Changes in the Return to Skills
and the Variance of Unobserved Ability

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Abstract

Changes in the return to unobservable skills are often inferred from changes in the variance of wages within groups of workers with common observable characteristics. The crucial assumption for such an approach to be valid is that the variance of unobservable skills within these groups remains constant over time. Motivated by the fact that the expansion of higher education has likely led to a pool of university-educated workers which is more heterogeneous in terms of their unobservable skills, we propose a new identification strategy which relaxes the assumption of constant within-group skill variance and can be implemented using longitudinal data, requiring only two observations per individual. We estimate the changes in the return to skills and the changes in the variance of unobserved skills within education-age groups over the period 1982-2012 using data from the Current Population Survey’s Merged Outgoing Rotation Group sample. We find that relaxing the assumption of constant within-group skill variance is crucial. Contrary to the conclusion drawn when this assumption is imposed, we find that the return to skills has fallen over our sample period, and that increases in within-group wage inequality are driven exclusively by increases in the dispersion of unobserved skills within groups, particularly among college graduates.

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1 Introduction

Increases in wage inequality over recent decades in many developed countries have sparked an active research agenda analyzing its sources and its changes over time. An important finding in this literature is that a large proportion of the increase in wage inequality can be attributed to the dispersion of wages within groups of workers defined by their education and age, rather than across these groups (Katz and Murphy (1992), Juhn, Murphy, and Pierce (1993), Acemoglu (2002)). Within-group wage inequality is in turn attributed to three different components: (i) heterogeneities across otherwise-similar workers in terms of their unobservable skills (for example because of differences in schooling quality, intrinsic ability or effort), (ii) the market return to those skills, and (iii) unobservable idiosyncratic labor market shocks or measurement error.

In this paper we estimate the relative importance of these components in driving observed changes in residual wage inequality. This type of estimation is important for our understanding of many key issues in labor economics. For example, the influential theory of Skill-Biased Technical Change suggests that increases in inequality are driven by an increase in demand for skilled workers due to the development of new technologies since the 1980s (e.g. Autor, Katz, and Krueger (1998)). One of the key implications of this theory is that the return to skills should be increasing over time.

Using wage data to estimate the different components of residual wage inequality is challenging, as neither the quantity of skills nor their return can be directly measured. In order to solve this identification problem, a common assumption explicitly or implicitly made in the literature is that the variance of skills within groups of workers defined by their education and experience level remains constant over time (e.g. Chay and Lee (2000), Lemieux (2006), Lubotsky (2011)). Based on this key assumption, changes over time in within-group wage inequality are exclusively driven by changes in the return to skills, and thus the changes in these returns can be easily inferred from the within-group inequality changes.

In this paper we relax this traditional assumption and replace it with the more nuanced assumption that the variance of skills remains constant (or evolves in a particular way) only over short horizons and only when conditioning on people who remain employed across consecutive years. This allows the variance of unobserved skills to differ across cohorts. We show how changes in the return to skills can be identified under this milder assumption using longitudinal data. Our identification strategy only requires two observations per individual, and thus we are able to use data from the Merged Outgoing Rotation Group (MORG) sample from the Current Population Survey (CPS) over the period 1982-2012 in order to estimate the relative role of price changes and changes in the variance of skills in accounting for changes in within-group residual wage variances.

Previous literature has recognized the restrictiveness of assuming that the variance of un-
observed skills is constant within groups (see Chay and Lee (2000) p.16, or Lemieux (2006) footnote 11). Essentially, by comparing groups of workers defined by their education and experience levels at two different points in time, the assumption that the variance of unobserved skills is constant rules out the possibility of any cohort effects. This is a very strong assumption which we challenge in this paper.

One reason why this assumption is problematic is the expansion of tertiary education which has occurred over recent decades. Consider the group of young university-educated workers. The traditional assumption implies that the dispersion of unobservable skills in this group in 2010 would be the same as in 1980. As the set of young college educated workers has grown, this group has become more diverse along many observable dimensions, which likely implies that they are drawn from a wider distribution of unobserved abilities and therefore the variance of unobserved skills for this group would be increasing over time. This may be due to an increase in the dispersion of intrinsic ability among college-goers or because of a wider dispersion in the quality of education (Hendricks and Schoellman (2014), Guvenen and Kuruscu (2010)). Under the traditional assumption, any change in the variance of wages for this group would be attributed exclusively to changes in the return to skills, rather than to changes in the variance of skills.

A second reason why the assumption of constant within-group variance of unobserved skills may be problematic is related to the business cycle. If individuals who lose their job during a recession are not randomly selected among those in a given demographic group, then the variance of ability among workers from that group in a recessionary year may be quite different from the variance during an expansionary year (see for example Blundell, Reed, and Stoker (2003)).

In this paper, we propose a novel identification strategy which takes advantage of the limited longitudinal dimension of the CPS. In a first specification, we assume that the variance of unobserved skills remains constant only for the subset of workers that is observed over two consecutive years and who remain employed in both years. This assumption allows the variance of unobserved ability to vary freely across cohorts. We then extend our model to also allow for changes in the variance of unobserved skills within cohorts over time, by introducing permanent shocks to ability.

As in Chay and Lee (2000), our identification strategy relies on changes in residual wage variances. Other contributions to the literature have taken different approaches to address the identification challenge. For example, Heckman et al. (1998) and Bowlus and Robinson (2012)

1Technological change may potentially be a driver of the changes in the selection patterns into different educational groups. See Hidalgo-Pérez and Molinari (2014) and Cortes (2014) for models in which technological change affects the composition of skills within different occupations.

2Note that the conditional mean of unobserved skills may also be changing over time due to these changes in the composition of university graduates, or due to newer “vintages” of workers receiving different amounts of value added through the education process (Bowlus and Robinson (2012), Carneiro and Lee (2011)). This does not affect the identification strategy described below.
identify a “flat spot” in workers’ skill profiles: a period over the life-cycle during which workers’ skill levels remain flat. Changes in the return to skills are identified by analyzing changes in the wage levels of workers over their flat spots, rather than wage variances. Another strand of the literature specifies an error components structure for residual wages and is primarily interested in identifying the variance of permanent and transitory shocks to earnings (e.g. Meghir and Pistaferri (2011), Moffitt and Gottschalk (2012), Blundell et al. (2014)), without particularly focusing on estimating the return to skills. An exception is Lochner and Shin (2014), whose paper is closely related to ours. The main advantage of our identification strategy over the one proposed by Lochner and Shin (2014) is that we only require two observations per individual, and therefore can take advantage of the large sample size in the CPS. The identification strategy in Lochner and Shin (2014) requires more observations per individual, and therefore can only be implemented with panel datasets such as the Panel Study of Income Dynamics, which tend to have much smaller sample sizes.

Our results suggest that relaxing the assumption of constant within-group variance is crucial. There is substantial variation over time in the variance of unobserved skills within education-age groups which, when ignored, generates misleading results about the changes in the return to skills. Based on our identification strategies, we find that the return to skills falls during the 1980s, then recovers somewhat throughout the mid-1990s, and remains fairly stable during the most recent decade. This implies that the main driver of the observed increases in within-group inequality among college graduates in particular, is an increase in the dispersion of skills within this group, rather than an increase in the return to skills. We show that this key result is in no way driven by the fact that we use a selected sample (workers who remain employed across two consecutive years), but rather is due to the change in the identification strategy.

When allowing for heterogeneous returns to skills across demographic groups, we only find evidence of increases in the return to skills over the 1982-2012 period among young workers (aged 25-34). Meanwhile, if we allow the variance of skills to change over the life-cycle, we find that the results are sensitive to the assumption made about the nature of the permanent shocks to skills. In particular, if the variance of these shocks is assumed to be common across age groups, we find that that the return to skills falls in the late 1980s and recovers quite strongly thereafter. When we instead assume that the variance of these shocks is common across education groups, we find strong decreases in the return to skills from the early 1990s until the end of our sample period.

The rest of the paper is organized as follows. In Section 2 we discuss our identification strategy and contrast it with the traditional strategy which assumes that the variance of unobserved skills remains constant over time within groups. Section 3 describes the dataset and discusses the details of the empirical implementation. Section 4 presents our main results, while Section 5 extends our identification strategy to account for permanent shocks to skills.
within cohorts over the life cycle, as well as idiosyncratic transitory shocks to earnings. Section 6 concludes.

2 Estimating the Return to Unobserved Skills

Suppose that log wages are determined by an error components model such as in Chay and Lee (2000) or Lemieux (2006):

\[ w_{it} = x_{it}b_t + u_{it} \]  
\[ u_{it} = p_t e_{it} + \nu_{it} \]

(1)  
(2)

\( w_{it} \) is the natural logarithm of the hourly wage rate for individual \( i \) at time \( t \), \( x_{it} \) is a vector of observed skills (such as education and labor market experience) and \( b_t \) is the return (or price) of observed skills. \( u_{it} \) represents residual wages, which are composed of unobserved skills \( e_{it} \), the return to those skills which is given by \( p_t \), and measurement error \( \nu_{it} \). We assume that the distribution of \( \nu_{it} \) is independent from \( e_{it} \). Note that \( \nu_{it} \) may also be interpreted as an idiosyncratic shock to wages (uncorrelated with ability).

Suppose that observable skills \( x_{it} \) can be fully characterized by individual’s education and age. For individuals with education level \( c \) and age \( a \), given the independence assumption for \( \nu_{it} \), the within-group variance of wages is given by:

\[ V_{a,c,t} = p_t^2 \sigma^2_{a,c,t} + \sigma^2_{\nu,a,c,t} \]

(3)

where \( \sigma^2_{a,c,t} \equiv \text{Var}(e_{it}|a,c,t) \) is the conditional variance of unobserved skills (conditional on education and age) and \( \sigma^2_{\nu,a,c,t} \equiv \text{Var}(\nu_{it}|a,c,t) \) is the conditional variance of the measurement error.

From equation (3) it is clear that changes over time in within-group wage inequality may be driven by three different factors:

i. Changes in \( p_t \), the price paid to unobserved ability (due for example to changes in technology which change the demand for skills).

ii. Changes in \( \sigma^2_{a,c,t} \), the dispersion of unobserved ability within groups (due for example to changes in the skills of workers selecting into particular education groups, changes in the dispersion of schooling quality, or changes in the relevance of on-the-job training).

iii. Changes in \( \sigma^2_{\nu,a,c,t} \), the variance of measurement error (due for example to methodological changes in the survey, or changes in the incidence of temporary idiosyncratic shocks).
We are interested in decomposing the relative importance of each of these components. Identification is difficult due to the fact that none of these components are directly observable. For now we will ignore measurement error and focus on the identification of the return to skills $p_t$ and the within-group variance of unobserved skills $\sigma^2_{a,c,t}$. We return to the issue of measurement error in Section 5.

One common way in which previous literature has achieved identification is by assuming that the within-group variance of unobserved ability among workers in a given education-age group is constant over time, i.e. $\sigma^2_{a,c,t} = \sigma^2_{a,c}, \forall t$. Under this assumption, we have that:

$$\frac{V_{a,c,t}}{V_{a,c,t-1}} = \frac{p_t^2}{p_{t-1}^2}$$

$$\Rightarrow \ln p_t - \ln p_{t-1} = \left(\frac{1}{2}\right) \ln \left(\frac{V_{a,c,t}}{V_{a,c,t-1}}\right) \quad (4)$$

Equation (4) implies that the evolution of the price of unobserved skills can be identified from the changes over time in the within-group variance of wages. Making the normalization $p_0 = 1$, we can back out the return to unobserved skills at time $t$ by computing:

$$\ln p_t = \left(\frac{1}{2}\right) \sum_{\tau=1}^{t} \ln \left(\frac{V_{a,c,\tau}}{V_{a,c,\tau-1}}\right) \quad \forall t > 0 \quad (5)$$

This is the price series implied by the Chay and Lee (2000) and Lemieux (2006) frameworks when abstracting from measurement error.\(^3\)

The identification of the returns to unobserved ability based on equation (5) relies on the crucial assumption that the within-group variance of unobserved skills is constant over time. As discussed earlier, this is a very strong assumption. Therefore, in what follows we relax this assumption in order to allow the variance of unobserved skills to change over time. In particular, we make the alternative assumption that the variance of unobserved skills remains constant only over short time frames and only when conditioning on the exact same subset of individuals who remain in a given experience group.

Suppose that we have access to longitudinal data. Denote the subset of individuals whose wages are observed over two consecutive years $t - 1$ and $t$ as $s^t$. Individuals in group $s^t$ must be employed in both periods. In Section 5 we allow for a deterministic life-cycle pattern in the variance of ability; for now we abstract from life-cycle considerations and make the

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\(^3\)More specifically, this is the price series implied by the Chay and Lee (2000) framework in what they call their “between-cohort” analysis. They also consider a “within-cohort” analysis where the identifying assumption is that the variance of unobserved skills remains constant for workers from a given cohort as they gain additional experience. This “within-cohort” assumption is closer, although not equivalent, to our identification assumption discussed in detail below.
assumption that, for this subset of individuals only, the variance of ability remains constant across periods $t - 1$ and $t$. That is:

$$\sigma_{a,c,t,s}^2 = \sigma_{a-1,c,t-1,s}^2$$ (6)

The overall within-group variance of unobserved skills for all individuals of age $a$ and education level $c$, $\sigma_{a}^2$, may change between periods $t - 1$ and $t$ because the individuals belong to different cohorts and thus their ability distribution are potentially different.

Assuming that the return to unobserved skills $p_t$ is common for all individuals, identification is achieved by considering the changes over time in the within-group variance of wages for the subset of individuals in $s^t$. That is:

$$\frac{V_{a,c,t,s^t}}{V_{a-1,c,t-1,s^t}} = p_t^2 - p_{t-1}^2$$

$$\Rightarrow \ln p_t - \ln p_{t-1} = \left(\frac{1}{2}\right) \ln \left(\frac{V_{a,c,t,s^t}}{V_{a-1,c,t-1,s^t}}\right)$$ (7)

If we make the normalization $p_0 = 1$, we can obtain an implied price series given by:

$$\ln p_t = \left(\frac{1}{2}\right) \sum_{\tau=1}^{t} \ln \left(\frac{V_{a,c,\tau,s^\tau}}{V_{a,c,\tau-1,s^\tau}}\right) \quad \forall t > 0$$ (8)

If changes in the variance of ability over time within education-age groups were unimportant, then the implied price series from equations (5) and (8) should be similar to each other.

Note that the discussion above revolves around the variance of within-group wages. It relies on the condition that workers can be divided into education-age cells which fully characterize observable skill groups. In practice, when taking this approach to the data, the number of cells that a sample can be divided into while maintaining a large enough sample size in each cell is limited. This is particularly true in the case of our identification strategy which relies on longitudinal data. When individuals are grouped into relatively coarse skill groups, a non-negligible amount of heterogeneity in terms of observable characteristics will remain within each cell.

One option to deal with this issue is to follow Lemieux (2006) and impose the additional assumption that $E(e_{it}|x_{it}) = 0$ and $E(\nu_{it}|x_{it}) = 0$. Under this assumption, a log wage regression can first be estimated to obtain residual wages and the return to unobservable skills can be identified based on the within-group variances of these residuals. We impose this assumption in what follows in order to avoid confounding the effects of changes in observable within-group heterogeneity. In Appendix A we provide further discussion of this exogeneity
3 Empirical Implementation

We use information from the Merged Outgoing Rotation Group (MORG) sample from the monthly Current Population Survey (CPS) for the period from January 1982 until December 2012. The CPS is the main source of labor market statistics in the United States. We take advantage of the fact that the CPS is a rotating sample: households included in the survey are sampled for four consecutive months, then leave the sample for eight months before returning for another four months. Earnings information is collected when households are in their fourth and eight months in the sample (i.e. when they are in the so-called Outgoing Rotation Groups), so there is information on earnings for the same household across the same calendar month in two consecutive years. Details regarding the algorithm used to match individuals over time can be found in Nekarda (2009).

We restrict the sample to individuals aged 25 to 64. To obtain residual wages, we regress log real hourly earnings on a set of calendar month dummies, education dummies, age bin dummies and interactions of education dummies and a quartic in age. This regression is run separately by gender for each calendar year.

We then categorize our observations into education-age-gender groups, using two education categories (high school or less including those with some college education, and college graduates), and four age categories. The set of “stayers” $s^t$ is defined as the set of individuals who are in month-in-sample 4 in period $t-1$ and month-in-sample 8 in period $t$, and have valid earnings data in both periods. We exclude outliers from this group, defined as individuals with residual wage changes greater than 60 log points. In the remainder of the paper, we focus on the results for men.

Figure 1 illustrates the structure of the data for the set of workers observed in year $t$. Approximately half of these workers will be interviewed for the first time in year $t$ (and re-interviewed in year $t+1$) and the other half will be in their second interview in year $t$. This latter group will not be re-interviewed in the following period. Some of them would have also been observed working in the previous year; these constitute the group $s^t$. Among those being interviewed for the first time in year $t$, a small number will not be re-interviewed in the following period due to attrition, and some will be re-interviewed but will not be working in year $t+1$. The remainder represent our group of stayers $s^{t+1}$, for whom we also have earnings data in the following year.

As mentioned above, our data is at a monthly frequency, but we observe the same individuals across the same calendar month in two consecutive years (rather than across consecutive months). Therefore, for each year $t$ in our dataset, we pool all of the monthly observations and, as illustrated in the figure, we calculate: (i) $V_{a,c,t}$, the variance of residual wages by
education-age group using all workers in year $t$, (ii) $V_{a,c,t,s^t}$ and $V_{a-1,c,t-1,s^t}$ using workers in group $s^t$, and (iii) $V_{a,c,t,s^{t+1}}$ and $V_{a+1,c,t+1,s^{t+1}}$ using workers in group $s^{t+1}$.

One limitation of the longitudinal dimension of the matched monthly CPS data is that there are certain months when there are breaks in the CPS identifiers which make it impossible to generate matches. Therefore, although for most years in our sample we have data for stayers from all 12 months within the year, there are some years for which this is not the case. Moreover, due to the change in the CPS identifiers in January 1994, we are not able to generate matches for any month in 1993. In our analysis, we assume that the price change for that particular year only is given by the change estimated for the full sample rather than the sample of stayers i.e., using equation (4) rather than equation (7).

Figure 2 plots the evolution of the within-group variance of residual wages $V_{a,c,t}$ for each of our demographic groups. Within-group variances tend to increase with age and education. Over time, within-group variances are generally increasing for college graduates, while they are stable or decreasing slightly for those without a college degree.

Recall that under the traditional assumption of constant within-group variance of skills, all of the changes over time in the series plotted in Figure 2 are interpreted as reflecting changes in the return to skills. In the next section of the paper, we estimate the changes over time in the return to skills under that assumption, as well as under our more nuanced identification assumption in order to understand the extent to which the changes in within-group variance in Figure 2 can in fact be attributed to price changes, and the extent to which they can be attributed to changes in the distribution of unobserved ability within each group.

4 Results

4.1 Estimated returns to skills

For each of our demographic groups, we use the computed variances of residual wages for stayers to estimate the return to skills based on equation (8). A weighted average of the ratio of within-group variances for stayers is computed across groups in each period in order to obtain a single price series. The results are presented in Figure 3, along with the prices estimated under the traditional assumption of constant within-group variance (i.e. using ratios of the within-group variance for all workers, rather than stayers). The series represent price changes relative to the year 1982, when log-prices are normalized to zero.

The figure shows a clear difference between the two identification methods. By relaxing the assumption of constant within-group variance we obtain a price series that is decreasing in the 1980s and then only moderately increasing thereafter. This contrasts sharply with the

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5 As an alternative, we consider the assumption that the price change between 1993 and 1994 is equal to zero. Our results do not change in any major way under this alternative assumption.
consistently increasing returns to skills obtained under the assumption of constant within-group variance. In fact, the series of returns to skills obtained under our identification strategy implies that the observed increases in within-group variance are driven by increases in the variance of unobserved ability within groups, rather than increases in the return to skills. This is depicted in Figure 4. The estimated increases in within-group variance are particularly strong between the mid-1980s and the mid-1990s among college graduates.

Note that our identification strategy relies on the changes over time for the sample of stayers, which are a deliberately selected sample for whom we can assume that the variance of unobserved skills remains constant across consecutive periods. One might be concerned that the patterns of within-group wage inequality for this group may be systematically different from those of the full sample, and that this may be the source of any potential differences in the identified return to skills. To show that this is not the case, Figure 5 plots the estimated return to skills when we apply the traditional identification strategy using our selected sample only (i.e. individuals who are observed twice and are working in both periods). The results clearly show that the reason why we find the fall in the estimated return to skills is not the sample selection, but rather the change in the identification strategy which allows for changes over time in the variance of unobserved skills.

4.2 Decomposition of the changes in residual wage variance

The results on the changes in skill prices and in the variance of unobserved skills can be summarized by performing a decomposition of the change in the within-group variance for each group into a price effect and a distribution effect. Specifically, the change in the within-group variance across two periods \( t \) and \( t' \) can be decomposed as follows:

\[
V_{a,c,t} - V_{a,c,t'} = p_t \sigma_{a,c,t}^2 - p_{t'} \sigma_{a,c,t'}^2
\]

\[
= \left( p_t^2 - p_{t'}^2 \right) \sigma_{a,c,t}^2 + p_{t'}^2 \left( \sigma_{a,c,t}^2 - \sigma_{a,c,t'}^2 \right)
\]

(9)

The price effect in equation (9) is the change in the within-group variance that would have occurred due to the change in the return to skills if the within-group distribution of skills had remained constant as in period \( t \). The distribution effect is the portion attributable to changes in the within-group skill variance, holding prices constant at their level for period \( t' \).

Table 1 shows the results from the decomposition for the changes in residual wage variance between the periods 1982-1984 and 2010-2012. Recall that, under the traditional assumption that the variance of skills within groups is constant, essentially all of the changes in the residual wage variances are attributed to changes in the return to skills. The decomposition results make clear how this changes dramatically when implementing our identification strategy.
Our findings show that the increases in within-group inequality observed across demographic groups mask a reduction in the return to skills coupled with an increase in the dispersion of skills within groups.

4.3 Allowing for heterogeneous returns to skills across demographic groups

Our identification strategy allows us to estimate different returns to skills for different groups. Figure 6 plots the estimated return to skills separately by education group in the top panel, and by age group in the bottom panel. The aggregate patterns shown above are most closely reflected in the series for the lower education group. Among college graduates, there is evidence of increases in the return to skills over the 1990s after a stronger fall in the 1980s. Over the entire 1982-2012 period, the change in estimated returns to skills for both groups is quite similar. Among age groups, the patterns are very similar among the groups of workers aged 35 and above. The patterns for the youngest group of workers is different. Here we find that after a similar decline in the returns in the 1980s, there is a fairly strong increase in the return to skills among workers aged 25-34. This could reflect heterogeneities in demand for skills among different age groups along the lines of Card and Lemieux (2001).

5 Extensions: Life-Cycle Patterns (Permanent Shocks to Skills) and Measurement Error (Transitory Shocks to Earnings)

So far our identification strategy has relied on the assumption that the variance of unobserved ability remains constant for the group of workers who remain employed over two consecutive years. In this section we relax this assumption further to allow for changes in the variance of unobserved skills among stayers due to permanent shocks to skills, which could be attributed to heterogeneities in the accumulation of human capital on the job, for example due to differences in training or on-the-job learning across occupations. We also re-introduce measurement error into the estimation.

Specifically, suppose that skills are subject to permanent idiosyncratic shocks.\textsuperscript{6}

\[ e_{it} = e_{it-1} + \mu_{it} \]

where \( e_{it} \) are individual \( i \)'s unobserved skills at time \( t \), as introduced in Equation (2). Assuming that \( \mu_{it} \) is mean zero and allowing its variance to potentially vary over time and across demographic groups, we can re-write the within-group variance of unobserved ability among stayers as:

\textsuperscript{6}A similar specification is considered by Lochner and Shin (2014) and follows a long tradition of modeling the error component of earnings as a combination of permanent and transitory shocks, as reviewed by Meghir and Pistaferri (2011).
\[
\text{Var}(e_{it}|a, c, t, s^t) \equiv \sigma_{a,c,t,s^t}^2 = \sigma_{a-1,c,t-1,s^t}^2 + \sigma_{\mu,a,c,t,s^t}^2
\]

Re-introducing measurement error (or idiosyncratic temporary and non-persistent shocks to earnings), we have that the within-group residual wage variance for stayers would be given by:

\[
V_{a,c,t,s^t} = \rho_t^2 \sigma_{a,c,t,s^t}^2 + \sigma_{\nu,a,c,t}^2
= \rho_t^2 \left( \sigma_{a-1,c,t-1,s^t}^2 + \sigma_{\mu,a,c,t,s^t}^2 \right) + \sigma_{\nu,a,c,t}^2
\]

Identification will depend on the assumption that is made about the heterogeneities in the variances of the shocks across groups.

**Age-specific shock variances**

Assume that the variance of the permanent shock to earnings and the variance of measurement error depend on age and vary over time, but do not depend on education:

\[
\sigma_{\mu,a,c,t,s^t}^2 = \sigma_{\mu,a,t,s^t}^2 \quad \forall c
\]
\[
\sigma_{\nu,a,c,t}^2 = \sigma_{\nu,a,t}^2 \quad \forall c
\]

In this case, the return to skills can be identified using a Wald-type identification strategy, as suggested by Chay and Lee (2000). Specifically, price changes are identified from changes in the college-high school differential in residual wage variances over time, that is:

\[
\frac{V_{a,\text{COL},t,s^t} - V_{a,\text{HS},t,s^t}}{V_{a-1,\text{COL},t-1,s^t} - V_{a-1,\text{HS},t-1,s^t}} = \frac{\rho_t^2 \sigma_{a,\text{COL},t,s^t}^2 - \rho_t^2 \sigma_{a,\text{HS},t,s^t}^2}{\rho_{t-1}^2 \sigma_{a-1,\text{COL},t-1,s^t}^2 - \rho_{t-1}^2 \sigma_{a-1,\text{HS},t-1,s^t}^2}
\]
\[
= \frac{\rho_t^2 (\sigma_{a-1,\text{COL},t-1,s^t}^2 + \sigma_{\mu,a,t,s^t}^2) - \rho_t^2 (\sigma_{a-1,\text{HS},t-1,s^t}^2 + \sigma_{\mu,a,t,s^t}^2)}{\rho_{t-1}^2 \sigma_{a-1,\text{COL},t-1,s^t}^2 - \rho_{t-1}^2 \sigma_{a-1,\text{HS},t-1,s^t}^2}
\]
\[
= \frac{\rho_t^2}{\rho_{t-1}}
\]

**Education-specific shock variances**

Alternatively, we could assume that the variance of the permanent shock to earnings and the variance of measurement error depend on education and vary over time, but do not depend
on age:

\[
\begin{align*}
\sigma_{\mu,a,c,t,s}^2 &= \sigma_{\mu,c,t,s}^2 \quad \forall a \\
\sigma_{\nu,a,c,t}^2 &= \sigma_{\nu,c,t}^2 \quad \forall a 
\end{align*}
\]

In this case, the return to skills can be identified using a Wald-type identification strategy, as suggested by Chay and Lee (2000). Specifically, price changes are identified from changes in the college-high school differential in residual wage variances over time, that is:

\[
\frac{V_{a,c,t,s} - V_{a',c,t,s}}{V_{a-1,c,t-1,s} - V_{a'-1,c,t-1,s}} = \frac{p_t^2 \sigma_{a,c,t,s}^2 - p_{t-1}^2 \sigma_{a',c,t,s}^2}{p_{t-1}^2 \sigma_{a-1,c,t-1,s}^2 - p_{t-1}^2 \sigma_{a'-1,c,t-1,s}^2}
\]

\[
= \frac{p_t^2 (\sigma_{a-1,c,t-1,s}^2 + \sigma_{\mu,c,t,s}^2) - p_{t-1}^2 (\sigma_{a'-1,c,t-1,s}^2 + \sigma_{\mu,c,t,s}^2)}{p_{t-1}^2 \sigma_{a-1,c,t-1,s}^2 - p_{t-1}^2 \sigma_{a'-1,c,t-1,s}^2}
\]

\[
= \frac{p_t^2}{p_{t-1}^2}
\]

The results are presented for the estimated return to skills under the two different assumptions are presented in Figure 7.

[Section to be completed.]

6 Conclusions

In this paper we develop and implement an identification strategy to estimate the return to skills which relaxes the assumption that the variance of unobserved skills is constant over time within groups of workers defined by their levels of education and age. This allows the dispersion of skills within groups to vary across cohorts due, for example, to changes in the characteristics of workers selecting into different education levels over time or changes in the dispersion of the quality of education. It also allows for changes in the dispersion of skills within cohorts over time due, for example, to changes in business cycle conditions, or heterogeneities in the prevalence of on-the-job training leading to heterogeneous permanent shocks to skills across individuals over the life-cycle. Our identification strategy requires only two observations per individual and thus can be implemented using a large scale dataset such as the CPS due to the rotating nature of the sample. Identification is achieved by considering changes in the within-group residual wage variance over time for workers who are observed
over two consecutive years.

Our results suggest that relaxing the assumption of constant within-group variance is crucial. There have been important increases in the variance of residual wages, particularly among college-educated workers since the 1980s. Under the traditional assumption of constant skill variance within groups, these are interpreted as increases in the return to skills. By allowing for changes in the variance of skills we instead find that the return to skills has in fact fell during the 1980s and only partially recovered thereafter. We find that the increases in the variance of residual wages are instead attributable to increases in the dispersion of unobserved skills over time among college graduates.

When allowing for heterogeneous returns to skills across demographic groups, we only find evidence of increases in the return to skills over the 1982-2012 period among young workers (aged 25-34). Meanwhile, if we allow the variance of skills to change over the life-cycle, we find that the results are sensitive to the assumption made about the nature of the permanent shocks to skills. In particular, if the variance of these shocks is assumed to be common across age groups, we find that the return to skills falls in the late 1980s and recovers quite strongly thereafter. When we instead assume that the variance of these shocks is common across education groups, we find strong decreases in the return to skills from the early 1990s until the end of our sample period.
References


Figure 1: Data structure: Workers in period $t$

All workers at time $t$

Interviewed in $t$ and $t+1$

Interviewed in $t-1$ and $t$

$V_{a.c.t}$

Not observed or not working in $t+1$

Working in $t$ and $t+1$: $s(t+1)$

$V_{a.c.t.s^{t+1}}$

$V_{a+1.c.t+1.s^{t+1}}$

Not observed or not working in $t-1$

Working in $t-1$ and $t$: $s(t)$

$V_{a-1.c.t-1.s^{t}}$

$V_{a.c.t.s^{t}}$
Figure 2: Within-group variance of residual wages

Within-group variance of residual wages
Men, by education and age

Age: 25–34
Age: 35–44
Age: 45–54
Age: 55–64

Men, by education and age
Within-group variance of residual wages

High School
College Grad
Figure 3: Estimated Return to Skills

Estimated Return to Skills, Men, 1982=0

Identification Strategy: Traditional Proposed

Identification Strategy:  
- Traditional
- Proposed
Figure 4: Estimated Changes in Variance of Unobserved Ability
Figure 5: Sample Selection

- Traditional Strategy: Full Sample
- Proposed Strategy: Selected Sample
- Traditional Strategy: Selected Sample
Figure 6: Group-specific returns to skills

Estimated Return to Skills by Education Group

Estimated Return to Skills by Age Group
Figure 7: Estimated Return to Skills Allowing for Permanent and Transitory Shocks

Panel A: Assuming variance of shocks is age-specific (identification from changes in residual wage variance between education groups)

Panel B: Assuming variance of shocks is education-specific (identification from changes in residual wage variance between age groups)
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Appendix A  Exogeneity assumption [Preliminary and incomplete]

In this section we analyze the issues that may arise if the exogeneity condition \( E(e_{it}|x_{it}) = 0 \) fails. This assumption may be problematic as it implies that observable and unobservable skills are orthogonal to each other.

Suppose that wages depend on education, age, and interactions of these variables. For simplicity, consider the case where education is a categorical variable which may adopt one of three values (high school dropout, high school graduate, college graduate), and the effect of age on wages is linear. The wage equation is given by:

\[
 w_{it} = \beta_0 t + \beta_1 t A_{it} + \beta_2 E_{2it} + \beta_3 E_{3it} + \beta_4 E_{2it} A_{it} + \beta_5 E_{3it} A_{it} + p_t e_{it} + \nu_{it} \tag{A.1}
\]

Suppose that unobservable skills \( e_{it} \) are correlated with education. Following Wooldridge (2002), we can write the linear projection of \( e_{it} \) onto the observable explanatory variables as:

\[
 e_{it} = \pi_0 t + \pi_1 t A_{it} + \pi_2 E_{2it} + \pi_3 E_{3it} + \pi_4 E_{2it} A_{it} + \pi_5 E_{3it} A_{it} + \epsilon_{it} \tag{A.2}
\]

Substituting equation (A.2) into (A.1) gives:

\[
 w_{it} = (\beta_0 t + p_t \pi_0) + (\beta_1 t + p_t \pi_1) A_{it} + (\beta_2 t + p_t \pi_2) E_{2it} + (\beta_3 t + p_t \pi_3) E_{3it} + (\beta_4 t + p_t \pi_4) E_{2it} A_{it} + (\beta_5 t + p_t \pi_5) E_{3it} A_{it} + p_t \epsilon_{it} + \nu_{it}
\]

The error term \( p_t \epsilon_{it} + \nu_{it} \) has zero mean and is uncorrelated with each regressor. The plim of the OLS estimators from the wage regression are therefore \( \beta_{kt} + p_t \pi_{kt} \), \( k = 1,2,\ldots,5 \). In other words, the residual wages that we obtain are:

\[
 \tilde{u}_{it} = p_t \epsilon_{it} + \nu_{it} = p_t (e_{it} - \pi_0 t - \pi_1 t A_{it} - \pi_2 E_{2it} - \pi_3 E_{3it} - \pi_4 E_{2it} A_{it} - \pi_5 E_{3it} A_{it}) + \nu_{it} \tag{A.3}
\]

rather than \( p_t \epsilon_{it} + \nu_{it} \).

The variance of this residual within an education-age group at a given time \( t \) would be given by:
\[ V_{a,c,t} \equiv \text{Var}(\tilde{u}_{it} \mid a, c, t) \]
\[ = \text{Var} \left[ p_t (e_{it} - \pi_{0t} - \pi_{1t}A_{it} - \pi_{2t}E_{2it} - \pi_{3t}E_{3it} - \pi_{4t}E_{2it}A_{it} - \pi_{5t}E_{3it}A_{it}) + \nu_{it} \mid a, c, t \right] \]
\[ = \text{Var} \left[ p_t (e_{it} - \pi_{a,c,t}A_{it}) + \nu_{it} \mid a, c, t \right] \]
\[ = p_t^2 \sigma_{a,c,t}^2 + \sigma_{\nu t}^2 - p_t^2 \pi_{a,c,t}^2 \text{Var}(A_{it} \mid a, c, t) \] (A.4)

where we have used the fact that education does not vary within a group, but there is some variation in age given that we aggregate workers into 10-year age groups. All terms that are constant across individuals within groups have been dropped as they do not affect the within-group variance at time \( t \). \( \pi_{a,c,t} \) is equal to \( \pi_{1t} \) for groups of high school dropouts, \( \pi_{1t} + \pi_{4t} \) for high school graduates and \( \pi_{1t} + \pi_{5t} \) for college graduates.

The key assumption for our identification strategy to be valid is \( \pi_{a,c,t} = \pi_{a,c} \forall t \). This means that the partial correlation of age with unobserved ability (conditional on education) is constant over time. We also require that \( \text{Var}(A_{it} \mid a, c, t) \) is constant over time. For our identification strategy, we only require this variance to remain constant over consecutive periods for stayers, which holds by construction. With these assumptions, and defining \( \sigma_{A,c}^2 \equiv \text{Var}(A_{it} \mid a, c, t) \forall t \), we have:

\[ V_{a,c,t} = p_t^2 \left( \sigma_{a,c,t}^2 - \pi_{a,c}^2 \sigma_{A,c}^2 \right) + \sigma_{\nu t}^2 \] (A.5)

and the identification approach using changes over time in within-group variances of residual wages in order to identify changes in the return to unobservable skills described in Section 2 goes through. The estimated within-group variance of ability now includes the term \(-\pi_{a,c}^2 \sigma_{A,c}^2\), but we focus on the changes in the within-group variance over time, so as long as this term is (approximately) constant, this does not affect our results.

Note that the above discussion requires that each of our education-experience groups is composed of workers who are homogenous in terms of their education level. In practice, due to the small number of high school dropouts in our sample (particularly for more recent years and when conditioning on experience level and on staying in the sample over consecutive periods), we pool high school dropouts and high school graduates. However, we have verified that our empirical findings go through if we exclude high school dropouts from our sample.

---

1 In the data, there are minor deviations for stayers due to measurement error in age.