Using “Opposing Responses” and Relative Performance To Distinguish Empirically Among Alternative Models of Promotions

by

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Abstract: Applying a simultaneous-equations estimation approach that accounts for both worker and firm behavior, I show that classic tournaments, market-based tournaments, and performance standards models can be empirically distinguished when promotions induce worker effort. I also show that market-based tournaments with effort choices can be distinguished from those with human capital investments. A key insight is that an empirical test can be based on the “opposing responses” property whereby workers and firms adjust their choice variables in opposite directions when the stochastic component of worker performance changes. Finally, I propose a new approach – also requiring simultaneous equations – for empirically distinguishing between classic tournaments and market-based tournaments with human capital investments, showing that the two models differ in their predictions regarding the average wage between job levels.

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1. Introduction

Promotions typically come with wage increases and with other features that workers desire.¹ For this reason, the prospect of future promotion creates incentives for workers to take actions that increase their chances of promotion. Such actions are usually performance enhancing and thus desirable from the employer’s perspective.² This role of promotions as an incentive mechanism is one important consideration for the firm as it makes choices associated with the design and management of promotion systems. Another important consideration for the firm is optimal assignment of workers to jobs, which sometimes conflicts with the incentive-creating function of promotions, as noted in Baker, Jensen, and Murphy (1988). Three types of core theoretical models of promotions have been developed to shed light on these issues (classic tournaments, market-based tournaments, performance standards). As indicated in Table 1, these models can be categorized by their assumptions about the nature of the job hierarchy, the choice variables of workers, and the extent to which the firm can pre-commit to compensation prizes given competitive pressure from other firms in the labor market.³ Distinguishing among these competing models empirically is difficult given that their testable implications overlap substantially. This difficulty motivates the present study. In this paper I show that the three core theoretical models can be empirically distinguished to a greater extent than previously thought.

As indicated in Table 1, both types of tournaments have hierarchies with fixed managerial job slots. For example, consider a two-level job hierarchy with one managerial position and two subordinate positions, so that only one subordinate can be promoted. This scarcity of managerial positions automatically creates a zero-sum internal promotion competition between the two subordinates. In contrast, if managerial job slots are flexible rather than fixed, as in the performance standards models, then it is possible that both subordinates can be promoted.

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¹ Several papers that document the large wage increases that accompany promotions include Murphy (1985), Gerhart and Milkovich (1989), Lazear (1992), Main, O’Reilly, and Wade (1993), Baker, Gibbs, and Holmstrom (1994a,b), and McCue (1996).
² These actions may in some cases be unproductive or even destructive, as in the case of sabotage (e.g. Lazear 1989, Garvey and Swan 1992, Drago and Garvey 1998, Konrad 2000, Chen 2003, Kräkel 2005, and Gürtler 2008) or influence activity (e.g. Milgrom and Roberts 1988, Meyer, Milgrom, and Roberts 1992, Schaefer 1998, and Fairburn and Malcolmson 2001).
to the level of manager, as long as their individual performances exceed a “performance standard” as discussed in Gibbs (1994, 1996).

Tournaments can be further classified into “classic” and “market-based” – using the terminology of Waldman (2012) – according to the mechanism by which wage spreads between job levels are generated. In classic tournaments, as first articulated in Lazear and Rosen (1981), firms strategically set wage spreads *ex ante* with the aim of eliciting the desired worker behavior. In market-based tournaments, wage spreads arise *ex post* as the outcome of a competitive bidding process in which an employer raises a promoted worker’s wage to prevent the worker from being raided by competing firms that interpret observed promotions as signals of worker ability. In such models, the worker’s choice variable that affects performance (and thereby the promotion probability) is sometimes effort (as in Gibbs 1995 and Waldman 2012) and sometimes a human capital investment (as in Zábojník and Bernhardt 2001 and Zábojník 2012).

A large empirical literature has endeavored to test the implications of the core theoretical models in Table 1. The bulk of this literature has focused on testing the implications of classic tournaments, and much of the evidence has been consistent with that model. The problem is that most of that evidence is also consistent with the other core models in Table 1. The near observational equivalence of the core models is the motivation for this study. The problem is well known. For example, Gibbs (1994) observes that the classic tournament model and the performance standards model are essentially observationally equivalent, and Waldman (2012) concludes that most of the existing evidence does not allow the classic and market-based tournament models to be distinguished.

I show that in the class of models in which promotions induce worker effort, two empirical tests are sufficient to distinguish the three core theoretical models in Table 1. The first

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6 However, Waldman does argue that in the existing empirical literature there are some pieces of evidence that point in one direction or the other.
test concerns whether promotions are determined by relative or absolute performance. When managerial job slots are scarce, an increase in worker i’s performance, *ceteris paribus*, reduces worker j’s chance of promotion, and *vice versa*. That would not be true with flexible managerial job slots in which any worker who performs sufficiently well is promoted. In that case, absolute performance determines promotions, so that an increase in worker i’s performance, *ceteris paribus*, has no effect on worker j’s chance of promotion, and *vice versa*. This situation occurs in the performance standards models. Thus, one can distinguish performance standards from either type of tournament by investigating whether promotions are determined by absolute performance or by relative performance. The idea for such a test was mentioned in Gibbs (1994), and it was implemented empirically in DeVaro (2006a,b).

The second test is based on a property I refer to as “opposing responses”. The basic idea is as follows. Consider a parameter, θ, in one of the core theoretical models. When the value of θ changes, this might induce adjustments in the choice variables of workers and the firm. If workers and the firm adjust their respective choice variables in opposite directions, the opposing responses property holds. Otherwise, it does not. If the property holds in some of the core models but not in others, this creates the basis for an empirical test to distinguish among models. This approach requires the existence of a parameter that is common to all models the researcher wishes to compare. I exploit the fact that in the core models individual performance typically includes a stochastic component. My focus is on the variance of that term, and henceforth I refer to that variance as θ.

Classic tournaments exhibit the opposing responses property, based on the following logic. Consider a two-player tournament with homogeneous workers in which the higher-performing worker wins the promotion, with the prize for promotion being the wage spread between the winner’s job and the loser’s job. Let θ denote the variance of the stochastic component of each worker’s performance. When θ is large, worker performances (and therefore promotion outcomes) are largely random and are not affected much by the workers’ (costly) effort choices. This decreases worker effort, given the wage spread. Anticipating this, the firm increases the wage spread to “sweeten the pot” and offset the depressed incentives. In this
context, the property of *opposing responses* arises from the fundamental assumption that firms strategically set wage spreads *ex ante* to induce effort.

If cross-sectional data on firms (or tournaments) are available that include information related to the choice variables of workers and firms, as well as information on θ across firms, the opposing responses property can be empirically tested. The primary obstacle to conducting this test is that data on θ are typically unavailable, given that θ is inherently difficult to measure. However, DeVaro (2006a,b) showed how to conduct the test for classic tournaments even in the absence of data on θ, via an implied prediction on the sign of the cross-equation error correlation in a simultaneous-equations econometric model. Whereas those papers focused only on testing the predictions of classic tournaments, in the present paper I determine whether the opposing responses property holds in market-based tournaments and in performance standards models. Thus, the present paper extends an existing empirical framework to answer a broader set of questions. The new theoretical results yield new interpretations of the empirical results from the earlier papers. Generating the new results concerning opposing responses requires deriving new predictions from existing theoretical models, because previous research has focused on aspects of these models other than the responses of choice variables to shifts in θ.

There are five new results. First, in the class of models in which promotions create incentives for effort, the opposing responses test and the relative-versus-absolute performance test together can distinguish among the three core promotions models in the literature. Of particular interest is that classic and market-based tournaments can be distinguished by the opposing responses property. Second, the performance standards model does not exhibit opposing responses. Third, the two alternative ways to model market-based tournaments (i.e. worker effort choices or worker human capital investments) can also be distinguished using opposing responses. Fourth, opposing responses cannot distinguish classic tournaments with effort choices from market-based tournaments with human capital investments. Fifth, motivated by the fourth result, I derive a new empirical test that can distinguish between classic
tournaments and market-based tournaments with human capital investments. The new test is based on the *average* wage between job levels instead of the wage *spread* between levels.\(^7\)

After establishing the first two results in section 2, I explain how to conduct the empirical tests in section 3, showing that the empirical framework developed in DeVaro (2006a,b) nests the core promotion models. Given the data that are typically available, the empirical framework must simultaneously account for both worker and firm behavior via a system of equations. In section 4 I review the previous empirical evidence and discuss how it can be reinterpreted in light of the first two new results. In section 5 I present the third and fourth results of the paper. In section 6 I show that operationalizing the fifth result requires developing a new systems-based estimation approach. A possibility that might complicate the interpretation of results from the tests in this paper is that the empirical evidence might be generated by a blend of the core models as opposed to just one of them. I discuss this issue in section 7 before concluding in section 8.

In related recent work, Waldman (2012) presents a detailed comparison of classic and market-based tournaments and provides the first evaluation of the extent to which the empirical evidence supports each of these core models. Given that Waldman focuses only on tournaments, he does not consider the relative-versus-absolute performance test since it is unhelpful for his purpose. Furthermore, whereas I focus on the “opposing responses” prediction, Waldman focuses on other predictions such as those concerning the number of tournament contestants and the convexity of the wage structure within the firm. Although Waldman concludes that most of the evidence does not allow the two types of tournaments to be distinguished, he discusses some ways in which they might be distinguished, and I build on his discussion here.

Given its focus on opposing responses to changes in $\theta$, this paper emphasizes the role of risk in the production environment in affecting worker and firm behavior and therefore observed outcomes. Although this paper concerns promotion schemes, the work relates in a natural way to a large literature that explores the effects of risk on the design of incentive contracts, following original theoretical papers by Holmström (1979) and Shavell (1979). The subsequent literature focused on the observable implications (in terms of the compensation contracts employers

\[^7\] In a two-level job hierarchy where $W_m$ denotes the wage in the higher-level managerial job and $W_s$ denotes the wage in the lower-level subordinate job, the equilibrium “average wage between job levels” is $L^* \equiv (W_m + W_s)/2$, whereas the equilibrium “wage spread between levels” is $S^* \equiv W_m - W_s$.\[^7\]
choose) of variation in the degree of risk in the production environment and why the empirical literature has failed to consistently support a tradeoff between risk and incentive pay.


2. Core Promotion Models with Endogenous Worker Effort

In this section I discuss each of the three core promotion models in which workers choose effort (i.e. classic tournaments, market-based tournaments, performance standards). I also briefly discuss performance standards models in which workers make no endogenous choice, though it is understood that the omission of worker choices in these models is merely a simplification that allows researchers to focus on aspects of promotions other than incentives (e.g. job assignment). The two new results of this section, stated in Propositions 1 and 2, are that in the performance standards model and in market-based tournaments the opposing responses property fails to hold. In contrast, the previous literature has established that the property holds in classic tournaments.

Throughout the paper, unless otherwise indicated, $P_i$ refers to worker $i$’s job performance (or output), $e_i$ is worker $i$’s effort choice, $e_i^*$ is worker $i$’s equilibrium effort level, and $u_i$ is the stochastic component of worker $i$’s performance, which has a variance of $\theta$. The wage for non-promoted workers is denoted $W_s$ (for “subordinate” wage) whereas the wage for promoted workers is denoted $W_m$ (for “managerial” wage), where $W_m > W_s$. The “wage spread”, $S$, is defined as $S \equiv W_m - W_s$, and its equilibrium value is denoted $S^*$. As in the preceding two sentences, to simplify notation I sometimes suppress a subscript $i$ indexing workers.

2.1 Classic Tournaments With Endogenous Worker Effort

Most of the classic tournament literature builds on the two-player model of Lazear and Rosen (1981), which assumes a two-level job hierarchy with one managerial job at the top and

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two subordinate positions at the bottom. For the purposes of the present paper, there are two important results regarding classic tournaments. First, promotions are determined by relative performance. Second, $\partial e^*/\partial \theta < 0$ and $\partial S^*/\partial \theta > 0$, so that the choice variables of the workers and the firm are adjusted in opposite directions when $\theta$ changes, i.e. the property of opposing responses holds. Both results are known from prior literature, and Appendix A provides the underlying details. Note that when workers are risk neutral, the decrease in effort that arises when $\theta$ increases (holding the spread constant) is exactly offset by the effort-enhancing effect of the increase in the spread. Thus, when we say that one requirement of opposing responses is that $e^*$ decreases in response to an increase in $\theta$, we mean “holding the spread constant”.

2.2 Market-Based Performance Standards With Endogenous Worker Effort

Gibbs (1994,1996) argues that there are sometimes good reasons for firms to use performance standards rather than tournaments to make promotion decisions. Such schemes are characterized by flexible job slots, so that all workers whose performance exceeds a fixed standard get promoted. This implies that promotions are determined by absolute performance. A seminal model of this type is Waldman (1984a), which was recently extended in DeVaro and Waldman (2012). In that framework, young workers are hired into the subordinate job in the first period. A worker’s first-period output is observed by his own employer but not by competing employers. On the basis of that observation, the worker’s employer decides whether to promote the worker in the second period or to retain the worker in the subordinate job. In either case, competing firms observe the worker’s second-period job assignment, interpreting this as a signal of the worker’s ability. Competing firms make wage offers to the worker on the basis of this job assignment. In equilibrium the worker remains with the original employer, and

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10 There are two points concerning the robustness of the opposing responses property. First, the result $\partial e^*/\partial \theta < 0$ assumes that noise is normally distributed, and though it can be generalized beyond the normal case, it cannot be generalized to all continuous, symmetric distributions. Second, the property requires that workers competing in the same tournament are roughly homogeneous in ability. See Appendix A for more details.

11 Another influential model of this type is Gibbons and Waldman (1999a) and the related analysis in Gibbons and Waldman (2006). These models assume symmetric learning, i.e. information about worker ability is revealed to all firms in the labor market at the same rate. In contrast, in models such as Waldman (1984a) and its various extensions, learning is asymmetric in that a worker’s current employer observes the worker’s productivity more accurately than competing employers do.
promoted workers (who are thought to be of high ability in the eyes of competing firms) receive a wage increase sufficient to prevent them from being raided by competing firms.

Recall that the opposing responses property requires $\partial e^*/\partial \theta < 0$ and $\partial S^*/\partial \theta > 0$. In performance standards models such as one of the models in Ghosh and Waldman (2010), the first of these conditions fails in a trivial sense given that there are no endogenous effort choices in the model, and thus no incentives. Such models do not exhibit opposing responses. But given that effort choices are omitted from these models primarily to simplify the analysis, the more interesting case to consider incorporates worker effort choices, and that case is also analyzed in Ghosh and Waldman (2010). Although that analysis did not consider the effect of $\theta$ on either $e^*$ or $S^*$, I show below that it yields the result $\partial S^*/\partial \theta = 0$. To facilitate a precise discussion in this subsection and the next, it is helpful to reproduce the model here. At the start of period 1 it is common knowledge that each worker’s innate ability, $A_i$, is $a_{H_i}$ with probability $\rho$ and $a_{L_i}$ with probability $1-\rho$, where $0 < a_{L_i} < a_{H_i}$. The worker’s effective ability in period $t$, $\eta_{it}$, is $\eta_{i1} = A_i$ and $\eta_{i2} = kA_i$, where $k > 1$ represents general human capital. Subordinate $i$ produces period-1 output of $y_{is1} = d_{i1} + c_s(\eta_{i1} + e_{i1} + u_{i1})$, where the subscript 1 denotes time and the subscript “s” denotes subordinate, $e$ is the effort choice, and $u$ is a mean-zero, normally-distributed stochastic term with variance $\theta$. In period 2, worker $i$’s output in the managerial job is $y_{im2} = (1+f)[d_m + c_m(\eta_{i2} + e_{i2} + u_{i2})]$ whereas the same worker’s output in the subordinate job is $y_{is2} = (1+f)[d_s + c_s(\eta_{i2} + e_{i2} + u_{i2})]$, where $f = F \geq 0$ if the worker remains with the original employer in the second period and $f = 0$ if the worker switches employers, so that $f$ captures firm-specific human capital. As in Rosen (1982) and Waldman (1984b), $0 < c_s < c_m$, and $d_m < d_s$, implying a job ladder in which output increases faster with ability in the managerial job than in the subordinate job. In both periods, workers can choose any effort level in $[e_{L_i}, e_{H_i}]$, where $e_{L_i} = 0$ is sometimes assumed. Worker $i$’s cost of exerting effort in period $t$ is $\alpha C(e_{it})$, where $\alpha > 0$ and $C(e)$ is defined as in the preceding subsection. Higher values of $\alpha$ lower the sensitivity of effort choices to incentives and frequently imply reduced effort.

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12 This is also true in Waldman (1984a), Gibbons and Waldman (1999a, 2006), and a number of other “performance standards” papers in the promotions literature, and it is true in DeVaro and Morita (2012), which is neither a performance standards model (because it exhibits a managerial job slot constraint) nor a tournament model (because it does not incorporate any endogenous worker choices).

13 The presentation here is abbreviated, with some details omitted and some changes in notation. Refer to the original paper for a more complete treatment.
The firm privately observes its workers’ outputs each period, whereas second-period job assignments are publicly observed. After observing worker i’s first-period output, $y_{i1}$, the worker’s employer forms an updated belief, $\eta^*(y_{i1})$ at the start of period 2 concerning the worker’s effective ability. The timing is as follows. At the start of the first period all firms make wage offers to each worker, and each worker chooses an employer. Then workers choose first-period effort, the value of $u$ is realized, and the worker’s output is observed by his employer. At the start of the second period, the firm gives the worker a job assignment that is publicly observed. Each firm then makes the worker a wage offer, and the worker’s original employer then makes the worker a counteroffer. Each worker then chooses an employer, switching firms only if offered a strictly higher wage. Then the worker chooses an effort level, a new value of $u$ is realized, and the worker’s employer privately observes the worker’s output.

Since the model has two periods, the worker’s second-period effort choice is zero as a result of the last-period problem. But in the first period, it may be in the interests of subordinates to exert effort levels beyond this minimum for the same reason as in the classic tournament model. That is, higher levels of first-period effort increase the subordinate’s first-period output, which the employer privately observes and uses as the basis for a promotion decision. Thus, higher levels of first-period effort imply higher promotion probabilities – and the accompanying wage increases – in the next period. The following new result can now be stated:  

**Proposition 1**: In the performance standards model with endogenous worker effort, the opposing responses property (i.e. $\partial e^*/\partial \theta < 0$ and $\partial S^*/\partial \theta > 0$) fails to hold, since $\partial S^*/\partial \theta = 0$.

The intuition for the result $\partial S^*/\partial \theta = 0$ follows from a winner’s curse property that characterizes this model. Given that a worker’s current employer is able to make a counteroffer to prevent the worker from separating to join a competing firm, this suggests that any worker a competing firm can successfully raid in equilibrium must be of low quality. As a consequence, competing firms will only be willing to offer a firm’s worker a wage equal to the lowest possible productivity (conditional on job assignment) that the worker would have in the same job in a competing firm. Given that competing firms are always bidding the lowest possible productivity

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14 All proofs are in Appendix B.

15 A winner’s curse property is common in models of this type that incorporate counteroffers. See Milgrom and Oster (1987) and also Greenwald (1986), Lazear (1986), and DeVaro and Waldman (2012).
conditional on job assignment, and given that this minimum productivity conditional on job assignment is not sensitive to changes in \( \theta \), it follows that the equilibrium wage spread will also not be sensitive to changes in \( \theta \). This severe adverse selection result is an extreme case that is not robust; under alternative assumptions that relax the winner’s curse property, if wages are determined by competing firms then job assignments reveal less information when there is more noise, so wage differentials will be smaller. I show this in the next subsection in the context of a market-based tournament model, and similar intuition should apply in the present context of a performance standards model. Note that this lack of robustness does not present a problem for the analysis; it simply means \( \frac{\partial S^*}{\partial \theta} = 0 \) in Proposition 1 could be replaced by \( \frac{\partial S^*}{\partial \theta} \leq 0 \), but in either case the opposing responses property fails to hold.

To summarize, there are three results of this subsection, the second and third of which are new. First, in the performance standards model with endogenous effort choices, promotions are determined by absolute performance rather than relative performance. Second, these models do not exhibit the opposing responses property given that \( \frac{\partial S^*}{\partial \theta} \) is zero rather than positive. Third, if the performance standards model does not incorporate endogenous worker effort then the first condition required by opposing responses, namely \( \frac{\partial e^*}{\partial \theta} < 0 \), obviously fails to hold given that \( \frac{\partial e^*}{\partial \theta} \) does not exist.

2.3 Market-Based Tournaments With Endogenous Worker Effort

If a managerial job slot constraint is imposed in the Ghosh and Waldman (2010) model described in the previous subsection, the model transforms to a market-based tournament model. Waldman (2012) presents a discussion and a partial analysis of this extension, which changes the original model in three key ways, and I build on his discussion here. First, there is now a single managerial position, so that relative performance determines promotions. The managerial job can either be staffed or left vacant. Second, the number of young workers hired in the first period is public information. To simplify the discussion, I assume that there are two subordinates. Third, following Greenwald (1986), with a small probability the worker is assumed to separate in the second period for exogenous reasons unrelated to worker ability. Whether the separation occurs is publicly observed in the second period after the worker’s original employer makes a wage counteroffer. This assumption eliminates or mitigates the winner’s curse property characterizing
the model in the previous subsection. Now, competing firms offer wages equal to the expected productivity of a worker in a given job assignment at a competing firm. In contrast, the winner’s curse in its extreme form implies that competing firms only offer a worker a wage equal to the minimum productivity the worker would have in a given job assignment at a competing firm.

Using the previous subsection’s notation, consider the case $\alpha < \infty$ so that first-period effort choices are potentially greater than the minimum. Given that introducing a managerial slot constraint significantly complicates the analysis, Waldman (2012) presents a partial analysis and informally discusses the main results, focusing on how the equilibrium wage spread varies with the number of subordinates competing for promotion in the second period. My focus is on how the equilibrium wage spread varies with $\theta$. As in Waldman’s discussion, mine assumes there is a sufficient amount of firm-specific human capital to ensure that there is no turnover other than the aforementioned turnover for exogenous reasons and that the production function is such that the employer always wants to promote a subordinate in the second period (as opposed to leaving the managerial position unfilled). Under these assumptions, the worker with the highest period-1 output is promoted, and if there is a tie (which happens with probability zero) the firm randomly selects which worker to promote.

Call the subordinates $i$ and $j$. Under the preceding assumptions, competing firms offer a promoted worker a wage equal to the expected productivity of that worker in the managerial position at a competing firm, i.e. $W_m = d_m + c_mE[\eta_{12} | y_{is1} > y_{js1}]$, assuming without loss of generality that subordinate $i$ is promoted. For the subordinate who is not promoted, competing firms offer a wage equal to the expected productivity of that worker in the subordinate position at a competing firm, i.e. $W_s = d_s + c_sE[\eta_{12} | y_{is1} > y_{js1}]$, assuming without loss of generality that subordinate $j$ is not promoted. The fact that workers cannot observe innate abilities (their own or their competitor’s) implies a symmetric equilibrium in which the workers make identical first-period effort choices. The following new result can now be stated:

**Proposition 2:** In the market-based tournament model with endogenous worker effort, the opposing responses property (i.e. $\partial e^* / \partial \theta < 0$ and $\partial S^* / \partial \theta > 0$) fails to hold, given that $\partial e^* / \partial \theta < 0$ and $\partial S^* / \partial \theta < 0$. 

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The logic for $\frac{\partial e^*}{\partial \theta} < 0$ is similar to that in the classic tournament model. That is, holding the spread constant, increases in the variance induce decreases in subordinates’ first-period effort, because the amount by which incremental effort increases the subordinate’s probability of winning diminishes as random determinants of performance (and therefore promotion) become more important. Given that $\frac{\partial S^*}{\partial \theta} < 0$, when $\theta$ increases, first-period equilibrium effort choices decrease for two reasons. The first effect depresses equilibrium effort because incremental effort increases the probability of victory by a smaller amount, and the second effect depresses effort because the equilibrium wage spread, or tournament prize, shrinks.

Note that $\frac{\partial S^*}{\partial \theta}$ differs in sign between the classic and market-based tournaments. The logic for $\frac{\partial S^*}{\partial \theta} < 0$ in the market-based model is as follows. When the variance increases, the quality of a firm’s information regarding the ability of its subordinates deteriorates, implying more frequent mistakes in second-period job assignments. These mistakes imply that, in the eyes of competing employers, the expected ability of promoted workers decreases, and the expected ability of workers who are not promoted increases. If $\theta$ approaches 0, then $u_{i1}$ and $u_{j1}$ disappear from the model, and $y_{is1} - y_{js1} = c_s(\eta_{i1} - \eta_{j1})$, so that the firm observes the worker’s ability and never makes mistakes in promotions decisions, i.e. the higher-ability subordinate gets the promotion. Thus, as shown in Appendix B, expected effective ability of promoted workers in the eyes of competing employers is

$$E(\eta_{i2} | i \text{ is promoted, } \theta \to 0) = k[(2\rho - \rho^2)a_H + (1 - \rho)^2a_L].$$

In contrast, when $\theta \to \infty$ so that the firm’s information about worker ability effectively disappears, then the firm often makes mistaken promotion decisions, and each worker’s probability of promotion approaches 0.5. In this case the expected effective ability of promoted workers in the eyes of competing employers is

$$E(\eta_{i2} | i \text{ is promoted, } \theta \to \infty) = k[\rho a_H + (1 - \rho)a_L] < E(\eta_{i2} | i \text{ is promoted, } \theta \to 0).$$

Analogous expressions could be given for workers who are not promoted, such that in the eyes of competing employers $E(\eta_{i2} | i \text{ is not promoted, } \theta \to 0) < E(\eta_{i2} | i \text{ is not promoted, } \theta \to \infty)$. Since promoted workers and those who are not promoted are less differentiated in the eyes of competing employers when the variance is high (due to mistaken job assignments), the wages that competing firms offer to these two types of workers are also less differentiated.
Comparing this extension to the model in the previous subsection helps to clarify the intuition for the result that the equilibrium wage spread in the previous subsection is insensitive to $\theta$. In the previous subsection, because of the winner’s curse, competing firms assume the worst about workers who separate from their initial employers, because their initial employers choose not to make counteroffers sufficient to keep them. In the present subsection, given the assumptions of exogenous turnover unrelated to ability and enough firm-specific human capital so that there is no other turnover, the winner’s curse disappears, so competing employers are willing to bid a wage equal to a worker’s expected productivity conditional on job assignment. As shown above, such expectations conditional on job assignments are sensitive to $\theta$ given that competing firms interpret $\theta$ as an indicator of how frequently the initial firm will make mistakes in its job assignments and will adjust their expectations about ability accordingly. Such adjustments in expectations by competing firms do not occur if, given a winner’s curse, they are always bidding the minimum productivity conditional on job assignment.

To summarize, there are two important points of this subsection, the second of which is new. First, promotions are determined by relative performance. Second, the model does not exhibit the opposing responses property. Although this result is derived under a particular model specification, the basic intuition underlying the result seems general so the result should be robust to alternative specifications. Thus, the classic and market-based models are distinguished by the effect of $\theta$ on the equilibrium wage spread. The fact that in classic tournaments workers and the firm adjust their choice variables in opposite directions in response to changes in $\theta$, whereas in market-based tournaments these adjustments are made in the same direction, is a testable implication that distinguishes these two models.

3. An Econometric Framework For Distinguishing Among the Core Promotion Models

The preceding section showed that in the class of models for which promotions create incentives for worker effort, the two tests based on opposing responses and relative-versus-absolute performance are sufficient to empirically distinguish among the three core promotion models (i.e. classic tournaments, market-based tournaments, and performance standards). The remainder of this section is in four parts. In the first I describe the data needed to conduct both
tests. In the second and third I discuss implementation of the two tests individually. In the fourth I discuss implementation of both tests simultaneously.

3.1 Data

Cross-tournament, worker-level data on each worker’s promotion history, pre-promotion job performance, the job performance of other workers in the same firm and job level as that worker (i.e. the worker’s competitors for promotion), the worker’s pre-promotion wage, and the worker’s post-promotion wage, are needed to conduct both tests. Note that data on \( \theta \) are not required and are typically unavailable, though if they are available they are helpful, as explained in the next subsection. Note also that the data must span tournaments, i.e. variation across tournaments identifies the key parameters, and this typically means the data must span firms. Most data sets fail to meet all of the preceding requirements, and two challenges in particular are worth highlighting. First, data on worker job performance by hierarchical level (so that pre-promotion job performance can be identified) are usually available only in single-firm datasets, whereas the tests require data that span tournaments; for example, the personnel records used in Baker, Gibbs, and Holmström (1994a,b) have unusually good information on worker job performance by level, but these data cannot be used because they do not span tournaments. One way to address this problem of missing performance data in cross-tournament samples is to exploit data on individual worker performance bonuses, which are available more commonly than direct data on worker performance. The idea is to infer an indirect measure of individual worker performance from the worker’s bonus payment, after netting out factors other than individual performance that might determine the bonus. Such an approach was used in Pekkarinen and Vartiainen (2006) and Gittings (2011). A second challenge is that post-promotion wage data are typically available only for promoted workers. However, as I discuss later in the section, a likelihood framework allows one to account for partially observed wage data (i.e. post-promotion wages observed only for promoted workers) in the same way missing wages are handled in the empirical labor supply literature.\(^{16}\)

To simplify the exposition, I ignore these two problems for the remainder of the paper by assuming the availability of a dataset for which the two problems do not arise. One such dataset

\(^{16}\) See Blundell and MaCurdy (1999), section 6.5.2 “Missing Wages”, page 1637.
is the Multi-City Study of Urban Inequality (MCSUI), on which the subsequent discussion is based. The MCSUI is a cross-sectional establishment telephone survey conducted between 1992 and 1995 in four metropolitan areas of the U.S. (Atlanta, Boston, Detroit, Los Angeles). These data were used in DeVaro (2006a,b) to conduct the two empirical tests, though the goal in those papers was only to test the implications of the classic tournament model rather than to distinguish among the three core promotion models. In light of the new theoretical results from section 2, the empirical results in the earlier papers can be given new interpretations, as discussed in the next section.

Each respondent employer in the MCSUI is asked questions pertaining to the establishment’s most recently hired worker. One question is whether the worker received a promotion by the time of the survey date. The data also include $P_i$, which is the employer-reported rating of worker $i$’s job performance in the job into which the worker was hired, where 0 is low, 100 is high, and 50 is average. Another variable, $P_{0i}$, is the employer-reported rating of the job performance of the “typical worker in that same job” (i.e. the original job held by worker $i$), on the same 100-point scale. This variable can be used as a proxy for the performance of worker $i$’s competitors at the same firm and job level. The data also contain the worker’s starting wage, which is the pre-promotion wage. The measurement of the post-promotion wage differs according to whether the worker was promoted by the survey data. If the worker was promoted, the worker’s current wage is used as the post-promotion wage. If the worker was not yet promoted, then the post-promotion wage is the employer’s answer to a question about what the worker’s wage (if promoted in the future) is expected to be. Worker $i$’s wage spread, $S_i$, is the difference between the post-promotion and pre-promotion wage. In addition to the key variables on promotions, $P_i$, $P_{0i}$, and $S_i$, the MCSUI data contain information on worker and firm characteristics that can be used as control variables.

### 3.2 Testing Whether Promotions Are Based on Relative or Absolute Performance

Using the preceding notation based on the MCSUI data on the most recently hired worker at each establishment, consider a probit model in which the dependent variable equals one if worker $i$ from establishment (or tournament) $i$ was promoted by the survey date and 0 if worker $i$ was not promoted. The key independent variables are $P_i$ and $P_{0i}$. If relative performance...
determines promotions, then the estimated coefficient of \( P_i \) should be positive, and that of \( P_{0i} \) should be negative, whereas if absolute performance determines promotions, then the coefficient of \( P_i \) should be positive and that of \( P_{0i} \) should be zero. I discuss empirical evidence from this test in section 4.

### 3.3 Testing the Opposing Responses Property

Given that the cross-tournament data that are typically available lack information on \( \theta_i \), testing the opposing responses property requires a simultaneous-equations empirical framework so that predictions regarding the unobserved \( \theta_i \) can be translated into a prediction about the sign of the cross-equation error correlation. I start by constructing an empirical model based on classic tournaments, given that this model motivates the largest share of the empirical literature on promotions, indicating throughout the discussion what parametric predictions would be implied by classic tournaments and what predictions would be implied by either of the other core models. Recall that the worker’s first-order condition in a classic tournament is \( g(0)S - C'(e^*) = 0 \) and that \( g(0) \) is inversely related to \( \theta \) when \( u \) is normally distributed. A linear approximation of this condition yields an estimating equation in which \( e^* \) is the dependent variable. But since data on effort are typically unavailable, whereas performance data are sometimes available, this linear approximation can be substituted into the production function, \( P = e^* + u \), yielding the following regression equation, where the equilibrium wage spread is \( S^* = W_m - W_s \):\(^{17}\)

\[
P_i = \alpha_0 + \alpha_1 S_i^* + \tau \theta_i + \epsilon_{1i}^*
\]

The opposing responses property requires \( \partial e^*/\partial \theta < 0 \), which implies \( \tau < 0 \).

Suppose the researcher has access to data on \( \theta \) across tournaments, even though this would rarely be the case. Then the researcher could estimate the preceding regression and then test the “first half” of the opposing responses property (i.e. the part pertaining to worker behavior) by testing the null hypothesis \( \tau = 0 \) against the alternative \( \tau < 0 \), using a one-tailed test. Rejecting the null would favor classic tournaments, market-based tournaments or the performance standards model with endogenous worker choices, whereas failing to reject it would favor the performance standards model with no worker choice.

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\(^{17}\) Control variables are suppressed throughout the discussion.
There are two problems with this test. First, data on $\theta_i$ spanning multiple tournaments would not normally be available given that $\theta_i$ is inherently difficult to measure, so $\theta_i$ would be subsumed in the regression disturbance term, yielding the model:

$$P_i = \alpha_0 + \alpha_1 S^*_i + \epsilon_{1i}$$

where $\epsilon_{1i} = \tau \theta_i + \epsilon_{1i}^*$. This regression is unhelpful for testing the “first half” of opposing responses, since $\tau$ cannot be estimated. Second, $S^*$ appears on the right-hand side of the regression even though it is endogenous in the theoretical model. This is a problem because factors unobserved to the econometrician are likely to affect both $S^*$ and $P_i$, biasing all of estimated coefficients in the model. This is a problem in both of the preceding regressions, but it is a particular problem in the more practically relevant second regression, because $\theta$ appears in $\epsilon_1$ and is correlated with $S^*$ by the first-order conditions of the classic model.

With the exception of DeVaro (2006a,b) the previous literature has not addressed this second problem and has estimated the preceding regression treating $S^*$ as exogenous. Although alone this regression is unhelpful for testing opposing responses, it has been used in previous work to test whether classic tournaments have incentive effects, i.e. $\alpha_1 > 0$ as implied by $\partial e^*/S > 0$. The typical finding in such studies is $\alpha_1 > 0$ which is interpreted as evidence that tournaments have incentive effects. Even setting aside the endogeneity problem, the prediction $\alpha_1 > 0$ is unhelpful for distinguishing among the core promotion models, because it is common to all of them. If promotions are associated with wage increases, and if workers can take performance-enhancing actions that increase the probability of promotion, then incentives are automatically implied regardless of whether managerial job slots are fixed or flexible and regardless of the mechanism generating wage spreads.

Whereas empirical tests of $\alpha_1 > 0$ are based on worker behavior, another body of previous empirical work on classic tournaments has focused instead on firm behavior.\(^\text{18}\) Recall that in classic tournaments the optimal wage spread is $W_m - W_s = 1/g(0)$, where $g(0)$ is inversely related to $\theta$ when $u$ is normally distributed. A linear approximation of this condition yields the following regression model:

\(^{18}\) Representative studies include O’Reilly, Main, and Crystal (1988), Lambert, Larcker, and Weigelt (1993), Main, O’Reilly, and Wade (1993), Eriksson (1999), and Bognanno (2001).
$S_i^* = \beta_0 + \varphi \theta_i + \varepsilon_{2i}^*$

The classic model predicts $\varphi > 0$, given that the first-order conditions imply $\partial S^*/\partial \theta > 0$. Thus, given data on $S_i^*$ and $\theta_i$ across tournaments, a researcher could test the “second half” of the opposing property (i.e. the part related to firm behavior) by estimating the preceding regression and testing the null hypothesis $\varphi = 0$ against the alternative that $\varphi \neq 0$, using a two-tailed test. Failure to reject the null would favor the performance standards model. Rejecting the null and finding $\varphi > 0$ would favor classic tournaments, whereas rejecting the null and finding $\varphi < 0$ would favor market-based tournaments. However, since data on $\theta_i$ are typically unavailable, the term $\varphi \theta_i$ is subsumed in the regression error, yielding:

$S_i^* = \beta_0 + \varepsilon_{2i}$ where $\varepsilon_{2i} = \varphi \theta_i + \varepsilon_{2i}^*$.\(^{19}\)

This regression alone is unhelpful for testing opposing responses, since $\varphi$ cannot be estimated.

The bottom line is that when $\theta_i$ is unobserved by the econometrician – which is the practically relevant case – neither the performance regression estimated alone nor the spread regression estimated alone allows the opposing responses property to be empirically tested. I now show that if the two equations are estimated jointly then under some identifying assumptions the parameter $\sigma_{12} \equiv \text{cov}(\varepsilon_{1i}, \varepsilon_{2i})$ can be estimated, and the opposing responses property combined with the fact that $\theta_i$ is an important common component of $\varepsilon_{1i}$ and $\varepsilon_{2i}$ allows a prediction on the sign of $\sigma_{12}$. Let $(\varepsilon_{1i}, \varepsilon_{2i})$ follow the bivariate normal distribution with mean vector $\mathbf{0}$ and covariance matrix $\mathbf{\Sigma}$. Note that $\sigma_{12} = \tau \varphi \text{Var}(\theta_i) + \text{cov}(\varepsilon_{1i}^*, \varepsilon_{2i}^*)$, where $\text{Var}(\theta_i)$ is the variance (across tournaments in the sample) of the variance (for a particular tournament in the sample) of $u$. The parameter $\theta$ plays a critical role in all of the core models, so from a theoretical standpoint $\theta_i$ can be expected to be the principal common component of both $\varepsilon_{1i}$ and $\varepsilon_{2i}$ given that the other important model components (i.e. the wage spread and worker performance) are already included in the model as observed data. When the simultaneous equations model is properly specified with a complete set of controls, then $\text{cov}(\varepsilon_{1i}^*, \varepsilon_{2i}^*) \cong 0$. Given $\text{cov}(\varepsilon_{1i}^*, \varepsilon_{2i}^*) \cong 0$, $\sigma_{12} \cong \tau \varphi \text{Var}(\theta_i)$, so the predicted sign of $\sigma_{12}$ hinges on the signs of $\tau$ and $\varphi$.

\(^{19}\)Note that linearity of $\varepsilon_{1i}$ and $\varepsilon_{2i}$ in $\theta_i$ is not essential.
The two requirements of opposing responses, i.e. \( \tau < 0 \) and \( \varphi > 0 \), imply \( \tau \varphi \text{Var}(\theta_i) < 0 \) and thus \( \sigma_{12} < 0 \). In contrast, in the performance standards model with worker effort or with no worker choice, \( \varphi = 0 \) implies \( \sigma_{12} = 0 \). And in the market-based tournament model, \( \tau < 0 \) and \( \varphi < 0 \) imply \( \tau \varphi \text{Var}(\theta_i) > 0 \), and thus \( \sigma_{12} > 0 \), distinguishing this model from classic tournaments.

Thus, even in the absence of data on \( \theta \), one can test the opposing responses property by estimating the P and S* equations jointly and then testing the null hypothesis that \( \sigma_{12} = 0 \) against the alternative \( \sigma_{12} \neq 0 \), using a two-tailed test. Rejecting the null in the direction of \( \sigma_{12} < 0 \) favors classic tournaments, rejecting the null in the direction of \( \sigma_{12} > 0 \) favors market-based tournaments, and failing to reject the null favors performance standards.

There are two points regarding the controls for worker and firm characteristics that would in practice be included in the regressions for P and S*. First, given that the models are linear, identification of the P equation requires that at least one exogenous variable appearing in the S* equation be excluded from the P equation, a point I discuss in more detail in the next section. Second, the ideal specification of the econometric model is one that minimizes unobserved heterogeneity (across tournaments in the sample) in dimensions other than risk (by including a sufficient set of controls) but *maximizes* heterogeneity across tournaments in the degree of risk. This is because identification of \( \sigma_{12} \) is based on cross-tournament variation in \( \theta_i \) that is unobserved by the econometrician but observed by economic agents. Samples that mix tournaments that are expected to differ widely in their levels of risk are ideal from the standpoint of testing the opposing responses prediction, since then \( \sigma_{12} \) will be large in magnitude under the alternative hypothesis, increasing the likelihood that the null of \( \sigma_{12} = 0 \) will be rejected when it is false. If the econometrician selects samples of tournaments that are too homogeneous with respect to risk, if the model includes insufficient controls, or if controls are included that proxy for risk, the test will be biased towards a finding of \( \sigma_{12} = 0 \). However, in the case of controls that proxy for risk being included in the model, the researcher’s attention should shift from a statistical test on \( \sigma_{12} \) to statistical tests on \( \tau \) and \( \varphi \) as discussed earlier in the subsection, since in such cases risk is (at least partially) observable.
3.4 Testing Relative Performance and Opposing Responses Simultaneously

As in the preceding two subsections, assume the setting involves MCSUI data on one worker (and tournament) per firm, with \( i \) indexing both the worker and the firm, though the method could also be applied using panel data or cross sectional data with multiple workers per firm. Consider the following system:

\[
P_i = \alpha_0 + \alpha_1 S_i^* + F_i \alpha_2 + X_i \alpha_3 + \epsilon_{1i}
\]
\[
S_i^* = \beta_0 + F_i \beta_1 + X_i \beta_2 + \epsilon_{2i}
\]
\[
T_i^* = \gamma_0 + F_i \gamma_1 + X_i \gamma_2 + \gamma_3 P_0 + \epsilon_{3i}
\]
\[
\text{Promote}_i = 1 \text{ if } P_i - T_i^* \geq 0 = 0 \text{ otherwise}
\]

where \( P_i \) denotes worker \( i \)'s performance in the lower-level job, \( S_i^* \) denotes the wage spread between worker \( i \)'s (low) job level and the next level up, \( T_i^* \) denotes the minimal performance threshold worker \( i \) must meet for promotion, Promote\(_i \) is a binary promotion indicator, \( X_i \) is a vector of worker characteristics, \( F_i \) is a vector of firm characteristics, \( P_0 \) denotes the performance of worker \( i \)'s competitors, and the \( \epsilon \)'s follow a multivariate normal distribution with mean vector 0 and covariance matrix \( \Sigma \). Note that for identification, at least one variable in either \( F_i \) or \( X_i \) must be excluded from the \( P_i \) equation. The model can be estimated via maximum likelihood. A likelihood framework easily allows the researcher to account for the fact that in most datasets \( S_i^* \) will be only partially observable, i.e. it is observed only for promoted workers. In this case, the wage spread can be integrated out of the likelihood function when it is unobserved.\(^{20,21}\)

If relative performance determines promotions then \( \gamma_3 > 0 \), whereas if absolute performance determines promotions then \( \gamma_3 = 0 \). Within this empirical framework, the first three columns of Table 2 illustrate the parametric predictions implied by the core promotion models. Two points are worth noting. First, if the goal is simply to conduct the relative-versus-absolute performance test, then the promotion probit can be estimated in isolation rather than jointly with

\(^{20}\) This problem did not arise in DeVaro (2006a,b), since in the MCSUI data a measure of \( S_i^* \) was available for all observations. For non-promoted workers, the post-promotion wage was based on a survey question asking the respondent employer what the most recently hired worker’s expected wage would be if this worker is promoted.\(^{21}\) Maximum likelihood estimation of the three-equation system (either for the case of partially observable or completely observable \( S_i^* \)) is straightforward in STATA using the command “cmp,” which stands for “conditional mixed process.” The command estimates multi-equation conditional (recursive) mixed process models. The command is simple and flexible, allowing the user considerable control in managing the optimization routine.
the P and $S^*$ equations, as in subsection 3.2. Second, the P and $S^*$ equations must be estimated jointly, since otherwise $\sigma_{12}$ cannot be estimated. An additional reason for conducting joint estimation is that if the P equation is estimated individually, as is typically done in the literature, then the estimate of $a_1$ (i.e. incentive effects of promotions) will be biased.

4. Empirical Evidence

DeVaro (2006a,b) used the MCSUI data to provides empirical evidence from both tests, interpreting the results only in the context of classic tournaments. Given the two new results from section 2, the evidence from both earlier papers can be given new interpretations. DeVaro (2006b) considered a subsample of 632 “skilled” workers, whereas DeVaro (2006a) considered a more highly-skilled sample of 215 professionals. As noted in section 3, identification of the P equation requires excluding at least one exogenous variable in the P equation that appears in the $S^*$ equation. DeVaro (2006b) considers two alternative exclusion restrictions that yield the same qualitative results. The first and least stringent of the two excludes one variable in the vector $F_i$ from the $P_i$ equation, namely the percent of the establishment’s workforce that is covered by a collective bargaining agreement. The rationale is that the effects of unions on the wage offerings of firms are clear, well-documented, and expected to be of first-order importance, whereas any direct effects of unions on worker effort – should they exist – are likely to be small by comparison. The second exclusion is a much stronger restriction, eliminating from the $P_i$ equation all firm characteristics ($F_i$) that appear in the $S_i^*$ equation. DeVaro (2006a) used only the stronger of the two restrictions, since the sample size was not large enough to support estimation using the less stringent one. Although this is a very strong identifying assumption, one justification for it derives from the fact that the data are a sample of recently hired workers. In addition to knowing little about the firm, the typical new hire will have experience with only a small number of previous employers, if any at all. In contrast, the firm possesses the relevant institutional history of the organization and knows how certain worker-types perform in given positions. This puts the firm at an advantage relative to a newly hired worker in assimilating information about firm characteristics into a decision function.

DeVaro (2006a,b) both found evidence of $\gamma_3 > 0$, suggesting that promotions are based on relative performance, so I restrict the following discussion to the two tournament models and to
the evidence concerning \( \sigma_{12} \). Both studies also found evidence of \( \alpha_1 > 0 \), consistent with the incentive effects of promotions that have been documented in the previous literature, and the simultaneous-equations estimation approach in these two studies allowed for unbiased estimation of this parameter.\(^\text{22}\)

First consider DeVaro (2006b), which found \( \sigma_{12} < 0 \) and interpreted this evidence as consistent with classic tournaments.\(^\text{23}\) In light of proposition 2, the empirical result \( \sigma_{12} < 0 \) conflicts with the prediction \( \sigma_{12} > 0 \) of market-based tournaments. Waldman (2012) surveys the extensive empirical literature testing the predictions of the classic tournament model and concludes that most of the findings in the literature are consistent with that approach but that all but one of those predictions are also consistent with the market-based approach. The one exception he notes is empirical evidence of a convex wage structure across hierarchical levels. This prediction arises naturally from the classic model as shown in the multi-round analysis by Rosen (1986). In contrast, Waldman notes that although there are no multi-round market-based tournament models, there is no clear reason why convex wage structures should emerge as a robust prediction of market-based tournaments. I am reluctant to interpret this difference between the two models as offering a distinguishing empirical test, for two reasons. First, the market-based approach has not been developed in a multi-round context that would yield a prediction regarding convexity or the lack of it; so whereas classic tournaments predict convexity, market-based tournaments currently offer no prediction. Second, Waldman (2012) predicts that even the market-based model could generate a convex wage structure if the signal associated with a promotion was larger for promotions higher up the job ladder. The present

\[^{22}\] Other representative studies that found evidence that performance is increasing in the size of the prize include Ehrenberg and Bognanno (1990a,b), Becker and Huselid (1992), and Audas, Barmby, and Treble (2004).
\[^{23}\] Specifically, DeVaro (2006b) found \( \sigma_{12} = -0.091, \gamma_3 = 1.496, \) and \( \alpha_1 = 0.591 \), where the estimate of \( \gamma_3 \) was statistically significant at the one percent level and the other two parameters were significant at the ten percent level. In subsequent work concerning the difference between nonprofits and for-profits, DeVaro and Brookshire (2007) found that when the 81 nonprofits were dropped from the sample of 632 used in DeVaro (2006b), the results strengthened slightly from the standpoint of classic tournament theory, i.e. \( \sigma_{12} = -0.118, \gamma_3 = 1.633, \) and \( \alpha_1 = 0.709 \), with the estimate of \( \alpha_1 \) attaining statistical significance at the five percent level and the other two estimates remaining at their original significance levels. The fact that the results are somewhat stronger when the sample is restricted to for-profits is consistent with the theory proposed in DeVaro and Brookshire (2007) that for-profits are more likely than nonprofits to rely on promotion schemes such as classic tournaments to create incentives.
analysis offers a clear distinction between the two types of tournaments, given that one predicts a negative sign on $\sigma_{12}$ and the other a positive sign.

Next consider DeVaro (2006a). That study also found $\sigma_{12} < 0$, though the result was statistically insignificant so the null hypothesis of $\sigma_{12} = 0$ could not be rejected. I will offer two interpretations of this result in light of the present analysis. The first interpretation is that the cross section represents a mix of the two types of tournaments. Estimating $\sigma_{12}$ requires data spanning multiple promotion systems, which would usually require data spanning multiple firms. Suppose that in the population, some firms conduct market-based tournaments (for which $\sigma_{12} > 0$) whereas others conduct classic tournaments (for which $\sigma_{12} < 0$). When the different types of firms are pooled in the cross section, their relative prevalence will determine the estimate of $\sigma_{12}$. The former type of tournaments would tend to increase $\sigma_{12}$ in estimation, whereas the latter type would tend to decrease $\sigma_{12}$, so that the two effects are roughly offsetting, producing a negative but statistically insignificant estimate of $\sigma_{12}$. This interpretation would say that in the more highly skilled subsample of professionals the market-based mechanism is a more powerful force than in the less skilled subsample. Now consider a second interpretation. I view the results in DeVaro (2006a) concerning $\sigma_{12}$ as less reliable than the corresponding result in DeVaro (2006b), because the latter study used a considerably larger sample size, a more complete set of controls, an econometric model that was generalized to allow for measurement errors in $S^*$ and $P$, and a less stringent identifying assumption. These factors could potentially explain the lack of statistical significance of $\sigma_{12} < 0$ in the professional subsample. In that case, the true results for professionals might match those for skilled workers, even though the statistical result is not strong enough to confirm this.

In summary, the evidence in DeVaro (2006b) was interpreted as consistent with classic tournaments. Propositions 1 and 2 allow a new interpretation. The evidence is now seen to be inconsistent with performance standards and also with market-based tournaments. Performance standards predict $\gamma_3 = 0$ and $\sigma_{12} = 0$ (or $\sigma_{12} \geq 0$), whereas $\gamma_3 > 0$ and $\sigma_{12} < 0$ were found in the data. And market-based tournaments predict $\sigma_{12} > 0$, whereas $\sigma_{12} < 0$ was found in the data. However, as shown in the next section, an alternative market-based tournament model in which
workers make human capital investments rather than choosing effort levels would also be consistent with the evidence in DeVaro (2006b).

5. Market-Based Tournaments with Endogenous Worker Human Capital Investments

This section is based on the original market-based tournament model, due to Zábojník and Bernhardt (2001) and recently extended in the closely related analysis of Zábojník (2012), in which the worker chooses a human capital investment rather than an effort level. There are two new results. First, the two alternative ways to model market-based tournaments (i.e. worker effort choices or worker human capital investments) can be distinguished using opposing responses. Second, opposing responses cannot distinguish classic tournaments from market-based tournaments with human capital investments, meaning the Zábojník-Bernhardt model can potentially explain the evidence in DeVaro (2006b). The theoretical foundation for both results is proposition 3, which takes a result from Zábojník and Bernhardt (2001) concerning two offsetting effects on the worker’s optimal human capital investment level in response to a change in \( \theta \) and generalizes it to a broader class of distributions.

Consider a two-level job hierarchy with one managerial position and two subordinate positions so that promotions are determined by relative performance. Firms and workers are risk-neutral with common discount factor \( \delta < 1 \). There are two periods. In period 1, worker i chooses a level of human capital investment, \( h_i \), to maximize expected utility. The investment is made at a cost \( C(h_i) \), where \( C \) is a standard cost function as defined in section 2. Subordinate i’s total human capital is given by \( h_i' = h_i + u_i \), where \( u_i \) is a stochastic component distributed uniformly on \( [\alpha, \beta] \), \( \alpha > 0 \), and \( \theta = \text{Var}(u_i) = (\beta - \alpha)^2/12 \). Subordinate j’s problem is symmetric, and \( u_i \) and \( u_j \) are independent and drawn from the same distribution. Zábojník and Bernhardt (2001) interpret the shocks \( u_i \) and \( u_j \) as reflecting human capital developed passively through learning by doing. Firms are characterized by productivity \( p \)arameters, \( V \), which can be thought of as the price at which output is sold, and that can assume either a high value, \( V_H \), or a low value, \( V_L \), where \( 0 < \)

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24 The importance of human capital investments in determining promotions was stressed earlier in Carmichael (1983) and Prendergast (1993). Empirical support for the idea was found in a study of promotions in a major American fast-food retailer (Campbell 2008). Zábojník (2012) notes that a reason for focusing on human capital rather than effort as the choice variable in market-based tournament models is that the conflict between the assignment and incentives functions of promotions (see the first paragraph of the introduction) is less pronounced in the case of human capital investments than in the case of effort. This is because promoting the highest-skilled worker fits well with both functions of promotions.
A subordinate who has chosen human capital $h'$ and remains with the original employer (with productivity parameter $V$) in the second period contributes $V y h'$ to the firm’s revenue, where $\gamma > 1$. If instead the subordinate switches in the second period to an employer with productivity parameter $V'$, the worker contributes $V' y h'$ to the new employer’s revenues, i.e. $\gamma - 1$ captures the firm-specific part of the worker’s human capital that is lost when switching employers.\footnote{The model features a blend of general and firm-specific human capital. In a market-based tournament, some of the human capital must be general so that competing firms value it and will be inclined to bid more for workers suspected of having acquired it, which is the key mechanism generating wage dispersion across hierarchical levels. This is also true in a market-based performance standards model with endogenous human capital investments, as in DeVaro, Ghosh, and Zoghi (2012). In contrast, if the wage setting mechanism is classic, so that firms can credibly pre-commit to wage spreads, then the human capital can be entirely firm specific as in the performance standards model of Prendergast (1993).}

At the end of the first-period, the firm privately observes each subordinate’s $h'$ and promotes the one with the higher value to the managerial position in the second period, retaining the other worker as a subordinate.\footnote{Other papers considering how asymmetric information in labor markets affects human capital investments are Waldman (1990), Chang and Wang (1996), and Acemoglu and Pischke (1998).} As in Waldman (1984a), competing firms do not observe each worker’s $h'$ but do observe second-period job assignments. Based on that observation, competing firms then formulate expectations about each worker’s level of accumulated human capital, $h'$, that directly affects the worker’s second-period productivity. Second-period wages for the manager and subordinate are determined by spot market contracting at the start of the second period. The worker’s current employer and competing firms simultaneously make wage bids, and workers accept the highest offer, switching firms only for a strictly higher wage. Under some parametric restrictions, there is no turnover in equilibrium because the worker’s employer increases the promoted worker’s wage sufficiently to prevent a separation, thereby generating a wage difference across hierarchical levels. Thus, in this market-based tournament model as in that of section 2, incentives are created by the anticipation of workers that wage spreads will emerge following a promotion, whereas in a classic tournament incentives arise from the firm’s pre-commitment to a wage spread before the workers invest.

Two important differences between this market-based tournament model and the other are worth highlighting. First, when the worker’s first-period choice variable is a human capital
investment, it directly translates into increased second-period productivity, whereas in the other market-based tournament the subordinate’s first-period effort choice does not directly translate into higher second-period productivity. This matters given that second-period wages are determined by the beliefs of competing firms concerning the workers’ second-period productivities. Second, like the worker’s first-period choice variable, the stochastic component of the worker’s first-period human capital is also persistent in that high values of \( u_i \) translate into high values of \( h_i^* \), which directly determine second-period productivity. The stochastic term is also strictly positive and thus always productivity-enhancing, consistent with its interpretation of “passive learning by doing”, whereas in the other model the \( u_i \) is a mean-zero “luck” term that affects the worker’s first-period output (either positively or negatively) but does not persist.  

The difference in interpretations of \( u_i \) between the two market-based tournaments is potentially important given that the empirical test based on opposing responses assumes that the interpretation of \( u_i \) is similar across models being compared, so that the effects of changes in \( \theta \) can be meaningfully compared across models. Given that the interpretation of \( u_i \) in the other market-based model matches that of the classic tournament model (i.e. both are mean-zero performance shocks that do not persist) whereas the interpretation of \( u_i \) in the market-based model of this section is different (i.e. it is strictly positive and persistent), for the purpose of comparing classic and market-based tournaments it might be more appropriate to focus on the market-based model with effort choices as in the preceding sections.

Zábojník and Bernhardt (2001) show that in equilibrium a worker’s second-period wage equals his expected productivity in a competing firm conditional on his job assignment with the first-period employer. The equilibrium wages for managers and subordinates are \( W_m(V) = V_H(h^*(V) + E[u_{(2)}]) \) and \( W_s(V) = V_H(h^*(V) + E[u_{(1)}]) \), where \( u_{(k)} \) denotes the \( k^{th} \) order statistic of two draws from the distribution of \( u \), so the equilibrium wage spread is \( S^* = W_m - W_s = V_H(E[u_{(2)}] - E[u_{(1)}]) \). Note that because the stochastic shocks are persistent in this model, expectations about these shocks influence the bids made by competing firms, which in turn

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\[27\] These distinctions between the two types of market-based models arise because both are two-period models, creating the possibility of differential impacts of first-period choice variables (and shocks) on second-period productivities.
influence the equilibrium wage spread. In the symmetric pure strategy equilibrium, both workers choose the same optimal level of h, denoted $h^*$ and defined by the first-order condition:

$$\delta(W_s - W_m)q(0) = C'(h^*),$$

where $q(0) \equiv \int_a^\beta \int_a^\alpha f(u) du \cdot$. This is the first-order condition from the classic two-player tournament model of Lazear and Rosen (1981), except that now the wage spread is determined \textit{ex post} by labor market competition rather than being set strategically by the firm \textit{ex ante} to induce effort.

As in the models discussed earlier, the worker’s choice variable is increasing in the wage spread, i.e. $\partial h^*/\partial(W_m - W_s) > 0$. This result says simply that wage spreads from promotion have incentive effects in that larger prizes motivate higher levels of investment on the part of subordinates. Now consider the responses of $h^*$ and $S^*$ to changes in $\theta$. Recall that in the classic tournament model, an increase in $\theta$ holding the spread constant implied a decrease in effort, i.e. $\partial e^*/\partial \theta < 0$. At the same time, the increase in $\theta$ implied an increase in the equilibrium wage spread, i.e. $\partial S^*/\partial \theta > 0$. Given that $\partial e^*/\partial(W_m - W_s) > 0$, this increase in the spread works in the direction of increasing $e^*$. In the classic model these two opposing effects on $e^*$ of an increase in $\theta$ were exactly offsetting. Zábojník and Bernhardt (2001) showed that given that $u$ has the uniform distribution, the same result of exactly offsetting effects holds in the market-based model, so that $h^*$ remains unchanged in response to changes in $\theta$.\textsuperscript{28} In the market-based tournament with endogenous human capital investments, an increase in risk has two effects. First, it decreases the marginal effect of human capital on the probability of promotion – i.e. decreases $q(0)$ – just as in the classic model it decreases $g(0)$, and this tends to decrease $h^*$. Second, it increases the spread, which tends to increase $h^*$. The reason why the spread is increasing in $\theta$ can be understood by recalling that the equilibrium spread is $V_{th}(E[u_{(2)}] - E[u_{(1)}])$ and noting that the difference in the expectations of the first two order statistics is increasing in the variance of the underlying random variable. Intuitively, if the variance of $u$ is small, then the expected difference in productivities between the tournament winner and loser will be small in the symmetric equilibrium, which in turn justifies a small wage spread.

\textsuperscript{28} Given the uniform distribution, the worker’s first-order condition can be rewritten as $C'(h^*) = \delta V/3$, in which $\theta$ does not appear.
Although the market-based model exhibits the same property of offsetting effects of changes in θ on the worker’s choice variable as the classic model, it does so under more restrictive distributional assumptions. In particular, Zábojník and Bernhardt (2001) assumed a uniform distribution on [α,β]. In the following proposition I generalize the result.

**Proposition 3**: Assume that u has an arbitrary, continuous, symmetric distribution on support [α,β]. Then the effects of changes in θ on q(0) and \( W_m - W_s \) are exactly offsetting, so \( h^* \) does not depend on θ.

The importance of Proposition 3 can be understood as follows. A main result is that the model of this section, like the classic tournament model, exhibits the property of opposing responses. Zábojník and Bernhardt (2001) showed that the same result holds in the market-based model if u is uniform on [α,β] and α is sufficiently positive, but absent Proposition 3 it would be unclear how robust this prediction is and whether it is an artifact of the uniform distribution (in which case it would be misleading to argue that the model exhibits the same property as classic tournaments). By weakening the distributional assumptions, Proposition 3 strengthens the conclusion that both models exhibit the opposing responses property.\(^{29}\)

I now summarize the key points of this section. First, in the market-based tournament model with endogenous human capital investments, promotions are determined by relative performance. Second, in contrast to the market-based tournament model with effort choices, the present one exhibits opposing responses, i.e. \( \partial h^*/\partial \theta < 0 \) and \( \partial S^*/\partial \theta > 0 \).\(^{30}\) One important implication of this is that the two different approaches to modeling market-based tournaments (i.e. worker effort choices versus worker human capital investments) are distinguished by the

\(^{29}\) The results still rely on a bounded support, in contrast to the classic tournament model or the market-based tournament model with effort choices. Zábojník and Bernhardt (2001) assume that α is strictly positive and sufficiently high so that there is no turnover in equilibrium, i.e. \( \alpha \geq \beta V_H / (\gamma V_L) \). Zábojník (2012) imposes a less stringent lower bound on α which allows it to be negative, i.e. \( \alpha \geq -C' (\delta V_H q(0)(E[u_{2}\mid q(0)] - E[u_{1}\mid q(0)]) \). The purpose of the latter assumption is to ensure that the worker’s human capital is always non-negative.

\(^{30}\) The sign of \( \partial S^*/\partial \theta \) differs between the two types of market-based tournaments because the interpretation of the stochastic shock, u, differs between them. In the market-based tournament with human capital investments, since the shocks are persistent (e.g. passive learning by doing) and competing firms know this, expectations about these shocks influence competing firms’ wage bids, which in turn influence the equilibrium wage spread. In contrast, in the market-based tournament with effort, since the shocks are not persistent they are not part of the expectations that competing firms form about the productivity the worker would have in a competing firm. Hence, the shocks do not affect competing firms’ expectations of workers’ productivities conditional on observed job assignments.
property of opposing responses. Another important result is that the relative-versus-absolute performance and opposing responses tests are insufficient for distinguishing market-based tournaments with human capital investments from classic tournaments with effort, and in the next section I propose a new way in which these two models can be distinguished in future work.

6. A New Test to Distinguish Between Classic and Market-Based Tournaments

The basic logic for the new test developed in this section resembles the opposing responses test in that I show that the two tournament models differ in how the choice variables of workers and the firm respond to shifts in a theoretical parameter. However, in contrast to the opposing responses test the parameter will no longer be \( \theta \), and we will need to consider a different function of the firm’s choice variables. Like the opposing responses test, systems-based estimation methods are needed, though the actual system of equations differs.

Focusing on two-player tournaments for simplicity, begin by noting that both tournaments feature a strictly convex cost function, \( C(z) \), with \( C(0) = 0 \) and \( C'(z) = 0 \), capturing the subordinate’s disutility of the choice variable, \( z \). Let \( \xi \) be a parameter that affects \( C(z) \), either increasing or decreasing the worker’s total and marginal costs of \( z \), and assume that \( \xi \) is observed by workers and the firm when their decisions are made. If \( \xi \) is incentives-enhancing then \( \partial C(z)/\partial \xi < 0 \) and \( \partial C'(z)/\partial \xi < 0 \), whereas if \( \xi \) is incentives-depressing then \( \partial C(z)/\partial \xi > 0 \) and \( \partial C'(z)/\partial \xi > 0 \). Without loss of generality, I consider the incentives-enhancing case henceforth. Note that \( \xi \) is a theoretical parameter that is assumed to vary across tournaments in cross sectional data. The test requires that the researcher have data on at least one measure of \( \xi \). As an example of such a variable, some companies (e.g. Google, Adobe, and SAS) provide their employees with free or subsidized snacks and meals throughout the day. One of the rationales for such practices is that they can be expected to lower workers’ costs of exerting productive effort. Thus, one measure of \( \xi \) would be the generosity of employer snack and meal subsidies.

Given a measure of \( \xi \), one might be tempted to follow the same approach as in earlier sections, but using \( \xi \) rather than \( \theta \). That is, one would first determine whether the responses of effort and the spread to changes in \( \xi \) differed between the two types of tournaments. Assuming the pattern of responses of the choice variables to changes in \( \xi \) differed
between the two models, one would then go to the data to see which of the models was empirically supported. It turns out that this approach would not work because, as I show below, the responses of effort and the spread to changes in \( \xi \) do not differ between the two models.

However, suppose that instead of focusing on the wage spread, i.e. \( S^* \equiv W_m - W_s \), we focus on the average wage between job levels, which is a subordinate’s expected wage, i.e. \( L^* \equiv (W_m + W_s)/2 \). The main result is given in the following proposition, showing that the response of \( L^* \) to shifts in \( \xi \) differs between the two models, producing a testable implication:

**Proposition 4**: When \( \xi \) is incentives-enhancing the following results hold concerning the equilibria of both tournament models: \(^{31}\)

(i) In both models, \( \partial z^*/\partial \xi > 0 \) and \( \partial S^*/\partial \xi = 0 \).

(ii) In the market-based model with human capital choices, \( L^* \) is a linear function of \( h^* \), and \( \partial L^* / \partial \xi = V_h \partial h^* / \partial \xi > 0 \), so that \( \xi \) influences \( L^* \) only via its effect on \( h^* \).

(iii) In the classic model with effort choices, \( L^* \) is a nonlinear function of \( e^* \), and \( \partial L^* / \partial \xi = C'(e^*) (\partial e^* / \partial \xi) + \partial C(e^*) / \partial \xi \), so that \( \xi \) influences \( L^* \) both via its effect on \( e^* \) and also directly via its effect on \( C(e^*) \) for a given \( e^* \).

Point (i) says that the responses of \( z^* \) and \( S^* \) to changes in \( \xi \) do not differ between the two models. This means that a distinguishing empirical test based on \( \xi \) cannot be based on the choice variables \( z^* \) and \( S^* \). Points (ii) and (iii) create the basis for a distinguishing test based on responses in the choice variable \( L^* \) (rather than \( S^* \)) to changes in \( \xi \). Point (ii) says that in the market-based model with human capital investments, changes in \( \xi \) influence the average wage only by affecting the worker’s optimal choice of human capital. In contrast, point (iii) says that in the classic model, via the worker’s participation constraint \( L^* = C(e^*) \), changes in \( \xi \) influence the average wage both by affecting the worker’s optimal effort choice and by directly affecting the worker’s effort cost function. \(^{32}\) The basic logic for the difference between the two models in

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\(^{31}\) The proposition could easily be recast to handle the incentives-depressing case.

\(^{32}\) For example, suppose \( C(z) = az^*/\xi \), where \( \xi > 0 \), \( a > 0 \), and \( \lambda > 1 \), so \( C(z) = a \lambda z^{\lambda-1}/\xi \) and \( \partial C(z)/\partial \xi = -a \lambda z^{\lambda-2}/\xi^2 \). In the classic model, \( L^* = ae^{\lambda-1}/\xi \) where \( e^* = [\xi S(0)/(a \lambda)]^{1/(\lambda-1)} \), so that \( \xi \) affects \( L^* \) both directly and also indirectly via its effect on \( e^* \). Given that (i) \( C(e^*) \) is zero at \( e^* = 0 \) and positive and increasing in \( e^* \) for \( e^* > 0 \), (ii) \( \partial C(e^*)/\partial \xi \) is positive, and (iii) \( \partial C(e^*)/\partial \xi \) is zero for \( e^* = 0 \) and negative for \( e^* > 0 \), we have that \( \partial L^* / \partial \xi \) is 0 when \( e^* = 0 \), then increasing in \( e^* \) up to a peak where \( \partial^2 L^* / (\partial e^* \partial \xi) = 0 \), then monotonically decreasing in \( e^* \) thereafter so that \( \partial L^* / \partial \xi > 0 \) for \( e^* \) sufficiently large. So in the classic model the effect on the subordinate’s expected wage of a marginal increase in \( \xi \), i.e. \( \partial L^* / \partial \xi \), is non-monotonic in \( e^* \). For sufficiently low levels of \( e^* \), it is positive and increasing in \( e^* \). Then it is positive and decreasing in \( e^* \), and ultimately negative and decreasing in \( e^* \). In contrast, in the market-based model \( \partial L^* / \partial \xi = V_h[\xi S(0)/(a \lambda)]^{1/(\lambda-1)} > 0 \), which is not a function of \( h^* \).
how $L^*$ responds to changes in $\xi$ is that in the classic model, unlike in the market-based model, the average wage is determined by the worker’s participation constraint, i.e. $L^* = C(e^*)$. From the participation constraint, the two channels of influence of $\xi$ on $L^*$ are immediately apparent, given that shifts in $\xi$ change the function, $C(e)$, and also the equilibrium effort choice, $e^*$. In contrast, in the market-based model, since in equilibrium the average wage is not determined by the worker’s participation constraint, the worker’s cost function is of no direct relevance to $L^*$. Thus, shifts in $\xi$ affect $L^*$ only by changing the equilibrium human capital investment, $h^*$.

I now translate Proposition 4 into a testable implication, which requires a systems-based estimation approach. To simplify the discussion, I suppress control variables. As a further simplification, I assume the case of completely observable $L^*$, i.e. $L^*$ is observed even for non-promoted workers, as in the MCSUI data described earlier. Starting with the market-based model, recall that in equilibrium $L_i^* = V_h(h^*(V) + 0.5(E[u_{i(2)}] + E[u_{i(1)}]))$, where $i$ indexes firms (or tournaments) in a cross section. Note in this expression that a subscript $i$ appears on the expected values of the order statistics of $u_i$ given that the variance of $u_i$, i.e. $\theta_i$, varies across tournaments in the sample. Thus, the term $0.5(E[u_{i(2)}] + E[u_{i(1)}])$ gets subsumed into the regression disturbance given that the econometrician typically cannot observe $\theta_i$, as was the case for $\theta_i$ in section 3. This yields the following regression:

$$L_i^* = \omega_0 + \omega_1 h_i^* + \varepsilon_{1i}$$

Since $h_i^*$ is endogenous, there is a second equation:

$$h_i^* = \eta_0 + \eta_1 S_i^* + \eta_2 \xi_i + \varepsilon_{2i}.$$  

The spread, $S_i^*$, is also endogenous, so there is a third equation describing it, as in section 3. Given point (i) of Proposition 4, $\xi_i$ does not appear in the equation for $S_i^*$. Substituting the equation for $S_i^*$ into the equation for $h_i^*$ yields the following reduced form:

$$h_i^* = \varphi_0 + \varphi_1 \xi_i + \varepsilon_{2i},$$  

where $\varepsilon_{2i}$ is redefined to subsume the disturbance from the $S_i^*$ equation.

The system involving the $L_i^*$ equation and the reduced form for $h_i^*$ is exactly identified given that $\xi_i$ appears in the $h_i^*$ equation but not in the $L_i^*$ equation, and this is implied by part (ii)

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33 The more typical situation in which $L^*$ is partially observed (i.e. observed only for promoted workers) is easily handled in a likelihood framework that includes a promotion probit as a third equation. The resulting system closely resembles the one discussed earlier, and both systems can be easily estimated using STATA’s “cmp” command, both for the cases of complete wage data and partially observable wage data.
of Proposition 4. If the econometrician has data on \( h_i^* \) this system can be estimated. But assume, as is typically the case, that the econometrician has data on worker performance but not worker choice variables. Then \( h_i^* = P_i - u_i \) can be substituted into both regressions, yielding the following “market-based system” where the disturbances \( \varepsilon_{1i} \) have been redefined to include \( u_i \):

\[
L_i^* = \beta_0 + \beta_1 P_i + \varepsilon_{1i}
\]

\[
P_i = \alpha_0 + \alpha_1 \xi_i + \varepsilon_{2i}
\]

This market-based system makes clear that, as in point (ii) of Proposition 4, \( \xi_i \) affects \( L_i^* \) only via its effect on \( h_i^* \) (and therefore \( P_i \)). There is no direct channel of influence of \( \xi_i \) on \( L_i^* \), hence \( \xi_i \)'s exclusion from the \( L_i^* \) equation.

Next consider the classic model, where the participation constraint implies \( L_i^* = C(e_i^*) \). A linear approximation of \( C(e) \) yields the following regression:

\[
L_i^* = \omega_0 + \omega_1 e_i^* + \omega_2 \xi_i + \varepsilon_{1i}
\]

Since \( e_i^* \) is endogenous, there is a second equation:

\[
e_i^* = \eta_0 + \eta_1 S_i^* + \eta_2 \xi_i + \varepsilon_{2i}.
\]

The spread, \( S_i^* \) is also endogenous, so there is a third equation describing it, as in section 3. Given point (i) of Proposition 4, \( \xi_i \) does not appear in the equation for \( S_i^* \). Substituting the equation for \( S_i^* \) into the equation for \( e_i^* \) yields the following reduced form:

\[
e_i^* = \phi_0 + \phi_1 \xi_i + \varepsilon_{2i} \]

where \( \varepsilon_{2i} \) is redefined to include the disturbance from the \( S_i^* \) equation.

In contrast to the market-based system, the system involving the \( L_i^* \) equation and the reduced form for \( e_i^* \) is not identified given that \( \xi_i \) appears in both equations via part (iii) of Proposition 4. Assume, as is typically the case, that the econometrician has data on worker performance but not worker choice variables. Then \( e_i^* = P_i - u_i \) can be substituted into both regressions. Furthermore, to resolve the identification problem, assume the econometrician has access to an exogenous variable, \( I_i \), that determines subordinate performance but that affects the average compensation between job levels only via subordinate performance. This gives rise to the following “classic system” where the disturbances \( \varepsilon_{1i} \) have been redefined:

\[
L_i^* = \beta_0 + \beta_1 P_i + \beta_2 \xi_i + \varepsilon_{1i}
\]

\[
P_i = \alpha_0 + \alpha_1 \xi_i + \alpha_1 I_i + \varepsilon_{2i}
\]

In this system, as in point (iii) of Proposition 4, \( \xi_i \) affects \( L_i^* \) both via its effect on \( e_i^* \) (and
therefore $P_i$) and also directly. Furthermore, and most importantly, the classic system nests the market-based system as the special case for which $\beta_2 = 0$.

To summarize, an empirical test based on Proposition 4 that allows classic tournaments to be distinguished from market-based tournaments with human capital investments would proceed as follows. First, assuming the disturbances $\varepsilon_{1i}$ and $\varepsilon_{2i}$ are jointly normal with mean vector $0$ and covariance matrix $\Sigma$, estimate the preceding classic system via maximum likelihood. Second, test the null hypothesis that $\beta_2 = 0$ against the alternative hypothesis that $\beta_2 < 0$ using a one-tailed test. As indicated in the fourth column of Table 2, rejecting the null favors the classic tournament model, whereas failing to reject favors the market-based tournament model with endogenous human capital investments.

I close the section with four observations. First, the preceding test assumes cross-sectional variation across tournaments so that $i$ indexes tournaments and there is one worker observed per tournament. The method could also be applied to data in which there are observations on multiple workers per tournament and/or panel data. Second, justifying the choice of instrument, $I_i$, is facilitated by the fact that the classic tournament model assumes that $W_m$ and $W_s$ (and therefore $L^*$) are chosen by the firm ex ante, before subordinates choose effort levels and, thus, before subordinate performance is determined. This timing provides a theoretical basis for exclusion restrictions that might otherwise not be obvious. Third, the fact that the test is based on $L^*$ rather than $S^*$ is interesting, since the previous literature has focused on the determinants of $S^*$ (given that the wage spread is what creates incentives) with little attention paid to the determinants of $L^*$. This section illustrates that for the purpose of distinguishing between the two types of tournaments, studying the average wage between levels is more helpful than studying the wage spread between levels. Fourth, like sections 3 and 4, this one highlights the usefulness of systems-based approaches that simultaneously account for worker and firm behavior in research that aims to distinguish among alternative theoretical models of promotions.

7. Discussion

The underlying logic of the methods proposed in this paper takes each of the core promotion models seriously as “stand alone” models. For example, suppose that the data
generating process consists entirely of market-based tournaments with human capital investments. What would we expect to find in the data? Applying the relative-versus-absolute performance test and the opposing responses test, we would expect $\gamma_3 > 0$ and $\sigma_{12} < 0$, and then applying the new test from section 6 we would expect $\beta_2 = 0$. So the overall approach asks what we would expect to see in the data in the extreme case in which one model was the sole data generating process. But the real world is painted in shades of grey rather than in black or white. In a broad cross section spanning many different markets and employer types, it is plausible that a blend of alternative promotion schemes might occur in the labor market. It is even possible that alternative schemes might operate simultaneously within the same firm. For example, a worker might get promoted only if (a) this worker won an internal promotion competition and (b) the worker meets some performance requirement that does not depend on the performance of peers. This would blend internal promotion competitions with performance standards models within a firm. Note that the canonical performance standards model exhibits constant returns to scale whereas tournaments with one winner promoted exhibit extreme decreasing returns to scale. Apart from the CEO’s position (where there is only one job slot) most jobs will exhibit less extreme decreasing returns to scale.\(^{34}\)

The fact that multiple promotion models could be operating at the same time would complicate the interpretation of results from the empirical tests proposed in this paper. There are a number of responses to this concern. One is that, to a large extent, the core promotions models do in fact exist in isolation in the theoretical literature. The theorists who proposed these models evidently believe that the models, though stylized, are reasonable descriptors of the real world, at least in some settings. Judging by the influence these theory papers have had on the promotions literature, it would appear that others agree. Thus, while few would argue that any particular model is complete in capturing all relevant features of promotion processes, the literature to date suggests that taking these models seriously as “stand alone” models is a reasonable first pass.

Furthermore, concerns about different promotion models being blended in real-world data can be mitigated by judicious choices of analysis samples. For example, if there are

\(^{34}\) For concreteness, consider the hiring problem in an economics department. Although there may be only one job opening, if a second really good candidate emerges, the department can appeal to the dean to request another position. Thus, if neither candidate is good, neither is hired; if both candidates are not great but at least one is acceptable, then the better one is hired, and if both are great then both are hired.
particular industries or settings in which it is likely that labor contracts can be written (or cannot be written), this would be helpful. Labor contracts are necessary for the “classic” wage-setting mechanism (in which the firm pre-commits to a wage spread) to operate, whereas contracts are not necessary for the market-based wage setting mechanism to operate. Alternatively, if there are settings in which external labor market competition is likely to be particularly intense, this would also be helpful, since in such settings the market-based models should be more likely to find support in the data. Such distinctions might be helpful for identifying settings in which each of the models, taken individually, are more or less likely to hold. For example, the tests in this paper are most likely to find support for “classic models” in empirical settings in which labor contracts are more likely to be seen and in which external labor market competitive pressure is likely to be less intense. Alternatively, settings in which labor contracts are less likely to be seen and in which external labor market competition is likely to be strong should be most likely to yield support for the market-based wage setting mechanisms. Similarly, focusing on particular types of jobs might be helpful. In a broad cross section that includes all types of jobs, distinguishing a pure effort-inducing tournament from one that induces human capital accumulation is complicated by the fact that both could be happening at the same time (even within a firm). But if the analysis sample is restricted to a certain job type in which human capital accumulation seems unlikely to be important, then it would be reasonable to apply the methods of section 3 to try to distinguish among alternative effort-based promotion models. The broad point here is that the theorists who developed the core promotion models surely recognize that the stylized models are likely to capture reality better in some settings (e.g., industrial, occupational, contractual, skill-level, etc.) than in others, and selecting analysis samples for empirical work with this in mind would both refine the empirical methods developed in this paper and make the resulting evidence more convincing.

Another response to concerns about the implications of blending of alternative promotion models for the approach taken in this paper is that the tests developed in this paper appear to be the best available so far. As noted in the introduction, researchers have been unsuccessful to date in distinguishing empirically among the core models of promotions. And while the possibility of blending of alternative models may complicate the interpretation of the tests in this paper, the tests are still more informative than the alternative. For example, take a sample of workers
governed (mostly) by market-based effort-inducing tournaments, and another sample of workers governed (mostly) by classic effort-based tournaments. Applying the methods of section 3 to these samples, one would expect to find $\sigma_{12} > 0$ in the first sample and $\sigma_{12} < 0$ in the second, whereas I am unaware of any alternative empirical tests in the literature that could be used to show that the two samples are generated via different theoretical mechanisms. Furthermore, in some cases the methods presented here can provide useful empirical distinctions even when blending is present. For example, consider a blend of market-based performance standards with effort (section 2.2) and market-based tournaments with effort (section 2.3), versus an alternative blend of classic tournaments (section 2.1) and market-based tournaments with human capital investments (section 5). The evidence in DeVaro (2006b) would be consistent with the second blend (which should predict $\gamma_3 > 0$ and $\sigma_{12} < 0$) but not the first blend (which should predict $\gamma_3 > 0$ and $\sigma_{12} \geq 0$).

A final point concerning the blending of isolated theoretical models is that the methods developed in this paper can help guide future efforts to integrate theoretical models. For example, consider an integration of classic and market-based (effort-inducing) tournaments as recommended in Waldman (2012), i.e. the firm might commit to some minimal wage spread *ex ante* and make adjustments to the spread *ex post* in response to labor market conditions. A recent example of such an integration is Zábojnít (2012) in which the firm’s ability to pre-commit to and control tournament prizes is constrained by the presence of an outside labor market. The present analysis suggests that an integrated model that combines both the classic and market-based features of tournaments might help to better reconcile certain theoretical predictions with the evidence. For example, consider an integration of the market-based model with effort choices (which predicts $\sigma_{12} > 0$, conflicting with the empirical evidence from section 4) and the classic tournament model. The classic mechanism in such an integration might allow the prediction $\sigma_{12} < 0$ to emerge empirically (or would at least weaken the prediction $\sigma_{12} > 0$) even though the strategic decisions of firms in setting wage spreads would be somewhat constrained by the actions of competing firms. The broad point is that empirical findings concerning $\gamma_3$, $\sigma_{12}$, and (from section 6) $\beta_2$ from a certain narrowly-defined occupational setting might suggest to theorists which integrations of individual theoretical models might be the most relevant and promising to develop for explaining behaviors in that setting.
8. Conclusion

This study shows that the core theoretical models in the promotions literature can be distinguished empirically to a greater extent than previously recognized. The following four results are of particular interest. First, in the class of models in which promotions create incentives for workers to exert effort the relative-versus-absolute performance test and the opposing responses test distinguish among classic tournaments, market-based tournaments, and performance standards. The distinction between classic and market-based models is particularly interesting given that those models imply opposite predictions for the sign of $\sigma_{12}$, and this is an important result given that, as noted in Waldman (2012), most of the existing empirical evidence does not allow the two models to be distinguished. Second, if we restrict attention to market-based tournaments, then the model with worker effort choices can be distinguished from the model with human capital investments, again due to opposite predictions on the sign of $\sigma_{12}$. Third, the two tests are insufficient for distinguishing classic tournaments from market-based tournaments with human capital investments, since both models exhibit the opposing responses property. However, fourth, these two tournament models can be empirically distinguished using a new test based on the average wage between job levels, and the new test provides guidance for future data collection and analysis.

A fundamental new insight of the paper is that the role of risk, which is an important common component of the core models of promotion, differs across those models. In effort-based promotion incentive systems, the effect of risk in classic tournaments is exactly the opposite of its effect in market-based tournaments. This insight provides the basis for distinguishing empirical tests. In earlier work, theoretical predictions concerning $\theta$ either had not been derived or they had been derived but not highlighted and translated into testable implications. An advantage of the opposing responses test is that it allows predictions about $\theta$ (which would usually be unobserved to the econometrician) to be translated into a prediction about the sign of the parameter $\sigma_{12}$, which can be and has been estimated. The opposing responses test requires accounting for worker and firm behaviors simultaneously using systems-based econometric methods. The new test proposed in section 6 also requires a (different) systems-based approach. In contrast, the empirical literature on promotions has historically relied
on single-equation, non-structural estimation techniques. The analysis highlights the value of systems-based methods in empirical work on promotions, and a priority for future research should be applying these methods to other data sets containing information on the choice variables of workers and firms, as well as collecting new data that will support these methods.

Appendix A

The formal argument establishing the opposing responses property in classic tournaments is due to Lazear and Rosen (1981) and is summarized here. The performances of two identical, risk-neutral subordinates, denoted with subscripts i and j, are given by $P_i = e_i + u_i$ and $P_j = e_j + u_j$, where $u_i$ and $u_j$ are distributed independently, symmetrically, and identically. The firm observes which subordinate has the higher performance and promotes that worker to the managerial position, retaining the other as a subordinate. Due to the fixity of the job slots, this promotion rule implies that promotions are based on relative performance.

Let $C(e)$ denote the worker’s cost of exerting effort level $e$, where $C(0) = 0$, $C'(0) = 0$, $C'(e) > 0$ for $e > 0$, and $C''(e)$ for $e > 0$. After observing $W_m$ and $W_s$, which are chosen by the firm ex ante to elicit the optimal subordinate effort choices, subordinate i chooses $e_i$ to maximize expected utility, i.e. $pW_m + (1 - p)W_s - C(e_i)$, where $p$ is the probability that i wins (i.e. that $P_i > P_j$) which can be expressed as $p = G(e_i - e_j)$, where $G$ is the cumulative distribution function for $u_i - u_j$, and $\partial p/\partial e_i = g(e_i - e_j)$. Worker j’s problem is symmetric. The first-order condition defining $e_i^*$ is: $(W_m - W_s)\partial p/\partial e_i^* - C'(e_i^*) = 0$. Symmetric equilibrium implies $e_i^* = e_j^*$, so the first-order condition can be rewritten as: $(W_m - W_s)g(0) = C'(e^*)$. Two implications of this condition are worth noting. First, $\partial e^*/\partial (W_m - W_s) > 0$. Second, $\partial e^*/\partial g(0) > 0$. When $u_i$ and $u_j$ are normally distributed, $g(0) = 0.5(\theta \pi)^{-0.5}$, so that $\partial e^*/\partial g(0) > 0$ implies $\partial e^*/\partial \theta < 0$. Intuitively, this result says that as random factors become more important (relative to effort) as determinants of performance outcomes – and therefore promotion outcomes – workers invest less effort. 

$^{35}$ The result is given on page 847 of Lazear and Rosen (1981). Although the result holds for the normal distribution, it does not hold for every continuous, symmetric distribution. The reason is that since $g(0)$ is the value of the density function at a particular point, whereas $\theta$ is a characteristic of the entire distribution, it is possible to have cases in which two distributions have similar values of their density functions at 0, yet an arbitrary order of variances. However, it can be expected that the result holds more generally than for the normal case, though it is difficult to show this analytically since for most common (non-normal) distributions the difference between two i.i.d. random variables (i.e. $u_j - u_i$) is non-standard and lacks a convenient closed form expression. The reason the result is likely
The firm chooses $W_m$ and $W_s$ to maximize expected profit, $E(\pi)$, subject to incentive compatibility and participation constraints, yielding the following first-order conditions in which the output price is normalized to 1:

$$\frac{\partial E(\pi)}{\partial W_m} = (1 - C'(e))\frac{\partial e}{\partial W_m} = 0$$
$$\frac{\partial E(\pi)}{\partial W_s} = (1 - C'(e))\frac{\partial e}{\partial W_s} = 0.$$  

These and the worker’s first-order condition together imply $W_m - W_s = 1/g(0)$. Letting $S^*$ denote the equilibrium wage spread, note that $\partial (W_m - W_s)/\partial g(0) < 0$, which implies $\partial S^*/\partial \theta > 0$ given that $u_j - u_i$ is normally distributed. Given that $(W_m - W_s)g(0) = 1$, any change in $g(0)$ induced by a change in $\theta$ is offset by a change in the spread. Intuitively, the firm increases the generosity of the prize to offset the depressed incentives created by an increase in the importance of luck in determining performance.  

Note that $e^*$ remains unchanged in response to changes in $\theta$, given that the effects of changes in $\theta$ on $g(0)$ and the spread are exactly offsetting. Concerning the robustness of the opposing responses property, as discussed in footnote 35 the result $\partial e^*/\partial \theta < 0$ assumes that noise is normally distributed, and though it can likely be generalized beyond the normal case, it cannot be generalized to all continuous, symmetric distributions. Furthermore, the result requires that workers competing in the same tournament are roughly homogeneous in ability.  

Appendix B

Proof of Proposition 1: Consider first the case without worker effort (i.e. $\alpha = \infty$). In this case, the subordinate always invests the minimum effort in the first period, so $\partial e^*/\partial \theta = 0$. In models that do not incorporate an effort choice, $\partial e^*/\partial \theta$ does not exist. From Proposition 1 of Ghosh and

to hold for a broader class of distributions than for the normal distribution is that the economic intuition underlying the result (i.e. workers do not try as hard when random factors become more important as determinants of their performances, so that promotions depend more on luck than effort) seems likely to hold in many settings.

Note that the result that the firm chooses the wage spread that yields the first-best level of effort relies on the assumption that workers are risk neutral. If the workers were risk averse then the employer would find it optimal to partially insure workers against income risk by choosing a smaller wage spread, which in turn results in an equilibrium effort level below the first best. However, even in this case the qualitative result of interest continues to hold in that increases in $\theta$ induce the employer to increase the wage spread.

Tournaments with heterogeneous contestants are considered in Lazear and Rosen (1981) and O’Keeffe, Viscusi and Zeckhauser (1984) and in the empirical analysis of Levy and Vukina (2004). Worker effort depends on $g(e_i - e_j)$, where $e_i - e_j$ is zero when contestants are homogeneous in ability but potentially nonzero when contestants are heterogeneous in ability. In the latter case, an increase in noise may increase $g(e_i - e_j)$ and equilibrium effort. Intuitively, consider the problem from the standpoint of the less able of two heterogeneous competitors. If there is little noise, the worker has virtually no chance of winning, so incentives to exert costly effort are low, whereas a significant amount of noise brings that worker back into the race and increases incentives to exert effort.
Waldman (2010), $S^* = W_m - W_s = [d_m + c_m \eta^+] - [d_s + c_s a_L k]$, where $\eta^+$ is a critical threshold (or standard) that determines whether a subordinate is promoted to a managerial position in the second period. After observing subordinate $i$’s first-period output, $y_{i1}$, the firm promotes the worker in period 2 if $\eta^+(y_{i1}) \geq \eta^+$ and retains the worker as a subordinate if $\eta^-(y_{i1}) < \eta^+$. The following equation defines $\eta^+$:

$$(1+F)[d_s + c_s(\eta^+ + e_L)] - [d_s + c_s(a_L k + e_L)] = (1+F)[d_m + c_m(\eta^+ + e_L)] - \max\{d_s + c_s(\eta^+ + e_L), d_m + c_m(\eta^+ + e_L)\}. $$

Since $\theta$ does not appear in this expression, $\eta^+$ (and thus $S^*$) is not a function of $\theta$, so $\partial S^*/\partial \theta = 0$. If $\alpha < \infty$ so that subordinates might exert more than the minimum effort level of $e_L$ in the first period, $\partial S^*/\partial \theta = 0$ continues to hold, with the slight modification that the parameter $e_L$ appears throughout the expressions. More precisely, the preceding expressions for $S^*$ and for the equation that defines $\eta^+$ are modified as follows:\footnote{The expressions for $W_m$, $W_s$, and $\eta^+$ in this case are given in the appendix of an earlier version of Ghosh and Waldman (2010). See page 32 of the version dated July 2006.}

$$S^* = W_m - W_s = [d_m + c_m(\eta^+ + e_L)] - [d_s + c_s(a_L k + e_L)]$$

$$\partial S^*/\partial \theta = 0 .$$

**Proof of Proposition 2:** First consider $\partial S^*/\partial \theta$. Using the expressions for the subordinates’ first-period output, the expressions for managerial and subordinate wages can be rewritten as:

$W_m = d_m + c_m k [\mu_m a_H + (1 - \mu_m)a_L]\]

$W_s = d_s + c_s k [\mu_s a_H + (1 - \mu_s)a_L],$

where $\mu_m = 2\rho(1-\rho)\Phi[(a_H - a_L)/(2\theta)^{0.5}] + \rho^2$, $\mu_s = 2\rho(1-\rho)\Phi[(a_L - a_H)/(2\theta)^{0.5}] + \rho^2$, and $\Phi$ denotes the standard normal cumulative distribution function. Note that $\partial \mu_m / \partial \theta = -2\rho(1-\rho)\phi[(a_H - a_L)/(2\theta)^{0.5}](a_H - a_L)(2\theta)^{-0.5} < 0$, and $\partial \mu_s / \partial \theta = 2\rho(1-\rho)\phi[(a_H - a_L)/(2\theta)^{0.5}](a_H - a_L)(2\theta)^{-0.5} > 0$, where $\phi(.)$ denotes the standard normal density function. Defining the equilibrium wage spread, $S^*$, as $S^* = W_m - W_s$, we have:

$S^* = (d_m - d_s) + k a_L[c_m - c_s] + k(a_H - a_L)[c_m h_m - c_s \mu_s].$ This expression makes clear that\

$\partial S^*/\partial \theta = k(a_H - a_L)(c_m \partial \mu_m / \partial \theta - c_s \partial \mu_s / \partial \theta) < 0.$

Next consider $\partial e^*/\partial \theta$. Worker $i$ chooses first-period effort $e_{i1}$ to maximize expected utility of $pW_m + W_s(1-p) - \alpha C(e_{i1})$, where $p$ is the probability that
worker i is promoted. The first-order condition defining worker i’s optimal effort, \( e_1^* \), is:

\[
(W_m - W_s) \partial p / \partial e_1^* - \alpha C'(e_1^*) = 0.
\]

Expressions for \( p \) and \( \partial p / \partial e_1 \) are as follows:

\[
p = \Phi[(a_h - a_l + e_{11} - e_{1j})/(2\theta)^{0.5}]p(1-p) + \Phi[(a_l - a_h + e_{11} - e_{1j})/(2\theta)^{0.5}]p(1-p) + \Phi[(e_{11} - e_{1j})/(2\theta)^{0.5}](1 + 2p^2 - 2p)
\]

\[
\partial p / \partial e_{11} = p(1-p)[\Phi[(a_h - a_l + e_{11} - e_{1j})(2\theta)^{0.5}]/(2\theta)^{0.5} + p(1-p)[\Phi[(a_h - a_l + e_{11} - e_{1j})(2\theta)^{0.5}]/(2\theta)^{0.5} + (1 + 2p^2 - 2p)[\Phi[(e_{11} - e_{1j})(2\theta)^{0.5}]/(2\theta)^{0.5} > 0
\]

From the preceding expression, it can be shown that \( \partial^2 p / (\partial e_{11} \partial \theta) < 0 \). Applying implicit differentiation to the worker’s first-order condition and assuming that the second-order condition is satisfied yields the result sign(\( \partial e^*/\partial \theta \)) = sign(\( \partial^2 p / (\partial e_{11} \partial \theta) \)). Thus, \( \partial e^*/\partial \theta < 0 \). Q.E.D.

**Proof of Proposition 3:** Let \( u \) be distributed symmetrically and continuously on the support \([\alpha, \beta]\), and let \( f_0(u) \) denote its density function. Assume that \( \alpha \) is sufficiently high to ensure that the equilibrium involves no turnover. Assuming \( C(h) = 0.5h^2 \) for simplicity, \( h^* \) is proportional to \( q(0)[E(u_{(2)}) - E(u_{(1)})] \). Let \( x \) be a random variable distributed continuously and symmetrically on \([0,1]\) with density function \( f_x \), and let \( u = (\beta - \alpha)x + \alpha \). Since \( x \) (and therefore \( u \)) is distributed symmetrically, we can assume \( \alpha = 0 \) without loss of generality since it is straightforward to show that neither \( q(0) \) nor \( E(u_{(2)}) - E(u_{(1)}) \) vary with \( \alpha \). First, note that \( E(u_{(2)}) - E(u_{(1)}) = E(\text{max}[u_{1}, u_{2}]) - E(\text{min}[u_{1}, u_{2}]) = E(\text{max}[\beta x_{1}, \beta x_{2}]) - E(\text{min}[\beta x_{1}, \beta x_{2}]) = \beta E(\text{max}[x_{1}, x_{2}]) - \beta E(\text{min}[x_{1}, x_{2}]) = \beta[E(x_{(2)}) - E(x_{(1)})] \), so that \( E(u_{(2)}) - E(u_{(1)}) \) scales linearly with \( \beta \). Second, it is straightforward to show that \( f_0(t) = (1/\beta)f_x(t/\beta) \), which implies:

\[
\int_0^\beta f_0^2(t)dt = \frac{1}{\beta^2} \int_0^\beta f_x^2(t/\beta)dt = \frac{1}{\beta} \int_0^\beta f_x^2(t)dt', \text{ where the last equality uses the substitution } t' = t/\beta.
\]

This shows that \( q(0) \) scales with \( 1/\beta \), canceling the scaling of \( E[u_{(2)}] - E[u_{(1)}] \) with \( \beta \). Q.E.D.

**Proof of Proposition 4:** In the classic model, implicit differentiation of the worker’s first-order condition, \( S \partial p / \partial e_{11}^* - C'(e_{11}^*) = 0 \), reveals that \( \partial e_{11}^*/\partial \xi > 0 \) given the properties of \( C(e) \) and satisfaction of the second-order condition. The equilibrium spread is \( S^* = 1/g(0) \), so \( \partial S^*/\partial \xi = 0 \) since \( \xi \) does not appear in \( g(0) \). In the market-based model the worker’s first-order condition is essentially the same as in the classic model, so a similar argument yields \( \partial h^*/\partial \xi > 0 \). The equilibrium spread is \( S^* = V(E[u_{(2)}] - E[u_{(1)}]) \), so \( \partial S^*/\partial \xi = 0 \) since \( \xi \) does not appear in \( E[u_{(2)}] - E[u_{(1)}] \). This establishes (i). In the market-based model, \( L^* = V_H[h^*(V) + 0.5(E[u_{(2)}] + E[u_{(1)}])] \), so
\( \frac{\partial L^*}{\partial \xi} = (\frac{\partial L^*}{\partial h^*})(\frac{\partial h^*}{\partial \xi}) = V h^* \frac{\partial h^*}{\partial \xi} > 0 \), establishing (ii). Point (iii) follows from the participation constraint binding in equilibrium, i.e. \( L^* - C(e^*) = 0 \), and from the properties of \( C(e) \). Q.E.D.
References


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Note: The first column simply lists some examples for each model type and is not an exhaustive list of papers.
## Table 2: Predictions of Alternative Models of Promotion

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