relatively important issue in micro-based econometric research.

2.2 The Haley Model

The human capital model assumes an individual’s potential earnings $Y_t^*$ (what a person could earn) in time period $t$ are directly related to human capital stock $E_t$. As such,

$$Y_t^* = RE_t$$

(1)

where for simplicity $R$ is assumed to be the constant rental rate per unit of human capital.12 Human capital stock is accumulated over one’s lifetime by prudent investments in oneself via schooling and on-the-job training (as well as health, job search and other

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12 Polachek (1981) assumes the rental rate can vary by type of human capital. Polachek and Horvath (1977) assume the rental rate can vary by geographic location. However, relaxing the assumption about a constant rental rate in these two ways is unnecessary for this application, nor is it a common practice in the human capital literature. As such, we assume the same rental rate for the entire population and that this rental rate is determined by supply and demand in the market.
earnings augmenting types of human capital). The rate of change in human capital stock, $E_t$, is expressed as the amount of human capital produced ($\dot{K}_t$) minus depreciation so that
$$E_t = \dot{K}_t - \delta E_t$$
where $\delta$ is the constant rate of stock depreciation. For simplicity, we assume individuals create human capital using a Cobb-Douglas production function such that
$$\dot{K}_t = \beta K^b$$
where $K_t$ is the fraction of human capital stock reinvested in time period $t$ and parameters $b \in [0,1]$ and $\beta$ are production function parameters. The parameter $b$ reflects the rate at which current human capital stock is transformed to new human capital. It reflects how one acquires new knowledge from old, and as such reflects how quickly one learns. We denote $b$ to depict the “scale” at which one learns. As such, because it measures how well one transforms past knowledge into new knowledge, it can be construed as related to the intellectual ability, perhaps what should be measured by psychological IQ type tests since it represents how well one transforms past knowledge into new knowledge. The parameter $\beta$ is the “technology” parameter. It represents “total factor productivity.” In reality IQ and aptitude tests measure a combination of $b$ and $\beta$.

The individual’s objective is to maximize discounted disposable earnings, $Y_t$, over the working life-cycle. This goal is achieved by choosing the amount of human capital $K_t$ to reinvest each year $(t)$ in order to maximize the present value of lifetime earnings.

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13 Specific training is also included because according to Kuratani (1973) in equilibrium workers receive remuneration for the exact same portion of specific training they pay for, which they finance by taking lower wages during the training period.
14 As already mentioned, we assume no other inputs other than one’s own human capital. Less simplified production functions could entail individuals employing “goods” inputs such as teachers, books, and study time. For example, Ben-Porath (1967) assumes $q_t = \beta K^b D^{bc}$ where $D_t$ equals other inputs. Later empirical analysis precludes taking account of these other factors of production because no data are available for these other inputs. Thus we adopt the above more simplified human capital production function used by Haley (1976).
15 As already mentioned, we abstract from labor supply.
\[
Max_{k_t} J = \int_0^N e^{-r} Y_t dt
\]  \hspace{1cm} (4)

where \( J \) is the total discounted disposable earnings over the working life-cycle, \( r \) is the personal time discount rate and \( N \) is the number of years after which one retires (assumed known with certainty).\(^{16}\) Disposable earnings are

\[ Y_t = R[E_t - K_t] \]  \hspace{1cm} (5)

Maximization of (4) subject to equations (2) and (3) can be done by maximizing the following Hamiltonian.

\[ H(K, E, \lambda, t) = e^{-rt} R[E_t - K_t] + \lambda_t [\beta K_t^b - \delta E_t] \]  \hspace{1cm} (6)

with constraints \( E_t - K_t \geq 0 \), which means one cannot invest using more human capital than one currently has (i.e., no borrowing); and the transversality condition \( \lambda_N = 0 \), which indicates a zero (labor market) gain from human capital investing in one’s final year at work. The solution involves three phases: (1) Specialization in human capital investment when \( K_t = E_t \) which can be defined as being in school since one is spending full-time investing; (2) “Working” which defines the time period when one both works and invests; and (3) Retirement when one ceases investing completely. We are concerned with Phase 2 since this is the only time period one can observe earnings. In school one plows back all one’s earnings potential into more human capital investment and hence has no net earnings. Likewise during retirement one does not work so there are no earnings then either.

This maximization yields a nonlinear (in the parameters) earnings function\(^{17}\)

\[ Y_t = A_0 e^{\delta(t^*-t)} + A_1 [1 - e^{\delta(t^*-t)}] - A_2 [1 - e^{(r+\delta)(t-N)}]^{\gamma/(1-b)} \]  \hspace{1cm} (7)

where

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\(^{16}\) For simplification because we have no data on individual investments prior to school we define \( t=0 \) to be the time when one begins full-time schooling.\(^{17}\) Appendix A contains the derivation. Note this differs a bit from the Haley specification because in our derivation we assume a one-term Taylor expansion whereas Haley uses a two-term Taylor expansion. Our approach yields a slightly simpler earnings function.
\[ A_0 = R\beta^{b(1-b)} \left[ \frac{1}{\delta} + \left( \frac{E_0^{1-b}}{\beta} - \frac{1}{\delta} \right) e^{\delta(b-1)t^*} \right]^{1/(1-b)} \]

\[ A_1 = R\beta^{b(1-b)} \frac{b}{r+\delta} \left[ \frac{1}{\delta} \right]^{b/(1-b)} \]

\[ A_2 = R\beta^{b(1-b)} \frac{b}{r+\delta} \]

and where \( t^* \) is the age at which one graduates from school (i.e., the age when Phase 1 ends), \( N \) is the anticipated retirement age which we take as 65, a reasonable assumption for this cohort, and \( E_0 \) is one’s human capital when one begins training. In reality, parents begin training their child at (or prior to) birth, but for our purposes we consider this time to be when one starts formal schooling because this is the point we know children spend full-time learning. Finally, given measurement error and other unobservable factors, one need add a time varying error term \( \epsilon_t \) for each individual.

One point about \( E_0 \) is noteworthy before we describe how we estimate (7). In the formal model (see Appendix A), \( E_0 \) corresponds to human capital stock when one begins specialization, that is when one begins school. This \( E_0 \) parameter is unidentified in the Mincer earnings function, which instead estimates potential earnings at the time formal school ceases and one begins work. However, with our model, one can also derive estimates for potential earnings when one just begins work. We do so by defining \( E_S \) as the amount of human capital upon completing school. \( E_S \) is computed by augmenting \( E_0 \) by the amount of human capital produced in each year of school. Multiplying \( E_S \) by the rental rate per unit of human capital yields potential earnings. Of course, at this stage of the life-cycle potential earnings exceed actual earnings because individuals are still heavily investing in human capital, though not full-time. Later in life, the gap between potential earnings and actual earnings should diminish as the proportion of available time spent investing declines. Later in the paper, when presenting our empirical estimates, we verify the validity of these predictions.

Haley estimates a variant of (7) using income by age data aggregated from the 1956, 1958, 1961, 1964, and 1966 CPS surveys. His estimates can be construed as
population averages. However, by employing sufficiently long panel data, equation (7) can be estimated person-by-person. To do so one can utilize nonlinear estimation techniques along with data on experience and earnings for each individual.

2.3 Identification Strategy

Equation (7) contains six parameters: $R$, $\beta$, $b$, $r$, $\delta$, and $E_0$. The parameters $r$, $\delta$, and $b$ all have no dimension. The parameters $r$ and $\delta$ are percents. The parameter $b$ is the output elasticity in the human capital production function (3). It reflects returns to scale of human capital. It also can be construed as an ability parameter since it measures the productivity of old human capital in creating new human capital. These parameters are technically observable. The parameters $\beta$, and $E_0$ are nominated in terms of units of human capital stock whereas $R$ is dimensioned as dollars per unit of human capital. Combining the two yields $E_0^{1-b} / \beta$, which is dimensionless. Thus we also treat $E_0^{1-b} / \beta$ as a single parameter. We write $R\beta^{1-b}$ as

$$\hat{w}_i = \left[ R^{(1-b)} \beta_i \right]^{1-b} = w_i^{*1-b}$$

(8)

where $w_i^{*} = R^{(1-b)} \beta_i$, which is the parameter we estimate. Finally, to conserve degrees of freedom and quicken convergence we assume a uniform human capital depreciation rate. Based on Haley (1976), we assume this to be 0.02.18 As a result of these identification restrictions we end up estimating four parameters: \( \hat{b}_i = b_i \), \( \hat{E}_i = \frac{E_0^{1-b}_i}{\beta_i} \), \( \hat{w}_i^* = R^{(1-b)} \beta_i \), and \( \hat{r}_i = r_i \) for each individual using nonlinear least squares for those individuals with at least twelve years of data.

For the estimation we employ a recently available parallel processing algorithm used by biologists in genetic research (Czarnitzki and Doherr, 2009). This algorithm reaches an optimum more efficiently than traditional Newton-Raphson hill-climbing.

18 Experimentation with other depreciation rates did not qualitatively alter our results.
techniques. We easily identify the individual-specific $b_i$ and $r_i$ parameters. To identify $\beta_i$ and $E_0$, we adopt the following approach: First, we specify $\beta_i$ to equal $\beta e_i$, where $\beta$ is the population average and $e_i$ is the individual deviation. Second, we rewrite (8) as

$$w_i^* = R^{(1-b_i)} \beta e_i.$$  \hspace{1cm} (9)

Taking the logarithm, yields

$$\ln \hat{w}_i^* = (1-b_i) \ln R + \ln \beta + \ln(e_i).$$ \hspace{1cm} (10)

Estimating (10) using each individual’s values obtained from the parameterization we employ to estimate (7) gives a population value of $R$ (the coefficient of $1-b_i$), the average $\beta$ (the constant term), and individual-specific values of $\beta_i$ obtained by taking the anti-log of the sum of the latter two terms in (10). Utilizing $b_i$ and $\beta_i$ values along with the coefficient $\hat{E}_i = \frac{E_0^{1-b_i}}{\beta_i}$ obtained from estimating (7) yields individual-specific $E_0$.

3. The Data

Nowadays there are a number of panel micro-data sets containing information on schooling, work experience, and earnings over the life-cycle. However, as far as we know, only the National Longitudinal Survey of Youth 1979 also contains extensive independent psychology-based information on ability. These include AFQT scores for much of the sample as well as various other ability tests administered to smaller samples of the NLS youth. For this reason we utilize the NLSY79 data in order to compare our own individual-specific ability parameters to the independent ability measures based on psychological tests.

As is well known, the NLSY79 is a nationally representative sample of young men and women aged 14 to 22 years old when first surveyed in 1979. The surveys have been conducted annually until 1994, and then performed every other year. We utilize the 2006 NLSY79, which contains up to 22 years of data for each respondent. The NLSY79
represents various groups such as men, women, Hispanics, blacks, non-Hispanics and non-blacks, as well as the economically disadvantaged. There are three subgroups comprising the NLSY79. The first is a cross-sectional sample representing non-institutionalized civilian youths living in the United States aged 14-22 in 1979. The second sample is a cross-sectional supplemental designed to oversample civilian Hispanic, black, and economically disadvantaged nonblack/non-Hispanics between 14 and 22 in 1979. The third is a cross-sectional military sample of youths that represent the population, aged 17-22 in 1979.19 We do not apply sampling weights since we are examining each individual separately rather than trying to use each individual’s data to build a nationwide mean.20 To estimate (7) we use data on weekly earnings (i.e., annual earnings divided by number of weeks worked) deflated by the 1982-84 urban CPI index,21 age, and years of schooling. From these we compute the experience level (t) from the time when schooling stopped (t*). Because our earnings function specification is designed for those who work continuously, we concentrate only on the males because females are more likely to have discontinuous labor force participation, making the measurement of experience (t) more difficult and resulting in a highly more nonlinear earnings equation (Polachek, 1975). In addition, current human capital acquisition is affected by future intermittent participation. Not being able to predict when and how long a woman will drop out precludes estimating female earnings functions, at least for the purposes of this paper. Further, we use data only on individuals that have completed school because those working while in school (or those working with the intention of going back to school) earn less than commensurately schooled individuals who completed their education (Lazear, 1977).

As was already mentioned, for the purposes of this study, the main virtue of the NLSY79 data is the information on ability which was obtained independent of economic and demographic variables. For most respondents this consists of at least one of 28 possible intelligence/aptitude tests. Of these we concentrate on the 1980 AFQT because it

19 The data and further explanations can be explored from the website http://www.bls.gov/nls/
20 We use the sample weights when we aggregate the results to get inferences about particular segments of the population.
21 We also estimated (7) using annual earnings data for full-time workers and found very little difference in the results.
is the most widely used test in the NLS-Y and is available for nearly all respondents. Nonetheless, to check robustness of our findings, we also present results using the scores on 15 additional tests reported in the NLS-Y. Detailed descriptions of each ability test are given in Appendix B. As already indicated, we compare these psychology-based cognitive ability scores respondent-by-respondent to the individual-specific ability parameters we estimated using (7) and (10).

4. Estimation Results

As discussed above, we use non-linear least-squares to evaluate (7) for each person with 12 or more years of data. We employ an algorithm (denoted as GA) used in genetic research (Czarnitzki and Doherr, 2007) which is less susceptible to getting stuck at local optima than traditional gradient optimization techniques. We estimate four crucial parameters. They are the ability parameter \( \hat{b}_i \), the discount rate \( \hat{r}_i \), and the composite parameters \( \hat{E}_i = \frac{E_0^{1-b}}{\beta_i} \) and \( \hat{w}_i^* = (R^{1-b}) \beta_i \). Table 1 contains estimates for the entire sample as well as for blacks and whites separately. White b, W, and E values exceed those of blacks, whereas, as is found in other studies, the black discount rate exceeds the

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22 The 1980 AFQT score differs slightly from the 1989 and 1993 scores because of the way each component is weighted.
23 We are not able to use all 25 tests because we drop individuals with less than twelve years of earnings data which we require to estimate the nonlinear earnings functions discussed above. The sixteen tests are: the Armed Forces Qualification Test (AFQT), the American College Test (Math), the American College Test (Verbal), the California Test of Mental Maturity, the Cooperative School and College Ability Test, the Differential Aptitude Test, the Henmon-Nelson Test of Mental Maturity, the Kuhlman-Anderson Intelligence Test, the Lorge-Thorndike Intelligence Test, the Stanford-Binet, the Otis-Lennon Mental Ability Test, the Preliminary Scholastic Aptitude Test (Math), the Preliminary Scholastic Aptitude Test (Verbal), the Scholastic Aptitude Test (Math), the Scholastic Aptitude Test (Verbal), and the Wechsler Intelligence Test for Children.
24 For most individuals we have between 17 and 22 years of data.
26 We eliminate Hispanics because there were far fewer observations for those with 12 or more years of earnings data which would make it difficult to form inferences for this group. Further, Hispanics tend to be more heterogeneous than whites or blacks given that many Hispanics immigrate from several Spanish speaking countries (e.g., Mexico, Puerto Rico, Cuba) where levels of development varies (Borjas and Tienda, 1985).
white discount rate. We shall discuss the implications of these parameter values shortly, but first we address statistical significance which we compute via bootstrap techniques. For this, we run 200 nonlinear regression replications utilizing up to 21 randomly drawn observations (with replacement) from the available 12-22 observations per person.\footnote{These computations took 275 hours using two i7-vpro chip parallel processor computers running ten STATA programs in tandem each utilizing the GA algorithm.} Mean values of the coefficient standard errors averaged across all individuals are given in row (2) of each panel and median values which de-emphasize outliers are given in row (3). On average, most observations contain coefficients that are statistically significant with the exception of r, for which a number outliers produce an abnormally high average standard error. These outliers are apparent when noting how the median standard error of these r coefficients decreases dramatically to .03 for the population.

Based on the identification strategy we described earlier, we get values for $b_i$, $\beta_i$, $E_0$, $E_s$, and $r_i$, and a population-wide value of $R$. Mean values across all individuals are given in Table 2 along with mean values for each of the 16 test scores contained in the NLS-Y that were mentioned above. Obviously, sample sizes vary among the non-AFQT scores because not all individuals took each test.

### 4.1 Consistency with Prior Population-Wide Estimates

Interestingly, mean values of our parameters compare favorably to past studies that estimate aggregate earnings functions. For example, we obtain an $r$ of 0.08 compared to Haley’s 0.06. We obtain a mean $b$ of 0.35 compared to Haley’s 0.57, Heckman’s (1975) 0.67, Heckman et al.’s (1998) 0.80, Song and Jones’s (2006) 0.5, and Liu’s (2009) 0.52. Similarly, we obtain a weekly rental rate per unit of human capital (R) of $24.7 compared to Liu’s $4.7. Of course, our results are based on weighted averages of individual values whereas the other studies examine one function for the population as a whole. Further, each uses slightly different human capital production functions.
Similarly our results are consistent with computations of Mincer’s “time-equivalent” post-school investment as well as with the prediction that time-equivalent investment decreases with age. Figure 1 plots potential and actual earnings for individuals who began work immediately following school.\textsuperscript{28} Actual earnings come from the data and as such are observed for each person. Potential earnings are computed by multiplying predicted human capital stock \( E_s \) by the population-wide market rental rate per unit of human capital stock \( R \), both of which are parameter estimates. Theory predicts potential earnings exceed actual earnings; and one can see this to be the case from the two distributions. The mode for actual weekly earnings is $100 per week (in 1992-3 dollars) and the modal value for potential earnings is about $250. This implies a “time-equivalent” investment for new entrants to be about 0.60 which compares favorably to the 0.7 range based on Mincer’s original earnings function regressions.\textsuperscript{29} Recomputing these two distributions for older workers (Figure 2) shows a definite narrowing of the distance between potential and actual earnings, as predicted by theory. In short, older workers reinvest less of their existing human capital as they age.

### 4.2 Consistency with Standardized Tests

Next we determine whether the economics-based individual ability parameters \( (b_i, \beta_i, E_0, \text{and } E_s) \) are correlated with standardized ability test scores.

One way to see how our economics-based ability measures compare to the psychologically-based test measures is to plot out kernel density functions of our measures and the psychologically based test scores. To do so, we must scale each test score because each has a different measurement range. For example, our ability measure \( b \) is an exponent in a production function. It ranges from almost zero to 0.50 with a mean of 0.36 and a standard deviation of 0.10. SAT scores are coded between 200 and 800, but for our sample they vary between 200 and 750 for math and between 200 and 770 for verbal. Each of the other ability tests also has unique scores. To compare the overall

\textsuperscript{28} These exclude those with very low schooling levels and those who took a year or more to find their first job.

\textsuperscript{29} One obtains 0.56 and 0.81 respectively when one solves for \( k_0 \) (the equivalent of our \( E_0 \)) using Mincer’s (1974) Gompertz specification \( G(2a) \) and \( G(2b) \), p. 92.
distributions each must be scaled to have the same range of values. To do so, we scale each measure \( x_i \) of test \( i \) by \( \frac{x_i - L_i}{H_i - L_i} \) where \( L_i \) is the lowest test score value and \( H_i \) is the highest test score value. This yields a scaling between zero and one, where \( i \) indexes each particular test. Figures 3-7 plot the kernel density functions for AFQT, \( b_i \), \( \beta_i \), \( E_{0_i} \), \( E_{0} \), and \( E_{0} \). The AFQT and \( b \) kernel densities are relatively similar. Both are left-skewed with modal values in the 70-85 range. The kernel density for \( \beta \) is bell-shaped with a slight right-skew, and \( E_{0} \) is bunched at the lower levels which makes sense since this is the amount of initial human capital before investing over the life-cycle.

Another way to see how our economics-based ability measures compare to the psychologically-based measures is to examine scatter plots of our ability estimates against AFQT. We do so in Figures 8-11 by plotting mean \( b \), \( \beta \), \( E_{0} \), and \( E_{S_i} \) levels associated with each AFQT score. For each of our ability measures, these yield strong positive correlations.

### 4.3 The Distribution of Ability by Race

Next, in Figure 12, we plot black and white differences in these kernel densities for \( b, \beta, E_{0} \), and AFQT using these same individuals. Generally blacks (solid line) fare worse than whites (dashed line) since for each test score the black distribution is further to the left than the white distribution. However, noteworthy is the large difference for AFQT compared to our estimated ability based on the life-cycle model. Thus we find smaller ability differences by race in our economics-based measure than is observed in the psychology-based measures. Kolmogorov-Smirnov tests for the difference in these distributions are given in Table 3. The race differences for each distribution are significantly different statistically, but the distance measure is largest for the AFQT.

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30 The AFQT test scores are given in percentiles ranging from 1 to 99. Because of this we compute raw scores based on summing the scores for each component part. We then scale these as indicated above. These rescaled scores are what are contained in Figure 1. To conserve space we do not present kernel density plots for the other psychology-based tests, but we later in the paper we present evidence how the other tests correlate with our human-capital based measures.
4.4 Patterns in the Ability Coefficients

Clearly, given the large number of individuals, we cannot present coefficients for each person. Instead we present aggregate estimates for various groups. These are given in Tables 4 for each of the sixteen achievement tests. Column 1 gives the mean score for each test; column 2 the number of respondents taking particular tests; columns 3-6 our estimates of the economics-based ability measures for these same individuals; column 7 our estimates of the time-discount rates; and finally column 8 these respondents’ AFQT scores. A number of patterns can be noted in the data. First AFQT scores are higher for whites than blacks. The same is true for the economics-based ability \((b, \beta, E_o, E_s)\) measures, but slightly less so. Second, \(b, E_o, E_s\) and AFQT appear to be positively correlated. AFQT scores as well as \(b, E_o, E_S\) values are larger for respondents with higher psychology-based test scores. Third, \(b, \beta, E_o, E_S\) and the achievement test scores are positively correlated. Respondents with higher psychology-based test scores tend to have higher values of \(b, E_o, E_S\) and AFQT. Fourth, AFQT and the psychology-based test scores are also positively correlated.

4.5 Consistency with Human Capital Theory

Our ability measures as well as AFQT are related to schooling in a predictable way. Table 5 contains coefficient averages by years of school. Again the table is divided between blacks and whites. Column 1 contains the number of observations, columns 2-5 contains the estimated human capital based ability coefficients, column 6 gives the estimated time discount rate, and column 6 presents the AFQT score of these individuals. Here, too, a few patterns are noteworthy. First, even within schooling groups, \(b, \beta, E_o, E_S\) values are higher for whites than blacks, but only marginally so, but there remain large differences in AFQT. Second, the \(b\) and \(\beta\) values rise significantly with years of school. AFQT scores also rise with years of school. Of course, a positive correlation between ability and schooling level is predicted by human capital theory because higher ability raises the amount of human capital one can produce per unit of time. Holding
rental rates per unit of human capital constant, this lowers the opportunity costs of going to school, thereby increasing the amount of school purchased. On the other hand, \(E_0\) does not rise with schooling level either for backs or whites. This is expected because an individual’s higher initial human capital substitutes for schooling, and as Ben-Porath (1967) predicts, leads one to stop schooling earlier. Finally, the estimated time discount rate (\(r\)) decreases with the level of school. This latter result is noteworthy because higher time discount rates should imply fewer years of schooling since individuals with high discount rates are more reluctant to put off the gratification of current market earnings given that they discount the future heavily.

These patterns are also presented in Table 6 which contains specific regressions. Row (1) indicates a higher ability score for whites for all ability measures. However, the race difference is about ten times larger for the AFQT than our human capital based measures. Row (2) indicates the positive relationship between \(b\) and schooling level, \(\beta\) and schooling level, \(E_0\) and schooling, and AFQT and schooling. However, as noted in Table 5, the same is not true for innate human capital \(E_0\). Also, as in Table 5, the estimated discount rate and schooling are negatively related. Thus, as predicted by human capital theory, individuals with high cognitive ability (\(b\) and \(\beta\)) positively sort, individuals with high innate human capital (\(E_0\)) negatively sort, and individuals with high discount rates negatively sort with schooling level.

4.6 Selectivity

Equation (7), from which we obtain ability parameters, are estimated only for those individuals with sufficiently long work histories. Clearly this sample is a select group because it does not include those respondents with shorter (or no) work histories. The “workers” we choose may be (and most likely are) different in ability (and probably other characteristics) than the “non-workers” we do not include. As such, making inferences about the whole population using our select sample may yield biased results because our ability estimates are not averaged over the entire population; but our purpose is not to

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31 Note that each ability measure is scaled in percentiles so comparisons can be easily made with AFQT scores.
obtain population-wide estimates of ability. Instead, our point is to use the difference in psychology-based and economics-based measures for the same workers to identify whether both methods of assessing ability differ from each other. In essence, we utilize the difference between the two measures holding observable and unobservable individual characteristics constant.

On the other hand, one might argue that black workers (working 12 or more years) are relatively more able than white workers (working 12 or more years) because only the relatively “better” blacks compared to whites are able to sustain such a long work history. One can assess this bias by utilizing the psychology-based test scores for non-workers of each race. If the “worker” compared to “non-worker” ability advantage is greater for blacks than whites, then our measures overstate black compared to white ability, and as a result understate the racial ability gap. In contrast, if the relative ability advantage is greater for whites, then the opposite is true, and as such, we then understate black-white ability differences. Table 7 presents differences in AFQT scores between those working 12 or more years (our sample) and the remainder of the population (those not meeting the work requirement) broken down by level of education. In seven educational categories the ability advantage for “workers” is higher for whites than blacks, while in five the ability advantage favors blacks. A t-test rejects the hypotheses that these differences are unequal. In short, those working 12 or more years tend to be more able than those working less than 12 years, or not at all; but the difference between black “workers” and “non-workers” is no different statistically than for white “workers” and “non-workers”. This result is consistent with small, if any, selectivity biases when considering racial differences in ability by concentrating on blacks and whites working at least 12 years of their lifetime.

4.7 Robustness Check
As described above, each NLS-Y respondent could have taken any of sixteen IQ-type tests. We divide the population into sixteen groups, each representing all respondents who took that particular test. For each respondent we have the psychology-based test scores as well as our own estimates of ability. A positive correlation between b and IQ is consistent with our interpretation that the parameters $b, \beta, E_0, E_s$ measure ability.

Table 8 contains three sets of four columns. The four columns in each set give correlation coefficients between each of our ability measures and the ability test of each row, the top row being the AFQT. The first set of columns depicts the correlation for the population of test takers independent of race. The second and third sets depict correlations separated by race. AFQT scores are strongly related to each of our ability measures. As a robustness check, in virtually all other cases our computed ability parameters and the psychologically-based ability test scores are also positively correlated. This means that despite being computed completely independently, the psychologically-based tests (founded on long history of measuring intelligence and achievement) and our ability parameters (based on a nonlinear estimation of individual-specific human capital production function parameters) correlate well. This positive association between our ability measure $s$ and all the test scores adds credence to our ability measures computed based on the life-cycle earnings model.

Several other patterns in Table 8 are noteworthy. Recall the interpretation of our estimated parameters $b, \beta, E_0, E_s$. The parameters $b$ and $\beta$ depict one’s ability to create new human capital from old. The $E_0$ parameter reflects innate human capital stock when one begins school. Finally $E_s$ is a combination of all three, reflecting human capital at the time one graduates school and enters the labor market. The psychology based ability tests vary in interpretation, but by and large they incorporate both intelligence and achievement, encompassing both knowledge and the ability to use this knowledge for logical reasoning. Test questions not only encompass vocabulary, simple reading passages, easy math concepts, but in addition include verbal relations tested via analogies as well as arithmetic and algebraic reasoning tested via math problems of various levels of sophistication. To the extent psychology-based tests measure both
achievement and intelligence, they should be most correlated with our \( E_s \) measure since that too estimates both achievement, based on what is learned in school, and intelligence, based on the ability to use this knowledge to create new knowledge. Clearly, the correlation between AFQT and \( E_s \) is higher than the correlation of AFQT and our other ability measures. This pattern is also generally true for all other psychology-based tests, though the magnitudes vary. Interestingly, \( E_0 \), which reflects innate knowledge at the time school begins, is least correlated with each psychology-based test, as might be expected if psychology-based test deemphasize this initial knowledge. The \( b \) and \( \beta \) correlations with AFQT are in between which also might be expected if \( b \) and \( \beta \) are purely intelligence based measures.

4.8 Explanatory Power

One final way to measure the importance of our estimated ability indicators is to examine how much additional variation in earnings is explained when incorporating these ability measures into a typical earnings function. Table 9 reports adjusted R\(^2\) measures for such earnings functions. AFQT increases the adjusted R\(^2\) by only .03 over the basic Mincer log\(_e\)-linear fit, whereas \( b \) and \( \beta \) increase adjusted R\(^2\) by .06 and .17 respectively, and by .27 together with \( E_0 \). Incorporating AFQT adds nothing to the explained portion of variance when including our three ability measures.32

5. Conclusion

Utilizing psychology-based test scores has the advantage of measuring ability early in one's lifetime. On the other hand, there is some controversy regarding how well such test scores really reflect cognitive ability. Further, there is a great deal of controversy regarding race differences in these measures (Fraser, 1995)

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32 We do not include \( E_s \) because that includes the effects of schooling.
Based on an entirely different paradigm, economists utilize concepts of ability in modeling earnings. In such models individuals create earnings power by producing human capital. Two of the parameters (b and $\beta$) reflect the intelligence necessary to create new human capital form old. A third parameter ($E_0$) depicts initial knowledge before formally entering school. A fourth parameter (r) depicts time-preference. We estimate these parameters for each individual in the National Longitudinal Survey of Youth who has sufficient earnings information. In addition, for each such respondent, we compute $E_S$ depicting the amount of human capital at the time this respondent finished school and just entered the labor force.

Aside from estimating these parameters, our aim in this paper is first to test whether these economics-based ability parameters are consistent with predictions from the human capital model, and second, to compare our economics-based ability measures with independently obtained psychology-based measures also contained in the NLS-Y data. Comparing both measures is important for a number of reasons. First, it serves as an independent check of the underlying paradigms upon which each is based. Second, it enables one to gain some insight into the question of possible racial biases inherent in psychologically-based tests, as is often alleged. Third, it serves as an example of how one can use newer and longer panels to measure aspects of the individual heterogeneity.

The results indicate an uncanny parallel between our economics-based ability measures and psychology-based measures from the data’s various achievement test scores. First, we find both measures to be positively correlated with each other. Second, we find both to indicate higher levels of measured ability for whites compared to blacks, though the correlation is weaker for our measure than for the standardized tests. From this we infer the possibility of greater racial biases in the psychologically-based measures than our economics-based measures. Third, as predicted by human capital theory, we find both intelligence measures (b and $\beta$) to be higher for those with greater levels of schooling. Obviously, $E_S$ is also positively correlated with school, but interestingly, as is also predicted, we find our measure of initial knowledge $E_0$ to be inversely correlated with levels of schooling completed. This means high intelligence individuals get more
education, but those with more initial raw knowledge do not. Fourth, we find an inverse correlation between our measure of time preference (r) and school, again as is predicted by human capital theory.

Of course, employing an economics-based model is not a panacea for measuring ability. Even if the economics-based approach provides a viable alternative to the psychologically based achievement tests, it is not informative early in one’s life because it requires earnings data for a period of time long after one terminates school. Further, both discrimination in the availability of high quality schooling, as well as discrimination in the labor market itself can cause racial biases in estimating ability parameters using earnings data. In this case racial discrimination in the labor market manifests itself in a similar way cultural biases might inculcate psychologically-based models.

Technical simplifications could also mar interpretation of our results. Underlying our approach are the typical assumptions incorporated in life-cycle models. Obviously, our results may be suspect if earnings are determined by other frameworks such as incentive contracts or deferred compensation schemes. In addition, for computational simplicity, we utilize a relatively simple human capital production function, which in our case only has two ability parameters. We envision more complicated versions incorporating non-cognitive skills in the human capital production function. These latter models would yield more complex earnings functions than the ones we already use. On the other hand, we feel strongly that our results are not simply verifying the well-known fact that high ability people simply earn more. Our ability measures are unrelated to earnings level. Instead, they arise from the curvature of the earnings profile.

Our results are promising enough to warrant pursuing the approach further. For example, identifying various types of ability might enable one to gain insights into occupational choice decisions including answering questions relating to gender differences in one's inclination to go into scientific professions.
Distribution of potential and actual earnings
With zero years of experience

Distribution of potential and actual earnings
Age group 40-45
Figure 3

**Distribution of raw AFQT**

Figure 4

**Distribution of b**
Figure 5

Distribution of beta

Figure 6

Distribution of Eo
Figure 11

Es and AFQT

Figure 12

Distribution of abilities (by race)
Solid: Blacks, Dash: Whites

AFQT

Es
Table 1
Earnings Function Parameter Estimates*

<table>
<thead>
<tr>
<th></th>
<th>b</th>
<th>W</th>
<th>E</th>
<th>r</th>
</tr>
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<tbody>
<tr>
<td><strong>All</strong></td>
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<tr>
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<td>4.262</td>
<td>2.613</td>
<td>0.080</td>
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<tr>
<td>SE(mean)</td>
<td>0.048</td>
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<td>1.133</td>
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<td>94.1</td>
<td>94.9</td>
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<tr>
<td><strong>Blacks</strong></td>
<td></td>
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<td>Average coefficient estimate</td>
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<td>SE(median)</td>
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<td>0.781</td>
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<td>95.0</td>
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* Row 1 of each panel gives average parameter estimates of (7) over the entire sample. Rows (2) and (3) give mean and median bootstrapped standard errors. Each observation is weighted by the NLS-Y weights when computing the averages and the medians.
Table 2

Descriptive Statistics (weighted):

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<th>Individual specific parameters and ability test scores</th>
<th>OBS</th>
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<th>SD</th>
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<tr>
<td>b</td>
<td>1992</td>
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<td>46.39</td>
<td>10.93</td>
</tr>
<tr>
<td>PSATVERBAL</td>
<td>239</td>
<td>40.48</td>
<td>9.86</td>
</tr>
<tr>
<td>SATMATH</td>
<td>160</td>
<td>484.26</td>
<td>116.07</td>
</tr>
<tr>
<td>SATVERBAL</td>
<td>159</td>
<td>423.22</td>
<td>107.45</td>
</tr>
<tr>
<td>STANFORD</td>
<td>15</td>
<td>86.18</td>
<td>40.53</td>
</tr>
<tr>
<td>WECHSLER</td>
<td>21</td>
<td>90.41</td>
<td>15.90</td>
</tr>
</tbody>
</table>

Source: Computed from NLS-Y. The values of b, beta, Eo, and r are averages parameter values obtained by estimating equation (7) separately for each individual. The remaining variables refer to specific achievement/ability test scores contained in the NLS-Y. The specific tests are described in Appendix B. Observations differ by type test because only the AFQT was administered to all respondents.

Table 3

Kolmogorov-Smirnov Test for the Difference in Distributions of Indicated Ability Variable for Blacks and Whites

<table>
<thead>
<tr>
<th>Distance</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0.116</td>
</tr>
<tr>
<td>beta1</td>
<td>0.212</td>
</tr>
<tr>
<td>Eo</td>
<td>0.113</td>
</tr>
<tr>
<td>AFQT</td>
<td>0.463</td>
</tr>
</tbody>
</table>

The p-value is P(D[m,n] > Do|Ho), for the hypothesis that black and white ability measures are from the same distribution, where Do is the observed value of the two sample K-S test statistic.
### Table 4: Descriptive Statistics by Race

<table>
<thead>
<tr>
<th>Test</th>
<th>Blacks</th>
<th></th>
<th>Whites</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IQ</td>
<td>b</td>
<td>beta</td>
<td>Eo</td>
</tr>
<tr>
<td>AFQT</td>
<td>23.45</td>
<td>0.34</td>
<td>0.65</td>
<td>2.16</td>
</tr>
<tr>
<td>AMERCOL</td>
<td>14.89</td>
<td>0.38</td>
<td>0.69</td>
<td>21.32</td>
</tr>
<tr>
<td>CALIF</td>
<td>82.41</td>
<td>0.35</td>
<td>0.66</td>
<td>2.41</td>
</tr>
<tr>
<td>COOP</td>
<td>52.75</td>
<td>0.32</td>
<td>0.65</td>
<td>1.83</td>
</tr>
<tr>
<td>DIFFEREN</td>
<td>30.75</td>
<td>0.34</td>
<td>0.67</td>
<td>2.39</td>
</tr>
<tr>
<td>HENMON</td>
<td>69.94</td>
<td>0.39</td>
<td>0.68</td>
<td>3.82</td>
</tr>
<tr>
<td>KUHLMAN</td>
<td>72.23</td>
<td>0.37</td>
<td>0.67</td>
<td>2.02</td>
</tr>
<tr>
<td>LORGE</td>
<td>81.22</td>
<td>0.35</td>
<td>0.65</td>
<td>2.59</td>
</tr>
<tr>
<td>OTIS</td>
<td>81.45</td>
<td>0.37</td>
<td>0.66</td>
<td>2.14</td>
</tr>
<tr>
<td>PSATMATH</td>
<td>39.35</td>
<td>0.37</td>
<td>0.79</td>
<td>2.61</td>
</tr>
<tr>
<td>PSATVERB</td>
<td>33.77</td>
<td>0.37</td>
<td>0.79</td>
<td>2.61</td>
</tr>
<tr>
<td>SATMATH</td>
<td>393.12</td>
<td>0.37</td>
<td>0.74</td>
<td>2.09</td>
</tr>
<tr>
<td>SATVERVAIL</td>
<td>324.46</td>
<td>0.37</td>
<td>0.74</td>
<td>2.09</td>
</tr>
<tr>
<td>STANFORD</td>
<td>65.24</td>
<td>0.20</td>
<td>0.57</td>
<td>1.49</td>
</tr>
<tr>
<td>WESCHLER</td>
<td>75.93</td>
<td>0.26</td>
<td>0.53</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Source: Computed from the NLS-Y. Average values across individuals taking indicated test based on estimation of equation (7) weighted by NLS-Y weights.

### Table 5: Ability Measures by Level of Schooling (using NLS weights)

<table>
<thead>
<tr>
<th>Schooling</th>
<th>b</th>
<th>beta</th>
<th>Eo</th>
<th>Es</th>
<th>r</th>
<th>AFQT</th>
<th>b</th>
<th>beta</th>
<th>Eo</th>
<th>Es</th>
<th>r</th>
<th>AFQT</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;12</td>
<td>0.32</td>
<td>0.59</td>
<td>2.02</td>
<td>10.48</td>
<td>0.10</td>
<td>12.12</td>
<td>0.32</td>
<td>0.66</td>
<td>3.18</td>
<td>14.28</td>
<td>0.10</td>
<td>26.77</td>
</tr>
<tr>
<td>12</td>
<td>0.33</td>
<td>0.63</td>
<td>2.26</td>
<td>14.78</td>
<td>0.09</td>
<td>17.83</td>
<td>0.35</td>
<td>0.73</td>
<td>3.02</td>
<td>19.13</td>
<td>0.09</td>
<td>47.06</td>
</tr>
<tr>
<td>13-15</td>
<td>0.35</td>
<td>0.65</td>
<td>1.84</td>
<td>17.84</td>
<td>0.06</td>
<td>30.10</td>
<td>0.37</td>
<td>0.75</td>
<td>2.82</td>
<td>23.77</td>
<td>0.07</td>
<td>61.37</td>
</tr>
<tr>
<td>&lt;=16</td>
<td>0.38</td>
<td>0.78</td>
<td>2.51</td>
<td>31.00</td>
<td>0.05</td>
<td>48.09</td>
<td>0.40</td>
<td>0.80</td>
<td>2.94</td>
<td>37.99</td>
<td>0.05</td>
<td>78.40</td>
</tr>
</tbody>
</table>

Source: Computed from the NLS-Y. Average (weighted by NLS weights) values across individuals based on estimation of equation (7).

### Table 6: Ability and Race: How Human Capital Production Function Measured Ability and AFQT Are Related to Each Other and Race for Un-scaled and Scaled Ability Measures

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>b</th>
<th>beta</th>
<th>Eo</th>
<th>Es</th>
<th>r</th>
<th>AFQT</th>
</tr>
</thead>
<tbody>
<tr>
<td>White dummy</td>
<td>2.606</td>
<td>2.905</td>
<td>2.754</td>
<td>3.77</td>
<td>-0.37</td>
<td>23.84</td>
</tr>
<tr>
<td>Schooling</td>
<td>1.840</td>
<td>0.753</td>
<td>-0.279</td>
<td>2.78</td>
<td>-1.56</td>
<td>6.077</td>
</tr>
<tr>
<td>Constant</td>
<td>39.402</td>
<td>11.244</td>
<td>10.788</td>
<td>-25.41</td>
<td>34.84</td>
<td>-72.61</td>
</tr>
</tbody>
</table>

OLS regressions of $b$, $\beta$, $E_0$, $E_s$, $r$, and AFQT on race and schooling level.

*** denotes significance at 0.01, ** denotes significance at 0.05, * denotes significance at 0.10.
## Table 7

### Average AFQT80 (weighted)

<table>
<thead>
<tr>
<th>Years of School</th>
<th>Difference in AFQT Black Workers and Black Non-Workers</th>
<th>Difference in AFQT White Workers and White Non-Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>-2.4</td>
<td>5.1</td>
</tr>
<tr>
<td>10</td>
<td>5.5</td>
<td>0.2</td>
</tr>
<tr>
<td>11</td>
<td>5.2</td>
<td>3.0</td>
</tr>
<tr>
<td>12</td>
<td>0.5</td>
<td>4.5</td>
</tr>
<tr>
<td>13</td>
<td>4.9</td>
<td>2.6</td>
</tr>
<tr>
<td>14</td>
<td>-1.5</td>
<td>4.1</td>
</tr>
<tr>
<td>15</td>
<td>-1.4</td>
<td>8.7</td>
</tr>
<tr>
<td>16</td>
<td>7.0</td>
<td>4.6</td>
</tr>
<tr>
<td>17</td>
<td>14.0</td>
<td>6.3</td>
</tr>
<tr>
<td>18</td>
<td>1.5</td>
<td>7.4</td>
</tr>
<tr>
<td>19</td>
<td>16.7</td>
<td>12.5</td>
</tr>
<tr>
<td>20</td>
<td>19.9</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Computed from NLS-Y
<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Blacks</th>
<th>Whites</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b</td>
<td>beta</td>
<td>Eo</td>
</tr>
<tr>
<td>AFQT80</td>
<td>0.2184</td>
<td>0.307</td>
<td>0.1068</td>
</tr>
<tr>
<td>AMERCOLLMA*</td>
<td>0.2509</td>
<td>0.102</td>
<td>0.1497</td>
</tr>
<tr>
<td>AMERCOLLVE*</td>
<td>0.1802</td>
<td>0.1249</td>
<td>0.0834</td>
</tr>
<tr>
<td>CALIF</td>
<td>0.1061</td>
<td>0.1066</td>
<td>0.0275</td>
</tr>
<tr>
<td>COOP</td>
<td>-0.0002</td>
<td>-0.1428</td>
<td>0.3564</td>
</tr>
<tr>
<td>DIFFEREN</td>
<td>0.0524</td>
<td>0.1334</td>
<td>0.2379</td>
</tr>
<tr>
<td>HENMON</td>
<td>0.1565</td>
<td>0.071</td>
<td>0.2989</td>
</tr>
<tr>
<td>KUHLMAN</td>
<td>-0.0732</td>
<td>0.3523</td>
<td>0.1336</td>
</tr>
<tr>
<td>LORGE</td>
<td>0.0462</td>
<td>0.0739</td>
<td>0.1544</td>
</tr>
<tr>
<td>OTIS</td>
<td>0.5672</td>
<td>0.3591</td>
<td>0.0543</td>
</tr>
<tr>
<td>PSATMATH</td>
<td>0.2242</td>
<td>0.1494</td>
<td>0.0983</td>
</tr>
<tr>
<td>PSATVERBAL</td>
<td>0.2524</td>
<td>0.0449</td>
<td>0.1463</td>
</tr>
<tr>
<td>SATMATH</td>
<td>0.1817</td>
<td>0.3092</td>
<td>0.0333</td>
</tr>
<tr>
<td>SATVERBAL</td>
<td>0.2623</td>
<td>0.2006</td>
<td>0.0889</td>
</tr>
<tr>
<td>STANFORD</td>
<td>0.0008</td>
<td>0.0112</td>
<td>0.2561</td>
</tr>
<tr>
<td>WECHSLER</td>
<td>0.0517</td>
<td>0.0556</td>
<td>0.0977</td>
</tr>
</tbody>
</table>

* For each test row (1) gives the correlation, row (2) the statistical significance, and row (3) the number of observations.
Table 9: Goodness of Fit Achieved by Incorporating Ability in an Earnings Function

<table>
<thead>
<tr>
<th>Variables in Earnings Function</th>
<th>Adjusted R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>s,exp,exp2</td>
<td>0.33</td>
</tr>
<tr>
<td>s,exp,exp2, AFQT</td>
<td>0.36</td>
</tr>
<tr>
<td>s,exp,exp2, b</td>
<td>0.39</td>
</tr>
<tr>
<td>s,exp,exp2,beta</td>
<td>0.50</td>
</tr>
<tr>
<td>s,exp,exp2,E0</td>
<td>0.33</td>
</tr>
<tr>
<td>s,exp,exp2,b,beta,E0</td>
<td>0.60</td>
</tr>
<tr>
<td>s,exp,exp2,b,beta,E0,AFQT</td>
<td>0.60</td>
</tr>
</tbody>
</table>

* Adjusted R² reported for Mincer-type loge-linear earnings functions of the form

\[ \ln Y_i = \alpha_0 + \alpha_1 S_i + \alpha_2 \exp_i + \alpha_3 \exp_i^2 + \alpha_4 Ability_i + \epsilon_{i} \]

where \( Y \) is weekly earnings, \( S \) is years of school, \( \exp \) is years of experience, and \( Ability \) constitutes each type ability measure described in the text.
Appendix A: Derivation of the Human Capital Earnings Function

Assume the individual’s objective is to maximize discounted disposable earnings, \( Y_t \), over the working life-cycle.\(^{33} \) This objective is achieved by choosing the amount of human capital \( K_t \) to reinvest each year in order to maximize the present value of lifetime earnings

\[
\begin{align*}
\max_{K_t} J &= \int_0^N e^{-r t} Y_t \, dt \\
\end{align*}
\]  

(A1)

where \( J \) is the total discounted disposable earnings over the working life-cycle, \( r \) is the personal discount rate and \( N \) is the number of years one works, assumed known with certainty. Disposable earnings are

\[
Y_t = R[E_t - K_t] 
\]  

(A2)

where \( E_t \) denotes human capital stock in time period \( t \) and \( K_t \) the amount of human capital stock reinvested in time period \( t \) to create new human capital. We assume the individual begins with an innate stock of human capital \( E_0 \) which can be augmented by investing all or part of this. The period-to-period change in human capital is denoted by

\[
\dot{E}_t = Q_t - \delta E_t - \beta K_t^b - \delta E_t 
\]  

(A3)

where we assume \( \delta \) is a constant rate of stock depreciation of existing human capital stock and where we assume individuals create human capital using a Cobb-Douglas production \( Q_t = \beta K_t^b \).

Maximization of (A1) subject to equations (A2), (A3) and (A4) entails maximizing the Hamiltonian

\[
H(K, E, \lambda, t) = e^{-r t} R[E_t - K_t] + \lambda_t [\beta K_t^b - \delta E_t] 
\]  

(A4)

with constraints \( E_t - K_t \geq 0 \), and making use of the transversality condition \( \lambda_N = 0 \).

The function \( \lambda_t \) is the marginal contribution to the total discounted disposable earnings if there is one more unit of human capital investment. Assuming that no corner solutions are binding, the necessary conditions are as follows.

\[
\begin{align*}
\frac{\partial H}{\partial K_t} &= 0 \\
\frac{\partial H}{\partial E_t} &= -\dot{\lambda}_t, \\
\frac{\partial H}{\partial \lambda_t} &= \dot{E}_t \\
\end{align*}
\]  

(A5.1)

(A5.2)

(A5.3)

\(^{33} \) As noted in the text, we abstract from labor supply considerations.
\[
\lambda_N = 0 \quad \text{(A5.4)}
\]

From equation (A5.2), we obtain \( \dot{\lambda} = -(\Re e^{-\nu} - \delta \lambda) \). Solving this differential equation and using the tranversality condition (A5.4) we obtain

\[
\dot{\lambda} = \frac{R}{r + \delta} e^{-\nu} \left[ 1 - e^{-(r + \delta)(N-t)} \right] \quad \text{(A6)}
\]

From (A6), \( \dot{\lambda} < 0 \), indicating a diminishing value of human capital over time.

From (A5.1)

\[
\frac{\delta H}{\delta K_t} = 0 = -\Re e^{-\nu} + \beta bK_t^{b-1}
\]

implying that

\[
K_t = \left[ \frac{\Re e^{-\nu}}{\lambda, \beta} \right]_{0}^{b-1} = \left[ \frac{b \beta e^{-\nu} \lambda}{R} \right]_{0}^{b-1} \quad \text{(A7)}
\]

Substituting (A6) into (A7) yields

\[
K_t = \left( \frac{b \beta}{r + \delta} \right)_{0}^{b-1} \left( 1 - e^{-(r + \delta)(N-t)} \right)_{0}^{b-1}, \text{ for } t \in [t^*, N]. \quad \text{(A8)}
\]

Of course, \( K_t = E_t \) during school since one devotes full-time to investing while in school.

To obtain human capital stock \( (E_t) \), we combine (A8) with (A3) and (A5.3) which yields a differential equation whose closed form solution entails an infinite hypergeometric series

\[
E_t = Be^{\delta (t^*-t)} + \left( \frac{\beta}{r + \delta} \right)_{0}^{b(1-b)} b^{b(1-b)} \sum_{j=0}^{\infty} \left( \frac{b / (1 - b)}{j} \right) \frac{e^{i(\delta + \delta)/(r + \delta)}}{j + (\delta / (r + \delta))} (-1)^j, \quad t \in [t^*, N]. \quad \text{(A9)}
\]

where

\[
B = \left[ \frac{\beta}{\delta} + \left( E_t^{1-b} - \frac{\beta}{\delta} \right) \right]_{0}^{b(1-b)} e^{\delta (b-1) t^*} - \frac{\beta^{b(1-b)}}{r + \delta} \left( \frac{b}{r + \delta} \right)_{0}^{b(1-b)} \sum_{j=0}^{\infty} \left( \frac{b / (1 - b)}{j} \right) \frac{e^{i(\delta + \delta)/(r + \delta)}}{j + (\delta / (r + \delta))} (-1)^j. \quad \text{(A10)}
\]
Haley shows that the infinite hypergeometric series converges to a particular value from the second term. In Haley’s derivation the convergence criterion is set for 6 decimal points. A simpler form can be obtained by setting the convergence at 4 decimal points level. Note that Haley’s convergence table shows that it converges from the first term (i.e. \( j=0 \)) at 4 decimal points. We use this slightly less stringent convergence criterion to construct the earnings function.

**VALUE OF THE INFINITE SUM FOR** \( j = 0, \ldots, 34 \) **AND** \( b = 1/8, \delta = 75, \) **EO= 50, R = .38, yj = .08, 5 = .03, N = 65, AND t = 1**

<table>
<thead>
<tr>
<th>J</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.537558</td>
</tr>
<tr>
<td>1</td>
<td>.537544</td>
</tr>
<tr>
<td>2</td>
<td>.537544</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>34</td>
<td>.537544</td>
</tr>
</tbody>
</table>

At \( j = 0 \), the infinite sum of the hyper-geometric series becomes

\[
\sum_{j=0}^{\infty} \left( \frac{b}{1-b} \right)^j \frac{e^{j(r+\delta)(t-N)}}{j+\delta (r+\delta)}(-1)^j = \frac{(r+\delta)}{\delta}
\]

Thus,

\[
B = \left[ \frac{\beta}{\delta} + (E_0^{(1-b)} - \frac{\beta}{\delta})e^{\delta(b-1)t^*} \right]^{1/(1-b)} \frac{\beta^{1/(1-b)}}{(r+\delta)} \left( \frac{b}{(r+\delta)} \right)^{b/(1-b)} \frac{(r+\delta)}{\delta}
\]

or,

\[
B = \left[ \frac{\beta}{\delta} + (E_0^{(1-b)} - \frac{\beta}{\delta})e^{\delta(b-1)t^*} \right]^{1/(1-b)} \frac{\beta^{1/(1-b)}}{\delta} \left( \frac{b}{(r+\delta)} \right)^{b/(1-b)} \frac{(r+\delta)}{\delta}
\]

A(11)

Thus the stock of human capital at time \( t \) can be expressed as

\[
E_t = Be^{\delta(t-t^*)} + b^{1/(1-b)} \left( \frac{\beta}{r+\delta} \right)^{1-b} \left( \frac{r+\delta}{\delta} \right)
\]

or

\[
E_t = Be^{\delta(t-t^*)} + \frac{b^{1/(1-b)}}{\delta} \left( \frac{b}{r+\delta} \right)^{b/(1-b)}
\]

or
\[ E_t = \left\{ \frac{\beta}{\delta} + (E_0^{1-b} - \frac{\beta}{\delta}) e^{\delta(b-1)t^*} \right\}^{1/(1-b)} - \frac{\beta^{1/(1-b)}}{\delta} \left( \frac{b}{r + \delta} \right)^{b/(1-b)} e^{\delta(t^*-t)} + \frac{\beta^{1/(1-b)}}{\delta} \left( \frac{b}{r + \delta} \right)^{b/(1-b)} \]

or

\[ E_t = \beta^{1/(1-b)} \left[ \frac{1}{\delta} + \left( \frac{E_0^{1-b}}{\beta} - \frac{1}{\delta} \right) e^{\delta(b-1)t^*} \right]^{1/(1-b)} e^{\delta(t^*-t)} + \frac{\beta^{1/(1-b)}}{\delta} \left( \frac{b}{r + \delta} \right)^{b/(1-b)} (1 - e^{\delta(t^*-t)}) \]

(A12)

Observed earnings can be expressed as following

\[ Y_t = R[E_t - K_t] \]

where, \( R \) is the rental rate of human capital. Thus,

\[ Y_t = A_0 e^{\delta(t^*-t)} + A_1 [1 - e^{\delta(t^*-t)}] - A_2 [1 - e^{(r+\delta)(t-N)}] \]

(A13)

where

\[ A_0 = R \beta^{b/(1-b)} \left[ \frac{1}{\delta} + \left( \frac{E_0^{1-b}}{\beta} - \frac{1}{\delta} \right) e^{\delta(b-1)t^*} \right]^{1/(1-b)} \]

\[ A_1 = R \beta^{b/(1-b)} \left[ \frac{b}{r + \delta} \right]^{b/(1-b)} \frac{1}{\delta} \]

\[ A_2 = R \beta^{b/(1-b)} \left[ \frac{b}{r + \delta} \right]^{1/(1-b)} \]
Appendix B: Description of Ability Measures Contained in the NLSY79

1. Armed Forces Qualification Test (AFQT)
   Armed Forces Qualification Test scores are calculated from some portions of Armed Services Vocational Aptitude Battery (ASVAB) which is administered by the Ministry of Defense. The main purpose of the AFQT is to determine the enlistment eligibility for branches of the Armed Services. The test itself comprises two chief parts which are the Math and the Verbal. The Verbal part contains Word Knowledge and Paragraph Comprehension and the Math contains Arithmetic Reasoning and Mathematics Knowledge. However, to calculate the score of AFQT, which is reported as a percentage, the Math sections are counted only once, whilst the Verbal parts are counted twice. In practice, for example, the percentiles below the 30th generally are not eligible for being part of any branches of the Armed Forces.1

2. American College Test (Math)
   The American College Test (ACT) is used to assess high school students’ performances in general education development and their abilities to complete a degree at the college level. This multiple-choice test consists of four parts: English, Mathematics, Reading, and Science. Also, there is an optional part of the test that is basically supposed to measure ability in planning and writing a short essay.
   As for the Mathematics part, the test contains Pre-Algebra, Elementary Algebra, Intermediate Algebra, Coordinate Geometry, Plane Geometry, and Trigonometry. The total number of questions for this part is 60, which is almost one-fourth of the 215 questions on the entire test.2

3. American College Test (Verbal)
   Another section of the American College Test is the Verbal. This part measures the English ability in the areas of Usage/Mechanics, Rhetorical Skills, etc. The total number of questions is 75. This section is weighted the most heavily. Additionally, the score of the whole test, every section combined, can range from 1 to 36, and the raw scores, the number of correct answers, would be converted to the scale scores before the final scores are reported.3

4. California Test of Mental Maturity
   The California Test of Mental Maturity (CTMM), administered by California Test Bureau, was primarily designed for students from Grades 7-14; its main objective is to gauge the mental abilities of students. This diagnostic evaluation is closely related to student success in a wide range of school activities, so that the teacher can be directly informed of who has learning difficulties (Carroll, 1982). Moreover, it provides comprehensive measurement of the functional capabilities essential to learning, problem-solving, and responding to new situations.4

5. Cooperative School and College Ability Test
   This ability test was designed to assess both verbal and mathematical abilities, primarily for students Grades 4-12. Rather than diagnosing individuals, its focus is on predicting student success in related areas of activity. There are two forms of the test, A
and B, which have been proven equivalent in terms of ability measurement and reliability. In terms of scores, percentiles and converted scores are reported for each grade level (Kaya, 1969).

6. Differential Aptitude Test
Differential Aptitude Test (DAT) was designed to measure an individual’s ability to learn or to succeed in various areas. This test consists of 8 areas: verbal reasoning, numerical ability, abstract reasoning, perceptual speed and accuracy, mechanical reasoning, space relations, spelling, and language usage. All of the DATs are multiple-choice, with time limits ranging from 12 to 25 minutes. In addition, one of the benefits of this test over others is that it provides a ranking for the student against national averages in the respective areas. The DAT results can be interpreted as an indicator of student progress with an identified future plan in pursuing a vocational program or college.

7. Henmon-Nelson Test of Mental Maturity
Henmon-Nelson Test of Mental Maturity was fundamentally designed to measure a variety of areas of mental abilities that are crucial for success both in academic work and outside the classroom. In detail, this test can be identified as four different levels: appropriate for Grades 3-6, Grades 6-9, Grades 9-12, and college level. It would be most accurate if the test taker’s age is between 12 and 18. The 90 multiple-choice questions are divided into three parts: word problems, number problems, and graphical representation. The overall score is believed to adequately represent the individual’s general cognitive abilities.

8. Kuhlman-Anderson Intelligence Test
Similar to other intelligence tests, Kuhlman-Anderson Intelligence Test was specifically designed to measure an individual’s academic potential by assessing general cognitive skills pertaining to the learning process. This test is a well-known standardized intelligence group test that can be given to Grades K-12. Originally developed in the 1920’s, it has been updated several times as the number of test-takers has increased. There are verbal and nonverbal items in this test whose scores can indicate performances among children by both chronological age and grade level.

9. Lorge-Thorndike Intelligence Test
The Lorge-Thorndike Intelligence test is another standardized, group-administered test suitable for Grades K-8 students. Its average score can be representative of the nationwide school population. According to the manual for this test, it was primarily intended to measure reasoning abilities, not the proficiency in particular skills taught in school. The test in general consists of two parts, which are verbal and nonverbal. Furthermore, it has been acknowledged that the Lorge-Thorndike Test is one of the best paper-and-pencil general intelligence tests (Jensen, 1973).

10. Otis-Lennon Mental Ability Test
The Otis-Lennon Mental Ability Test is the fourth generation of Otis series, which dates back to 1918. This revised edition is a substantial improvement on its
predecessors but still focuses on educational settings. Raw scores are easily converted to various types of normative scores, and normative data are reported both by age- and grade-based reference groups (Grotelueschen, 1969). There are three types of abilities that are meant to be measured by this test: comprehension of verbal concepts, quantitative reasoning and reasoning by analogy. Suitable for students in Grades 8-9, this test is also a group intelligence test whose norms can be updated annually.¹¹

11. Preliminary Scholastic Aptitude Test (Math)

Administered by the College Board and National Merit Scholarship Corporation, the Preliminary Scholastic Aptitude Test (PSAT) is a standardized test that is usually given to high school juniors. Not only does this test provide an opportunity to practice for the SAT, but it can also pinpoint the test-taker’s weaknesses. Furthermore, if the scores are high enough, they might qualify for a scholarship from the National Merit Scholarship competition. Like other standardized aptitude tests, PSAT is designed to measure a variety of skills. This multiple-choice test is comprised of three primary parts: Critical Reading, Mathematics, and Writing Skills.

Focusing on the Math part, it contains 28 multiple choice and 10 “grid-in” questions that aim at testing skills in basic math, algebra, geometry, measurement, data analysis, and statistics, as well as basic probability.¹² There are five possible answers provided for the multiple choice questions, while the grid-in questions require the test takers to determine their own answers. Generally, the strategies used for PSAT – Math are same as for SAT – Math, but the allotted time for PSAT is somewhat shorter than SAT for Math and the other sections.

12. Preliminary Scholastic Aptitude Test (Verbal)

As for the Verbal part, this test contains 48 multiple questions that focus on both sentence completions and critical reading skills (passage-based reading). The Verbal questions are arranged in random order; however, the test structure has 13 questions for sentence completions and 35 questions for the passage-based reading. The total amount of time allowed to complete this section is 50 minutes.¹³

13. Scholastic Aptitude Test (Math)

The Scholastic Aptitude Test (SAT) is perhaps the nation’s most widely accepted college-entrance exam, and is administered by the College Board. The SAT is typically taken by high-school juniors and seniors. It can reflect how well students are in terms of skills and knowledge they have acquired in and outside of the classroom, as well as how they think, communicate, and solve problems. This test is used by most schools as one of the best predictors of how successful the students are in college. Similar to the PSAT, the SAT comprises three parts: Critical Reading, Mathematics, and Writing. Each section of the SAT is scored on a scale of 200-800.¹⁴

The types of Math questions are five-choice multiple-choice and student-produced responses. In further detail, this section aims at testing skills of students in the following areas: exponential growth, absolute value, functional notation, linear functions, manipulations with exponents, properties of tangent lines, estimation, and number sense.¹⁵
14. Scholastic Aptitude Test (Verbal)

The SAT Verbal part, currently known as the critical reading section, is also similar to the PSAT Verbal, assessing critical and sentence-level reading. More specifically, it tests students reading comprehension, sentence completions, and paragraph-length critical reading. Questions may be based on one or two reading passages. Some questions, on the other hand, are not based on passages; instead, students need to complete sentences.16

15. Stanford Achievement Test

First published in 1926, the Stanford Achievement Test Series measures elementary and secondary school students' academic knowledge, and as such provides a measure of achievement. It was its content is designed to reflect school curricula and best instructional practices based on state and national standards. Each item is designed to measure up to four achievement parameters: content cluster, process cluster, cognitive level and instructional standard. The tests include three types of questions: multiple choice, short answer, and extended response. Besides requiring a written answer of five or six sentences, the extended response may also require the student to graph, illustrate or show work. Such answers are usually included within the areas of science or mathematics.

16. Wechsler Intelligence Test for Children

The Wechsler Intelligence Test for Children was originally developed by David Wechsler in 1949 to measure the individual’s intelligence, especially for children aged 6 years to 16 years and 11 months. Theoretically, it is believed that human intelligence is complex and multifaceted, so this test is designed to reflect this belief through testing both verbal and nonverbal (performance) abilities. The verbal IQ score is derived from scores on 6 subtests: information, digit span, vocabulary, arithmetic, comprehension, and similarities. The nonverbal score is from 6 subtests: picture arrangement, block design, object assembly, coding, mazes, and symbol search. In addition to its uses in intelligence assessment, this test is also used in neuropsychological evaluation, specifically with regard to brain dysfunction. Substantial differences in verbal and nonverbal scores may indicate some potential problems of brain damage.17

5 York University, http://www.yorku.ca/psycentr/tests/iq_test.html
7 Department of Psychology, The College of New Jersey, http://psychology.department.tcnj.edu/documents/Test_InventoryList.001.doc
References


