Abstract

This paper develops a multi-sector job search model with endogenous education choice to quantify the social cost from degree-inflation. Although the unskilled workers perform only the tasks without skill requirement, the skilled workers can perform both skilled and unskilled tasks, depending on their qualifications. It demonstrates that the over-supply of skilled workers crowds out demand for unskilled labor, suppresses unskilled wages, raises skill premia, and, in turns, reinforces college attainment. Our numerical experiments based on estimates from Canadian data report that the crowding out effect due to degree-inflation creates a significant adverse effect on output growth.

Keywords: Degree Inflation, Task Assignment, Returns to Education

JEL Classification: I25, J31, J64, O41
1 Introduction

Over the last few decades, in contrast to the standard demand and supply analysis such that the increasing supply of college graduates supposedly lowers return to college education, college attainment as well as college premium has steadily increased in many countries. People may argue that it was a natural and transitional phenomenon in economic development, but what makes them puzzled is that the persistent growth of college attainment coupled with college premium has been observed so far especially in the developed countries such as Canada, Japan, and United States. Motivated by so called ‘degree inflation’, the phenomenon that more and more college graduates work at unskilled positions which were previously held by less educated workers, this paper sheds lights on through what channels and to what extent the over-supply of college degree holders crowds out demands for less educated labor force, raises college premium, and, thus, self-reinforces college attainment.

Skill biased technological change (SBTC hereafter), ‘a shift in the production technology that favors skilled over unskilled workers by increasing its relative productivity and therefore demand’ has been considered as one of the most plausible explanations for the coupled growth of college attainment and premium from late 1970’s to early 1990’s. Indeed, numerous studies provided empirical evidence for the shift-out of labor demand which was caused by technological progress and integration of the world economy during the ‘Cold War’. However, we argue that while SBTC can seemingly account for these patterns before the end of the Cold War, SBTC is not the main driver of the Canadian labor market from early 1990’s to mid 2000’s. In fact, SBTC premises a positive technological shock that is accompanied by output and productivity growth, which are missing in the Canadian economy over recent two decades.

It is important to point out that the remarkable growth of college graduates in the aforementioned period has led many of them to work at relatively unskilled jobs, most of which are not necessarily high paying. In Canada, the number of Canadian students graduating from Canadian post-secondary institutions with bachelor’s degrees or equivalents has nearly doubled from approximately 89,000 to 160,600 since 1992. However, about one out of every five people having college degrees was said to be seemingly overqualified for their job. According to Quintini (2011), although there is significant variation across countries and socio-demographic groups, for instance, 35 percent of the workforce seemingly overeducated for their jobs in Sweden, while only 10 percent in Finland, most of the OECD countries are located between these two numbers. Indeed, we pay attention to the fact that more and more college graduates work at the traditional sectors rather than modern sectors in Canada after 1990’s, but
the former had a much smaller portion of college graduates than the latter in 1980’s.

To account for ‘degree inflation’, this paper develops a two-sector job search model by incorporating workers’ endogenous education decision and firms’ hierarchical labor demand. Although unskilled workers perform unskilled tasks (tasks without skill requirement) only, the skilled workers can perform both skilled and unskilled tasks, depending on their actual qualifications. Given this hierarchical structure of skills, firms upon positive productivity shocks create more vacancies in the labor market for college graduates than in the labor market for less educated workers. As a result, the measure of college graduates at unskilled positions (hereafter, cross-skill match) increases. An increase in supply of college graduates crowds out the demand for less educated workers, suppresses the unskilled wages, and enlarges returns to college education. In turns, it reinforces the college attainment rate and college premium together in spite of complementarity between the skilled and unskilled labor in the production technology. However, at the same time degree inflation, gauged by the measure of cross-skill matches hinders the enhanced college attainment from fully contributing output growth. We calibrate the model using the Canadian labor market data, which also reveals strong growth patterns of college attainment, college premia, and cross-skill matches.

The observation that many countries are oversupplied with college graduates is not a novel one. Hecker (1992) noted, in research for the Bureau of Labor Statistics, that there were more job seekers with college degrees than job openings requiring a degree. Gottchalk and Hansen (2003) find that the proportion of college-educated workers in non-college occupations declined from the mid-1980s to the mid-1990s in the U.S. Furthermore, they find no evidence to support the notion that the increasing proportion of overqualified college graduates was being forced to accept noncollege jobs. In a sharp contrast, Wolff (2006), in a detailed examination of employment data, showed that there is a significant disconnect between the growth in the number of highly educated workers and the job requiring high levels of skills. Beaudry, Green, and Sand (2013) also address the supply of high education workers taking the lower-skilled occupations and pushing low-skilled workers down the occupation ladder, and finally, out of the labor force all together. Beaudry, Green, and Sand (2013) focus on the post-2000 period where the U.S. economy saw a great demand reversal for the skilled-workers through an endogenously generated boom and bust cycle, after skill-biased technological changes in pre-2000. Building on this observation, we look into the possible channel that reinforces the process of the highly educated work force moving down the occupation ladder, namely the degree inflation channel.

This paper is organized as follows. Section 2 describes our data and the labor market patterns in Canada. Section 3 provides the model that we use to motivate the empirical exercise. Section 4 explains our estimation protocol and results. Section 5 briefly discusses policy implications from the exercise and Section 6 concludes.

2 Canadian Labor Market

We pull together several data sources to understand recent labor market and education attainment in Canada. First, we use several series from Statistics Canada to under-
Figure 1: Educational Attainment and College Premium in Canada

Note: Panel (a) is based on LFS micro-data. Panel (b) is based on the data constructed by the Statistics Canada, which combines information from Census and LFS. Mean real hourly wages are constructed by dividing annual labor compensations by annual hours worked for each education category (both business and non-business sectors are included). There are 2 education groups: high school graduates or less; and post-secondary education or more. To calculate the real wage, the nominal wages are deflated using the all-items CPI (CANSIM Table 326-0021).

Panel (a) in Figure 1 shows that the attainment of higher education (at least some post-secondary education) in Canada has increased steadily, while the fraction of employees without a high school diploma dropped significantly. In 2009, about 65.4 percent of Canadians workers had trade certificates, college diplomas, or university degrees, which is more than double of the percentage in 1981. The percentage of Canadians workers having any of post-secondary degrees was 30.4 in 1981. This is consistent

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5The growth of high-education employees has accelerated especially from 1990s. This might be partially due to the definition change in the LFS's education question. See “Labour and Household Surveys Analysis Division Staff Report: THE IMPACT OF THE 1990 CHANGES TO THE EDUCATION QUESTIONS ON THE LABOUR FORCE SURVEY” by Statistics Canada for more details. We will conduct robustness check in the next version of the paper, by switching the definition of ‘high-education’ being university degree or above.
with the decrease in the percentage of workers without post-secondary certification. Panel (b) in Figure 1 shows the real hourly wages of highly educated (skilled) and less educated (unskilled) workers in Canada. Despite the increased supply of highly educated workers shown in panel (a), the college (skill) premium measured by “gap” has been increasing. The college premium measured by “ratio” has risen significantly from 1980s (20 percent on average) to early 1990s (36 percent on average) due to a gradual decline in unskilled wages. As the average unskilled wage in the denominator has increased from late 1990’s, the premium measured by the ratio has stayed stable, which is a contrasting feature of Canadian labor market, in comparison to the U.S and other developed countries.

In Figure 2, we observe that share of high education employees for services sectors as well as non-agricultural sectors (panel (a)) have steadily increased over the data period, at faster speed than other sectors (panel (b)). This implies that the skilled worker ratios in the traditional sectors (unskilled tasks intensive sectors) has grown further than those in the modern sectors (skilled tasks intensive sectors). This is an evidence consistent with the prediction of this paper’s model, the degree inflation. Furthermore, using the LFS micro-data to understand sectoral-difference in wage gap and employment for different educational attainment level, we find that (1) overall, most of the sectors, excluding agricultural, forest and fishing sectors, the high-education employees increased steadily and that (2) the sectoral wage gap between high education and low education, in other words, sectoral (high) education premium, not only increased for all sectors, but diverged - some sectors showed more dramatic increase in real wage gaps compared to other sectors.

The next set of evidence shows that SBTC might not fully explain the recent evolution of Canadian labor market. In particular, we search for evidence of skill-biased technological change, which could have reduced labor demand for particular skill-groups, improved labor productivity, and improved output growth. In short, the evidence did not support this hypothesis. Panel (a) in Figure 3 plots the GDP growth rate in Canada from 1961 to 2009. The growth rate of GDP per capita also shows the same pattern. The declining growth rate implies that SBTC might have not been
Figure 3: Behavior of Key Macroeconomic Variables in Canada

Note: Panel (a) plots the GDP growth rate (annual %) in Canada, using the World Development Indicators of the World Bank. Panel (b) plots the ratio of capital input over GDP, using CANSIM Table 383-0021. Panel (c) plots the unemployment rate for total population using the World Development Indicators of the World Bank, and non-college graduates and college graduates, with quadratic trend lines using CANSIM Table 282-0004. Panel (d) plots the college graduates ratio within each sector and place them from the sector with the lowest ratio in the left end to the sector with the highest ratio in the right end, using LFS micro-data.

materialized, thus not supporting the hypothesis. Moreover, the capital-output ratio plotted in panel (b) shows that the growth of capital (K) outpaced the growth of the output (Y) overtime. Panel (c) in Figure 3 plots the unemployment rate in Canada for total from 1981 to 2009, and for each education group (high and low) from 1991 to 2009. This figure also shows the declining unemployment rates, despite of a declining GDP growth rate. These two pieces of evidence indicate that inputs (capital and labor) has increased but the output has not increased as much, which implies that the input factors are utilized in a less efficient way than before.

To be more precise, we search for evidence of any negative TFP during the past two decades, which could have generated labor demand to shrink, leading to the skilled workers to take the jobs of the unskilled workers and sometimes forcing the unskilled workers out of the labor force. However, by looking at the capital stock and investment activity in Canada over this period, as well as the unemployment share by education
attainment, we conclude that such negative TFP was not likely to have existed during this period. First, given the sharp increase in the capital-output ratio during the corresponding period (panel (b) in Figure 3), it is difficult to conclude that there existed some negative TFP shock that could have influenced the labor market demand for the skilled workers. Similarly, total unemployment rate shows a clear and steady decline ((panel (c) in Figure 3)), which is difficult to reconcile with negative TFP in Canada. This reduction in total unemployment rate is also driven by the drop in unemployment rate of low education group. Over the same period, the unemployment rate in high education group seem to stay constant, if not slightly increasing.

Moreover, Panel (d) in Figure 3 reveals an interesting aspect of the evolution of the Canadian labor market. First, it sorts out 43 sectors by the college graduates ratio within each sector and place them from the sector with the lowest ratio in the left end to the sector with the highest ratio in the right end. By this sorting strategy, for instance, fishing and hunting, agricultural, mining, accommodation and food services, and transport equipment sectors are located in the left side and finance, computer and electronic, science and technology, and education services in the right side. Panel (d) plots the cumulative sectoral employment share on the horizontal axis and the cumulative high education employees ratio on the vertical axis. As the sectoral variations in the shares of high education employees become smaller, the curve becomes close to the 45-degree line. In fact, Panel (d) tells us that from 1981 to 2001, many college graduates (high education group) were employed in the sectors which had not hired many college graduates previously. Since it is hard to imagine that skill biased technological affected the sectors listed in the left side (unskilled tasks intensive sectors), it can be inferred that many college graduates are employed at unskilled positions.

Lastly, using data from the National Occupational Classification (NOC), we also look at the number of university-educated who were overqualified for their job. We see that the number of highly educated workers who were overqualified for their job was nearly thirty percent higher in 2001 than in 1993. An estimated 331,100 workers had experienced this situation at some point in 2001, up from 251,600 in 1993. Proportionally, however, due to an increase in the stock of workers with university degrees between 1993 and 2001, their share of all university-educated people in the workforce remained virtually unchanged. These people accounted for about one-fifth (19 percent) of all the all university-educated people in the workforce in 2001, up marginally from 18 percent in 1993. A significant proportion of university-educated workers had worked in a job for which they were overqualified during their entire work period. As you compare the over-qualification in ages, younger workers were more likely to work in a position for which they were overqualified. However, older workers had higher chances of remaining overqualified during the entire work period, once they were in an overqualified situation. Another Statistics Canada study, “I still feel overqualified for my job”\(^6\), showed that 33 percent of young workers aged 20 to 29 felt overqualified, compared with 23 percent of their counterparts aged 30 to 64, according to a data from the 2000 General Social Survey. However, once workers were in a situation in which they were overqualified, the older workers showed a tendency to stay there. In other words, the incidence of being overqualified all the time increased with age.

3 The Model

3.1 Environment

We consider an economy with two sectors, traditional sector \((i = 1)\) and modern sector \((i = 2)\), which is populated by continuum of entrepreneurs and workers. Workers make their own schooling decision to acquire skills before entering the labor market. The labor market is subject to search and matching friction as in Mortensen and Pissarides (1994). Entrepreneurs create jobs both with and without skill requirement as in Albrecht and Vroman (2002). There are three types of matches unskilled matches \((j = u)\) of unskilled workers and jobs, skilled matches \((j = s)\) of skilled workers and jobs, and cross-skill matches \((j = c)\) of skilled workers and unskilled jobs, in each sector. We proceed with a continuous time model having common discount rate \(r\).

Final Goods  The final goods, denoted by \(Y_t\), are assembled from the two sectoral intermediate goods using the following production technology.

\[
Y_t = (y_{1t}^{\frac{1}{\sigma}} + y_{2t}^{\frac{1}{\sigma}})^{\frac{\sigma - 1}{\sigma}}, \tag{1}
\]

where \((y_{1t}, y_{2t})\) represent the quantities of sectoral products from the traditional and modern sectors, and \(\sigma(>1)\) the constant elasticity of substitution. Those final goods are consumed or used as inputs in the production process for the sectoral intermediate goods through the perfectly competitive final goods market. Let \(p_{it}\) and \(P_t\) be the price of the intermediate goods \(i\) and the final goods, respectively. For each \(i \in \{1, 2\}\), the demand for each intermediate goods to produce one unit of the final goods is characterized by

\[
y_{it} = p_{it}^{-\sigma} p_t^{\sigma - 1}, \text{ where } P_t = \left(\frac{1}{p_{1t}^{1-\sigma} + p_{2t}^{1-\sigma}}\right)^{\frac{1}{1-\sigma}}. \tag{2}
\]

In particular, the final goods are treated as the numéraire, the price of which is normalized to be one, \textit{i.e.} \(P_t = 1\) at every \(t \in [0, \infty)\). Since there is a unit measure of homogenous entrepreneurs in each sector, the aggregate supply of each sectoral product is given by \(y_{it}\). By equating the aggregate supply and demand and reordering, we get the following market clearing condition. For each \(i \in \{1, 2\}\),

\[
\frac{p_{it} y_{it}}{p_{1t} y_{1t} + p_{2t} y_{2t}} = \left(\frac{p_{1t}^{1-\sigma} + p_{2t}^{1-\sigma}}{p_{1t}^{1-\sigma} + p_{2t}^{1-\sigma}}\right)^{\frac{1}{1-\sigma}}. \tag{3}
\]

It implies that the price ratio reflects the revenue ratio of each sector. Together with the normalization assumption \(P_t = 1\), equation (3) determines \((p_{1t}, p_{2t})\).

Intermediate Goods  Each intermediate goods are produced by the entrepreneurs in each sector. Production technologies are given by

\[
y_{it} = \alpha_i^k l_{iit}^{\beta k} l_{ist}^{\beta s} (\lambda_i l_{ict} + l_{iut})^{\beta u} \tag{4}
\]

where \((k_{it}, l_{iit}, l_{ict}, l_{iut})\), respectively, the capital stock, the number of skilled, cross-skilled, and unskilled matches in sector \(i\) at time \(t\). Parameter \(\alpha_i\) represents the productivity of sector \(i\) (or TFP) and \(\lambda_i(\geq 1)\) reflects the fact that the skilled workers are
more productive than the unskilled workers at the same unskilled positions. Parameters $(\beta_{ik}, \beta_{is}, \beta_{il})$ represents the intensity of each factor and $\beta_{ik} + \beta_{is} + \beta_{il} = 1$.

The entrepreneurs in sector $i$ purchase investment goods $x_{it}$ at the fixed (real) price of $p_{ixt} = p_{ixt}P_t^{-1}$ through international financial market. The law of motion of the capital stock is given by

$$\dot{k}_{is} = -\eta_i k_{is} + \kappa_i^{-1} x_{is}^\kappa,$$  \hspace{1cm} (5)

where $\eta_i$ is the depreciation rate. As the qualified worker ratio rises, capital investment becomes more efficient in each sector. It can be interpreted as a proxy of the infrastructure of the sector or as knowledge spillover among qualified workers.

Let $(v_{ist}, v_{iut})$ be the number of skilled and unskilled vacancies, respectively. While unskilled vacancies are filled by unskilled job searchers at rate $q(\theta_{it})$, skilled vacancies are contacted by skilled job searchers at rate $q(\theta_{st})$ and filled with qualifying probability $\chi_i$ per contact. The skilled candidates who fail in the qualification test are assigned at the unskilled positions (cross-skill matches). All matches are resolved due to exogenous separation shocks at rate $\delta$ or retirement shocks at rate $\rho$. For each $j \in \{s, c, u\}$

$$\dot{l}_{ijt} = \begin{cases} 
-(\delta + \rho) l_{ist} + \chi_i q(\theta_{st}) v_{ist} & \text{if } j = s \\
-(\delta + \rho) l_{ict} + (1 - \chi_i) q(\theta_{st}) v_{ist} & \text{if } j = c \\
-(\delta + \rho) l_{iut} + q(\theta_{ut}) v_{iut} & \text{if } j = u 
\end{cases}$$  \hspace{1cm} (6)

Parameter $\chi_i$ captures the probability that a skilled worker passes the qualification test for the skilled position. If a worker fails in the qualification test, the worker is assigned at the unskilled position. Since the contact itself has rent due to search friction, the entrepreneur does not have incentives to dissolve the contact. Instead, the entrepreneur assigns the less qualified workers to the unskilled position. The profit flow of each entrepreneur is given by

$$\pi_{it} = p_{iut} y_{it} - p_{ixt} x_{it} - w_{ist} l_{ist} - w_{ict} l_{ict} - w_{iut} l_{iut} - \gamma_s v_{ist}^\xi P_t - \gamma_u v_{iut}^\xi P_t,$$  \hspace{1cm} (7)

where $(\gamma_s v_{ist}^\xi P_t, \gamma_u v_{iut}^\xi P_t)$ represent the nominal cost of creating each type of vacancies. It is assumed that $\xi > 1$. Using her profit flow, the entrepreneur consumes the final goods.

Denote by $J_{it}(K_{it}, L_{ist}, L_{ict}, L_{iut})$ the expected value of the entrepreneur having $(K_{it}, L_{ist}, L_{ict}, L_{iut})$ measures of each matches at time $t$. The entrepreneur decides to how many vacancies to create in order to maximize

$$J_{it}(K_{it}, L_{ist}, L_{ict}, L_{iut}) = \int_0^\infty e^{-r(\tau-t)} \pi_{i\tau} P_t^{-1} d\tau$$  \hspace{1cm} (8)

subject to (6) and her initial endowment $(K_{it}, L_{ist}, L_{ict}, L_{iut})$. The entrepreneurs make investment and vacancy creation decision such that

$$x_{it}^{1-\kappa} = \frac{\kappa_i^{-1} \kappa}{p_x} \int_0^\infty e^{-(r+\eta_i)\tau} \frac{\partial \pi_{i\tau}}{\partial K_{i\tau}} P_t^{-1} d\tau$$  \hspace{1cm} (9)

$$\xi \gamma_s v_{ist}^{\xi-1} = q(\theta_{st}) \int_0^\infty e^{-(r+\delta+\rho)\tau} \frac{\partial \pi_{i\tau}}{\partial I_{ist}} P_t^{-1} d\tau = q(\theta_{ut}) \frac{\partial J_{it}}{\partial I_{iut}} \text{ and}$$  \hspace{1cm} (10)

$$\xi \gamma_u v_{iut}^{\xi-1} = q(\theta_{st}) \int_0^\infty e^{-(r+\delta+\rho)\tau} \left[ \chi_i \frac{\partial \pi_{i\tau}}{\partial I_{ict}} + (1 - \chi_i) \frac{\partial \pi_{i\tau}}{\partial I_{iut}} \right] P_t^{-1} d\tau$$  \hspace{1cm} (11)
Equations (9), (10) and (11) show that the optimal investment decision and vacancy creation decisions are forward-looking. The left hand side of equation (9) represents the marginal cost of investment at time $t$ and the right hand side the persistent marginal benefit from doing so. In equations (10) and (11), the left hand sides represent the marginal cost of creating a vacancy, the right hand sides stand for the marginal benefit of doing so. The detailed derivations are presented in Appendix A. In equations (10) and (11), $\frac{\partial J_{it}}{\partial l_{ijt}}$ evolves as follows.

$$
\frac{d}{dt} \left( \frac{\partial J_{it}}{\partial l_{ijt}} \right) = -\frac{\partial \pi_{it}}{\partial l_{ijt}} P_t^{-1} + (r + \rho + \delta) \frac{\partial J_{it}}{\partial l_{ijt}},
$$

where $\pi_{it}$ is the job finding rate by an unskilled worker, which will be discussed in details later. The left-hand side in equation (14) can be interpreted as the opportunity cost of holding asset, unskilled unemployment at every instant. The terms on the right-hand side represent the benefit flow from holding the asset $V_{ut}$ which consists of the dividend flow from the asset, the potential loss from retirement, the potential gains from job finding, and the gains from changes in valuation of the asset, respectively. In addition, we get the asset value equation for the skilled unemployed workers $V_{st}$ as follows.

$$
\frac{d}{dt} \left( \frac{\partial J_{st}}{\partial l_{ist}} \right) = q(\theta_{st}) \left[ \chi_i \frac{\partial J_{st}}{\partial l_{ist}} + (1 - \chi_i) \frac{\partial J_{st}}{\partial l_{ict}} \right].
$$
our paper, the advanced degree plays a role as an imperfect signal, but it is different from the signaling device in Spence (1973) in the sense that the academic degree increases the uncertainty of qualification of the degree holder. For each \( j \in \{s, c, u\} \), \( w_{ijt} \) represents the wage payment at the \( ij \)-type match. The employed workers are laid off when they are hit by separation shock at rate \( \delta \). The HJB equation for the employed is characterized by

\[
rW_{ijt} = w_{ijt}P_t^{-1} - \rho W_{ijt} + \delta(V_{jt} - W_{ijt}) + \dot{W}_{ijt}, \quad \text{for each } j \in \{s, c, u\},
\]

where \( V_{ct} = V_{st} \).

In the presence of the labor market friction, the entrepreneurs should create their vacancies first and wait for job searches. Let \( u_{st} \) and \( u_{ut} \) be the measures of the skilled unemployed and the unskilled unemployed at time \( t \), respectively. The vacancies and unemployed workers that are matched at any point in time are randomly selected by the constant returns to scale matching function of \( (v_{1jt} + v_{2jt}, u_{jt}) \) in each submarket. Define the labor market tightness as \( \theta_{jt} := (v_{1jt} + v_{2jt})/u_{jt} \) for each \( j \in \{s, u\} \) at every instant. Given the homogeneity of the matching technology, the job-finding and vacancy-filling rates are denoted by functions of the market tightness \( \theta_{jt} \) only. In our empirical study, we borrow the matching technology from den Haan, Ramey, and Watson (2000), which results in

\[
f(\theta_{jt}) = \theta_{jt}(1 + \theta_{jt}^{\kappa_j})^{-1/\kappa_j} = \theta_{jt}q(\theta_{jt}), \quad \text{for each } j \in \{u, s\}.
\]

The economy is populated by \( L_t \)-measure of workers. Let \( (L_{ist}, L_{ict}, L_{iut}) \) be total masses of the employed workers at skilled, cross-skilled, and unskilled positions respectively in sector \( i \) at time \( t \). To abstract from the fertility decision, we assume that the measure of newly-born workers are that of retirees at every moment. The dynamics of worker flows are summarized as follows.

\[
\begin{align*}
\dot{L}_{ist} &= -(\rho + \delta)L_{ist} + \chi_i\ell_{ist}f(\theta_{st})u_{st} \\
\dot{L}_{ict} &= -(\rho + \delta)L_{ict} + (1 - \chi_i)\ell_{ist}f(\theta_{st})u_{st} \\
\dot{L}_{iut} &= -(\rho + \delta)L_{iut} + \ell_{ist}f(\theta_{st})u_{st} \\
\dot{u}_{st} &= -(\rho + f(\theta_{st}))u_{st} + \delta(L_{1st} + L_{1ct} + L_{2st} + L_{2ct}) + \rho s_t \\
\dot{u}_{ut} &= -(\rho + f(\theta_{st}))u_{ut} + \delta(L_{1ut} + L_{2ut}) + \rho(1 - s_t)
\end{align*}
\]

**Wage Determination** Wages are determined by the internal bargaining mechanism proposed by Stole and Zwiebel (1996). Let \( \phi \in (0, 1) \) be the share of the marginal surplus given to the worker at each match. Then, the entrepreneur keeps \( (1 - \phi) \) portion of the marginal surplus. That is,

\[
(1 - \phi)(W_{ijt} - V_{jt}) = \phi \frac{\partial J_{it}}{\partial l_{ijt}}, \quad \forall t \in [0, \infty),
\]

where \( V_{ct} = V_{st} \) again. There is an implicit equilibrium restriction such that in any equilibrium \( W_{ijt} - V_{jt} \geq 0 \) and \( \frac{\partial J_{st}}{\partial l_{ijt}} \geq 0 \) for each \( j \in \{s, c, u\} \) and \( t \in [0, \infty) \). As the Stole and Zwiebel (1996) bargaining rule is true at each instant, then continuity implies that

\[
(1 - \phi)(\dot{W}_{ijt} - \dot{V}_{jt}) = \phi \frac{\partial}{\partial t} \left( \frac{\partial J_{st}}{\partial l_{ijt}} \right), \quad \forall t \in [0, \infty).
\]
The wage setting rule in the aggregate consistency requires that the vacancy creation decision of the individual entrepreneur creates the optimal number of vacancies at every moment. Newly born workers optimally choose their schooling level.

We finally finish this section by characterizing the equilibrium of our model. The following definition summarizes the general picture of our model.

**Definition** An equilibrium consists of choice rules $\{x_t, v_{ist}, v_{iut}\}_{i=1,2}$, labor market tightness parameter $\{\theta_{sit}, \theta_{uit}\}$, value equations $\{W_{ist}, W_{1st}, W_{ist}, W_{2st}, W_{2ut}, W_{ist}, W_{ut}, V_{ist}, V_{ut}\}$, and measures $\{L_{ist}, L_{1st}, L_{ist}, L_{2st}, L_{2st}, L_{2ut}, u_{ist}, u_{ist}\}$ at every $t \in [0, \infty)$ such that:

(i) newly born workers optimally choose their schooling level.

(ii) entrepreneur creates the optimal number of vacancies at every moment.

(iii) the aggregate consistency requires that the vacancy creation decision of the individual entrepreneur should be consistent with the definition of market tightness $\{\theta_{sit}, \theta_{uit}\}$.

(iv) The wage setting rule in (25) determines the wage payment to each type of matches at every $t$. The market clearing condition in (3) determines the price of each intermediate goods.

Figure 4: The Worker Flow

Guess $w_{ijt} = A_{ijt} (\partial y_{it} / \partial x_{ijt}) + B_{ijt}$ for each $j \in \{s, c, u\}$ and $t \in [0, \infty)$. Combining (12), (14), (15), (16) together with (23) and (24) yields differential equations. Plugging the guess form into the differential equations and applying the undetermined coefficient methods results in

$$
\begin{align*}
    w_{ijt} &= \begin{cases}
        \phi_{ijt} \frac{\partial w_{ijt}}{\partial x_{ijt}} + (1 - \phi) bP_t + \phi \theta_{sit} \xi \gamma_s (\ell_{ist} v_{ist}^{\xi-1} + \ell_{ist} v_{ist}^{\xi-1}) & \text{if } j = s \\
        \phi_{ijt} \frac{\partial w_{ijt}}{\partial x_{ijt}} + (1 - \phi) bP_{ct} + \phi \theta_{cst} \xi \gamma_s (\ell_{cst} v_{cst}^{\xi-1} + \ell_{cst} v_{cst}^{\xi-1}) & \text{if } j = c \\
        \phi_{ijt} \frac{\partial w_{ijt}}{\partial x_{ijt}} + (1 - \phi) bP_{ut} + \phi \theta_{uct} \xi \gamma_u (\ell_{uct} v_{uct}^{\xi-1} + \ell_{uct} v_{uct}^{\xi-1}) & \text{if } j = u
    \end{cases}
\end{align*}
$$

(25)

The first terms on the right hand side of equation (25) indicate that the wage payment is proportional to the marginal product of labor. The second term captures the labor market condition.

**Equilibrium** We finally finish this section by characterizing the equilibrium of our interest. The following definition summarizes the general picture of our model.
(v) The evolution of the entire system is recursively governed by the law of motions of (14)-(16) and (18)-(22), given initial $\{L_{is0}, L_{ic0}, L_{iu0}\}_{i=1,2}$ and $\{u_{s0}, u_{u0}\}$.

### 3.2 Characterization of Dynamic Adjustment Path

At every instant, $\{u_{jt}, V_{jt}\}_{j=s,u}$, and $\{L_{1jt}, L_{2jt}, W_{1jt}, W_{2jt}\}_{j=s,c,u}$ are given by the law of motions summarized in equilibrium condition (v). Suppose that $\{k_{it}\}_{i=1,2}$ are given at every instant. From (4), we get $(y_{it}, y_{jt})$, and from equations (2) and (3), $(p_{it}, p_{jt})$ at every $t \in [0, \infty)$. From (13), $s_t$ is obtained at every instant. Given $\{W_{1jt}, W_{2jt}\}_{j=s,c,u}$ and $\{V_{st}, V_{ut}\}$, $\{\partial J_{jt}/\partial l_{1jt}, \partial J_{jt}/\partial l_{2jt}\}_{j=s,c,u}$ are obtained from (23) at every instant. Equations (10) and (11) together with the definition of $\theta_{it}$ determine $\{v_{jt}, v_{2jt}\}_{j=s,u}$ and $\{\theta_{st}, \theta_{ut}\}$ at every $t$. Finally, $\{k_{it}\}_{i=1,2}$ should be consistent with equation (9). As a result, once the dynamic path of $\{L_{1jt}, L_{2jt}, W_{1jt}, W_{2jt}\}_{j=s,c,u}$ and $\{u_{jt}, V_{jt}\}_{j=s,u}$ are given, the dynamic path of the entire system are obtained.

The entire system of differential equations is an autonomous control problem, it can be pinned down to a boundary value problem, which is governed by the law of motions in equilibrium condition (v) and the initial values of $\{L_{1s0}, L_{2s0}, L_{1c0}, L_{2c0}, L_{1u0}, L_{2u0}, u_{s0}, u_{u0}\}$ and $\{W_{1s}, W_{2s}, W_{1c}, W_{2c}, W_{1u}, W_{2u}, V_{s}, V_{u}\}$. In other words, if we can solve for the initial values, we can solve for the entire system.

Now, we focus on the steady states to characterize the initial values. The stationarity condition dictates that

$$
\dot{L}_{ist} = \dot{L}_{ict} = \dot{L}_{iut} = \dot{u}_{st} = \dot{u}_{ut} = \dot{W}_{ist} = \dot{W}_{ict} = \dot{W}_{iut} = \dot{V}_{st} = \dot{V}_{ut} = 0. 
$$

Suppose that $(v_{1s}, v_{1u}, v_{2s}, v_{2u})$ are given on the ex post steady state. The values of $\{\ell_{is}, \ell_{iu}\}_{i=1,2}$ are immediately followed. Given $(\theta_s, \theta_u)$, the optimal education decision, $s$, is obtained by

$$
s = \left[1 + \exp \left(\frac{\varepsilon - \phi \xi (\gamma_s \theta_s (\ell_{1s} v_{1s}^{\ell_{1s}-1} + \ell_{2s} v_{2s}^{\ell_{2s}-1}) - \gamma_u \theta_u (\ell_{1u} v_{1u}^{\ell_{1u}-1} + \ell_{2u} v_{2u}^{\ell_{2u}-1}))}{(1 - \phi) P(r + \rho) \zeta}\right)^{-1}. 
$$

Since $l_{ij} = L_{ij}$, we obtain from (6) that for each $i \in \{1, 2\}$,

$$
L_{is} = \chi_i q(\theta_s) v_{is}/(\delta + \rho),
$$

$$
L_{ic} = (1 - \chi_i) q(\theta_s) v_{is}/(\delta + \rho), \quad \text{and}
$$

$$
L_{iu} = q(\theta_u) v_{iu}/(\delta + \rho).
$$

Plugging (27)-(30) into (21) and (22), making the left hand sides of them zero, and applying the definition of $(\theta_s, \theta_u)$ yields a 2-dimensional system of nonlinear equations,

$$
(v_{1s} + v_{2s})/\theta_s = [\delta (L_{1s} + L_{1c} + L_{2s} + L_{2c}) + \rho s]/(\rho + f(\theta_s)) \quad \text{and}
$$

$$
(v_{1s} + v_{2s})/\theta_u = [\delta (L_{1u} + L_{2u}) + \rho (1 - s)]/(\rho + f(\theta_u)),
$$

which solves for $(\theta_s, \theta_u)$. From equation (9), we can get

$$
k_{i}^{1-\beta_{ik}} = \frac{(1 - \phi) \alpha_i \beta_{ik} (p_{it}/P_t)}{p_{ik} (1 - \phi \beta_{ik})} l_{ist}^{\beta_{is}} (\lambda t_{ict} + l_{ilt})^{\beta_{tu}}, \quad \text{for each } i \in \{1, 2\}
$$

---

7Although we are not able to get an explicit formula for the transition dynamics, we can at least characterize the transition path numerically. In particular, we, following from Ishimaru, Oh and Sim (2014) iterate forward- and backward shooting algorithms to characterize the transition path.
by combining it with (28)-(30). Once we get \{L_{1j}, L_{2j}\}_{j=s,c,u} and \{k_{it}\}_{i=1,2}, we can solve for \( (y_1, y_2) \) and \( (p_1, p_2) \) using (2), (3) and (4). Finally, \( \{v_{is}, v_{iu}\}_{i=1,2} \) should be consistent with each entrepreneur’s vacancy creation decision, which implies that for each \( i \in \{1, 2\} \),

\[
1 = \frac{q(\theta_u)[1 - \phi] \frac{\partial y_u}{\partial l_u} - (1 - \phi)b - \theta_u \phi \gamma_u \xi(\ell_1 u v_1^{\xi - 1} + \ell_2 u v_2^{\xi - 1})]}{(r + \delta + \rho) \gamma_u \xi v_1^{1 - \xi}} \quad \text{and} \quad (34)
\]

\[
1 = \frac{q(\theta_s)[1 - \phi] \frac{\partial y_u}{\partial l_u} - (1 - \chi_i) \frac{\partial y_u}{\partial l_u} - (1 - \phi)b - \theta_s \phi \gamma_s \xi(\ell_1 s v_1^{\xi - 1} + \ell_2 s v_2^{\xi - 1})]}{(r + \delta + \rho) \gamma_s \xi v_1^{1 - \xi}}. \quad (35)
\]

As a result, the ex post steady state is pinned down to the 6-dimensional system of equations described in (27) - (35) for six unknowns \( (v_{is}, v_{iu}, v_{2s}, v_{2u}, \theta_s, \theta_u) \).

It is useful to compare the equilibrium outcomes of the models of skill biased technological changes and degree inflation. Figure 5 presents a brief sketch of the equilibrium outcomes of both models. First, the argument based on skill biased technological changes documents that even though the relative supply of skilled workers has increased over time, technological change has shifted labor demand toward skilled workers to a larger extent. Then, the shifted labor demand increases skill prices and employs more skilled workers. The dashed line in Panel (a) represents the reinforced labor demand in
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.05</td>
<td>Discount Rate</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.025</td>
<td>Retirement Rate</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>1.0</td>
<td>TFP of Traditional Sector (Normalization)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.335</td>
<td>Separation Rate</td>
</tr>
<tr>
<td>$(\nu_s, \nu_u)$</td>
<td>(0.46, 0.46)</td>
<td>Elasticity Parameter of Matching Function</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.46</td>
<td>Bargaining Power of Workers</td>
</tr>
<tr>
<td>$\xi$</td>
<td>2.0</td>
<td>Convexity of Vacancy Creation Cost Function</td>
</tr>
<tr>
<td>$(\eta_1, \eta_2)$</td>
<td>(0.1260, 0.1021)</td>
<td>Capital Depreciation Rate</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.5</td>
<td>Parameter of Investment Function</td>
</tr>
</tbody>
</table>

the skilled labor market. Of course, although all demand and supply curves should be adjusted after the technological changes, for simplicity, we adjust only the key driving force, the demand curve for the skilled workers. The two arrows in the skilled labor market tell us that employment and prices of the skilled workers increase together. In contrast, the dashed line in Panel (b) shows ‘supply shift’, which is caused by the increase in the number of college degree holders in the unskilled labor market. It lowers wages for both high school graduates and college graduates. However, the former declines to a greater extent, which increases wage differentials between high school graduates and college graduates. Moreover, since the college graduates get employment opportunity in the unskilled labor market as well as in the skilled labor market, the employment share of college graduates also rises. It crowds out the high school graduates from the unskilled labor market, which is captured by the shaded area in Panel (b).

4 Calibration

The purpose of this section is to provide quantitative assessment on the degree inflation occurred in Canada from 1990 to 2010. Section 4.1 provides the estimation procedure. Section 4.2 discusses the identification strategy and targeted time series that drive the results. Section 4.3 provides estimation results that match the Canadian labor market and patterns of educational attainment. Section 4.4 provides results of the counterfactual exercise to see how degree inflation matters for labor market outcomes and educational attainment.

4.1 Basic Parameterization

The parameters that are independently identified from the outside of the model but hardly estimated within the model are needed to be determined in advance. The base unit of time interval is normalized to be one year, which sets the discount rate to
Targeted Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{t}y_{1t}(p_{t}y_{1t} + p_{2t}y_{2t}) )</td>
<td>Sectoral Share of GDP (Traditional Sector)</td>
</tr>
<tr>
<td>( \sum_{j=s,c,u} L_{ijt}/(\sum_{i=1,2} j=s,c,u L_{ijt}) )</td>
<td>Sectoral Share of Employment (Traditional Sector)</td>
</tr>
<tr>
<td>( \sum_{i=1,2} j=s,c L_{ijt}/(\sum_{i=1,2} j=s,c L_{ijt}) )</td>
<td>Sectoral Share of College Graduate Employment (Traditional Sector)</td>
</tr>
<tr>
<td>( \sum_{i=1,2} j=s,c L_{ijt}/(\sum_{i=1,2} j=s,c,u L_{ijt}) )</td>
<td>Proportion of College Graduate Employees (in total)</td>
</tr>
<tr>
<td>( u_{st}/(u_{st} + \sum_{i=1,2} j=s,c L_{ijt}) )</td>
<td>Unemployment Rate of College Graduates</td>
</tr>
<tr>
<td>( u_{st}/(u_{st} + \sum_{i=1,2} j=s,c,u L_{ijt}) )</td>
<td>Unemployment Rate of Non-college Graduates</td>
</tr>
<tr>
<td>( \sum_{j=s,c,u} w_{1jt} L_{1jt}/(p_{1t} y_{1t}) )</td>
<td>Labor Income Share of the Traditional Sector</td>
</tr>
<tr>
<td>( \sum_{j=s,c,u} w_{2jt} L_{2jt}/(p_{2t} y_{2t}) )</td>
<td>Labor Income Share of the Modern Sector</td>
</tr>
<tr>
<td>( \sum_{i=1,2} w_{ut} L_{uit}/(\sum_{i=1,2} L_{uit}) )</td>
<td>Mean Wage for Postsecondary Graduates</td>
</tr>
<tr>
<td>( \sum_{i=1,2} w_{ut} L_{uit}/(\sum_{i=1,2} L_{uit}) )</td>
<td>Mean Wage for Non-Postsecondary Graduates</td>
</tr>
<tr>
<td>( \sum_{i=1,2} w_{ut} L_{uit}/(\sum_{i=1,2} L_{uit}) )</td>
<td>(Normalized by Mean Wage for Non-college Graduates in 1990)</td>
</tr>
<tr>
<td>( \sum_{i=1,2} w_{ut} L_{uit}/(\sum_{i=1,2} L_{uit}) )</td>
<td>(Normalized by Mean Wage for Non-Postsecondary Graduates in 1990)</td>
</tr>
</tbody>
</table>

\( r = 0.05 \). The retirement rate is also exogenously fixed at \( \rho = 0.025 \), which implies that an individual worker is expected to work for 40 years in the labor market. Due to recoverability issue, the discount rate and retirement rate are fixed rather than estimated in other papers as well. The productivity parameter of the traditional sector, \( \alpha_1 \), is fixed at one as normalization, while the counterpart of the modern sector is endogenously estimated.

Regarding the labor market parameters such as the bargaining power of workers, separation rate, and elasticity parameter of the matching technology, we follow Zhang (2008). The arrival rate of separation shock \( \delta \) is set to be 0.335, which yields total annual separation rate of 0.36 joint with the retirement. The elasticity parameter of the matching function and the bargaining power parameter of workers are set to be 0.46 following the common practice in literature. Zhang (2008) also equalizes those parameters by invoking the Hosios rule in Hosios (1990). Finally, we set the convexity of vacancy creation cost function at \( \xi = 2.0 \). Table 1 shows the parameters exogenously fed and the values of them.

### 4.2 Identification

Most time series of our interest are available from 1990 to 2012, except GDP share (from 1990 to 2010), wages (in 1990, 1995, 2000, and 2010), and the price of investment goods.
We classify industries into the modern or traditional sector on the basis of the employment share of college graduates in each industry. The industry having the share of college graduate employees higher (lower) than the national average of the share in 2010 is classified as the modern (traditional) sector.

By numerically solving the model, we simulate the dynamic transition path from the initial state of Canada in 1990 to the steady state. In the simulation, we update the price of investment goods in each sector every five years, which is anticipated by all economic agents from the beginning. For example, price of investment good from 1990 to 1995 is given by the average price from 1990 to 1994. Since the price data is available only until 2008, price of investment good from 2005 to the steady state is given by the average price from 2005 to 2008.

The GDP and employment shares of the traditional sector and the time series behavior of them are adopted to capture the magnitude of the technology shock in the modern sector \( \alpha_2 \) and the elasticity of substitution \( \sigma \) in workers’ preferences. The time series behavior of the sectoral share of college graduate employees determines qualification probability \( (\chi_1, \chi_2) \) by capturing complementarity and substitutability between workers with and without postsecondary education. The dynamic behavior of the proportion of college graduate employees in total employment are adopted to determine the education cost \( \epsilon \) and the sensitivity parameter of the education decision \( \zeta \). The unemployment rates among skilled and unskilled labor forces are exploited to fix the scale of matching technology \( (\mu_s, \mu_u) \). The dynamic paths of the labor income share in each sector, together with the sectoral share of college graduate employees are exploited to determine the input share \( (\beta_{1k}, \beta_{2k}, \beta_{1s}, \beta_{2s}) \). At last, the replacement ratio, which is targeted at 0.6 following Zhang (2008), determines the value of unemployment benefit \( b \). We finalize the parameters by minimizing the sum of squared deviation between two series in 1990, 1995, 2000, 2005, and 2010. [Table 2] presents the auxiliary moments that we adopt.

[Table 3] shows the parameter values finalized by estimation exercise. Roughly speaking, the parameter estimates are consistent with our intuition. The parameter values of \( (\beta_{1k}, \beta_{2k}, \beta_{1s}, \beta_{2s}) \) shows that the modern sector is more capital- and skill-intensive than the traditional sector. Also, the qualification probability, which implies the probability that a college graduate worker can work at the position with skill requirement conditioning on being employed, is much tougher in the traditional sector. Also, the values of \( (\mu_s, \mu_u) \) tell us that the college graduates use more efficient

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8In the data, the price index and amount of investment in chained dollars of each investment goods (ICT Equipment, Non-ICT Equipment, Building Structure, and Engineering Structure) for each industry are available, although the data are limited on business firms. We compute the chained Fisher price index of all investment goods in each sector, by taking the price index as ‘price’ and the amount of investment in chained dollar as ‘quantity’.

9The industry definition is mostly based on two digit NAICS code, except that it’s based on three digit in manufacturing division and some two-digit industries are treated as single industry in the dataset. Industries classified into the modern sector are Utilities (NAICS Code 22), Paper Manufacturing (Code 322), Petro/Coal Products (Code 324), Chemical Manufacturing (Code 325), Machinery Manufacturing (Code 333), Computer/Electronic Product Manufacturing (Code 334), Finance/Insurance/Real Estate/Rental and Leasing (Code 52 and 53; treated as single industry in the dataset), Professional/Scientific/Technical Services (Code 54), Educational Services (Code 61), Health Care/Social Assistance (Code 62), Information/Culture/Recreation (Code 51 and 71; treated as single industry), Other Services (Code 81), and Public Administration (Code 91). The others are classified into the traditional sector.

10It may not be identified well. Finalized \( \sigma \) is very close to the initial guess in the minimizing experiment.
technology in searching for jobs. The average education cost of 2.081 is roughly 80 percent of the average annual income. For example, when a resident in Wisconsin goes to any universities own by the state or by the neighboring states, such as University of Wisconsin, University of Illinois, or University of Minnesota, she/he pays around 30,000 U.S. dollars as tuition for 8 semesters, which is not so much different from the 80 percent of the GDP in United States.

5 Conclusion

This paper develops a job search model with task assignment and endogenous education choice to analyze, so-called, ‘degree-inflation’. In the model, the academic degree plays a significant role as an imperfect signal on the individual worker’s qualification. In particular, the workers, who acquire the academic degree and relevant skills through higher education, can perform skilled tasks with probability. The workers, who turn out to be not qualified for the skilled tasks are assigned to the unskilled tasks which have been performed by the workers without advanced degrees. It demonstrates that as more and more workers with advanced degree work at unskilled positions, the unskilled wages decline and the workers without academic degrees are crowded out by the degree holders, which raises college premium and college attainment again. Without significant improvement on overall qualification of the degree holders, the enhanced college attainment would push out more and more degree holders to take the unskilled jobs in the vicious circle.

We match the labor market developments and educational attainment trends in Canada over the last two decades, where the college premium and attainment rate have steadily increased, while the average real wages for both group have continuously declined. We show that the positive technology shock based on the recent development of information technology has initiated and aggravated the vicious circle by enforcing a substantial portion of the advanced degree holders to take unskilled jobs in the
traditional sector such as agricultural, fishing, mining, and so on, which did not hire many degree holders previously. It implies that the improvement of educational quality cannot catch up the improvement of the technological progress.

Unfortunately, those seemingly overqualified workers working at unskilled positions with advanced degrees are growing concerns of not only Canada but also other well-developed countries. In the OECD on average, about one in four workers are reported as a mismatch in terms of their academic degree and job description. To stop degree inflation, it should be essential to achieve the separating equilibrium without cross-skill matches by improving the quality of higher education. Also, the career counseling and internship program for university students could be helpful in reducing cross-skill matches.
References


A Mathematical Derivation

A.1 The Optimal Vacancy Creation Decision

We proceed with the optimal control by the entrepreneur in sector $i$. The entrepreneur chooses the schedule of $(v_{i\tau s}, v_{i\tau u})$ at every $\tau \in [t, \infty)$ to maximize

$$J_d(k_{it}, l_{ist}, l_{ict}, l_{iut}) = \int_t^\infty e^{-r(\tau-t)} P_\tau^{-1} d\tau$$

(A1)

subject to (6) and the initial condition $(k_{it}, l_{ist}, l_{ict}, l_{iut}) = (k_{i0}, l_{ist0}, l_{ict0}, l_{iut0})$. The Hamiltonian for the above problem is

$$\mathcal{H} = e^{-r(\tau-t)} P_\tau^{-1} - \mu_k[\eta_h k_{it} - \kappa_0^{-1} \eta_{it}^0] - \mu_s[\delta + \rho]l_{ist} - \chi q(\theta_{st})v_{i\tau s} - \mu_c[\delta + \rho]l_{iut} - q(\theta_{ut})v_{i\tau u}$$

The maximum principle implies that

$$x_{i\tau} = e^{-r(\tau-t)} p_x = \mu_k \kappa_0^{-1} \kappa x_{i\tau}^{-1}$$

(A2)

$$v_{i\tau s} = e^{-r(\tau-t)} \gamma_s x_{i\tau} = \mu_s q(\theta_{st}) + \mu_c (1 - \chi_i) q(\theta_{st})$$

(A3)

$$v_{i\tau u} = e^{-r(\tau-t)} \gamma_u x_{i\tau} = \mu_u q(\theta_{ut})$$

(A4)

$$k_{i\tau} = \dot{\mu}_k = -e^{-r(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} P_\tau^{-1} + \mu_k \eta_h$$

(A5)

$$l_{i\tau s} = \dot{\mu}_s = -e^{-r(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial l_{i\tau s}} P_\tau^{-1} + \mu_s (\delta + \rho)$$

(A6)

$$l_{i\tau u} = \dot{\mu}_u = -e^{-r(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial l_{i\tau u}} P_\tau^{-1} + \mu_u (\delta + \rho)$$

(A7)

$$l_{i\tau c} = \dot{\mu}_c = -e^{-r(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial l_{i\tau c}} P_\tau^{-1} + \mu_c (\delta + \rho)$$

(A8)

Solving differential equations (A6)-(A8) yields that

$$\mu_j = \int_t^\infty e^{-(r+\delta+\rho)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial \pi_{i\tau}} P_\tau^{-1} d\tau + C_j e^{(\delta+\rho)(\tau-t)}, \text{ for each } j \in \{s, u, c\}.$$  \hspace{1cm} (A9)

Since $\mu_j$ cannot diverge, the integral constant $C_j$ should be zero. Plugging (A9) into (A3), and (A4) yields

$$\gamma_s x_{i\tau}^\xi = q(\theta_{st}) \int_t^\infty e^{-(r+\delta+\rho)(\tau-t)} \left[ \kappa_i \frac{\partial \pi_{i\tau}}{\partial l_{i\tau s}} + (1 - \chi_i) \frac{\partial \pi_{i\tau}}{\partial l_{i\tau c}} \right] P_\tau^{-1} d\tau$$

(A10)

$$\gamma_u x_{i\tau}^{\xi-1} = q(\theta_{ut}) \int_t^\infty e^{-(r+\delta+\rho)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial l_{i\tau u}} P_\tau^{-1} d\tau$$

(A11)

By the same reasoning, solving differential equation (A5) and combining with (A2) yields

$$x_{i\tau}^{1-\kappa} = \frac{\kappa_0^{-1} \kappa}{p_x} \int_t^\infty e^{-(r+\eta_i)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} P_\tau^{-1} d\tau$$

(A12)