Allocating Household Time:

When Does Efficiency Imply Specialization?

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ABSTRACT

When does efficiency in the household imply specialization? More specifically, if we recognize two sectors, “market” and “household,” when does efficiency imply "sector specialization"? In this paper I clarify the roles that household technology and human capital play in reaching conclusions about specialization.

The critical assumption that leads to the specialization conclusion in Becker's *Treatise on the Family* is that spouses' time inputs are perfect substitutes in household production. With no further assumptions (other than efficiency and the absence of process preferences) perfect substitutes imply specialization. Although some of Becker's proofs appear to rely on households optimally adjusting spouses' stocks of market and household human capital, actually, the specialization conclusion does not. With perfect substitutes, efficiency implies specialization even when each spouse's stocks of human capital are fixed, regardless of the levels at which they are fixed. Other assumptions about household technology also imply the specialization conclusion. I prove that (again in the absence of process preferences) if the household technology is "additive" and exhibits constant returns to scale, then efficiency implies specialization. A technology is additive if the output of each spouse in the household sector is independent of the time that the other spouse allocates to the household sector. The specialization conclusion implied by additivity and constant returns to scale, like the specialization conclusion implied by perfect substitutes, is independent of any assumptions about human capital.

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1. **Introduction**

When does efficiency in the household imply specialization? More specifically, if we recognize two sectors, "market" and "household," when does efficiency imply "sector specialization"? Becker (1981, 1991) raised the issues of the division of labor and specialization in his *Treatise on the Family*.[1] This paper revisits the issue of specialization.

In the *Treatise*, Becker identifies a class of household technologies for which efficiency implies specialization, namely, technologies in which the time inputs of husbands and wives are perfect substitutes. In this paper I show that there is an additional class of household technologies for which efficiency implies specialization. Specifically, I show that if the household technology is "additive" and exhibits constant returns to scale, then efficiency implies specialization.[2] [3]

Human capital *appears* to play a crucial role in Becker's analysis of specialization. Actually, it does not. If spouses' time inputs are perfect substitutes, then efficiency implies specialization even if each spouse's stocks of human capital are fixed at arbitrary levels and regardless of the strength of human capital effects on wages and on productivity in the household. Similarly, if the household technology is additive and exhibits

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[1] Hereafter I cite the *Treatise* as Becker (1991). Chapter 2 of the *Treatise*, entitled "Division of Labor in Households and Families," appeared in the 1981 edition. Many papers credit Becker (1965), "A Theory of the Allocation of Time," with raising the specialization issue, perhaps because in retrospect the division of labor and specialization seem obvious grist for the household production mill. But what seems obvious in retrospect was not obvious in prospect. Becker (1965) devotes only a single paragraph to multiple-person households; the rest of that paper, like Chapter 1 of the *Treatise*, assumes single-person households, so issues of specialization and the division of labor do not arise. A decade later, Pollak and Wachter (1975) missed the opportunity to develop the household production model in this "obvious" direction.

[2] I also show that Becker's "efficiency implies specialization claim" (the "specialization claim," for short) does not hold for all household technologies. If the specialization claim held for all technologies, then the observed pattern of widespread nonspecialization (i.e., both husbands and wives work in both the market sector and the household sector) would be evidence of widespread inefficiency.

[3] Following Becker, I focus on married couples. With some modification, the analysis applies to other types of multiple-person households.

constant returns to scale, then efficiency implies specialization regardless of assumptions about human capital.

If efficiency did imply specialization, then egalitarian marriages would be inefficient and an equity-efficiency tradeoff inescapable. Two assumptions link distribution in marriage to the specialization claim: (1) the assumption that distribution between spouses depends on bargaining in marriage, and (2) the assumption that bargaining power depends on earnings or wages. If equity depends on parity in bargaining power, and if bargaining power depends on earnings or wages, then equity requires that both spouses work in the market. But if the household is efficient and both spouses work in the market, then specialization implies that one spouse must do all the housework. Hence, if efficiency implies specialization, these two linking assumptions imply that efficient couples must choose between unequal bargaining power and an inequitable division of household work: equity and efficiency would be incompatible.[4] These distributional implications account for the continuing ability of the specialization claim to generate controversy.

To begin, we need a definition of specialization. With two sectors, we distinguish between "strong specialization" and "weak specialization." With strong specialization, each spouse allocates time to only one sector (e.g., one spouse works exclusively in the

[4] Although the core of Becker's specialization argument is gender neutral, Becker famously argues that the efficient pattern of specialization is gendered, with wives allocating time to household production and husbands allocating time to the market. Becker's argument is two-pronged. The first prong is the claim that gender serves as a focal point for premarital investments in specialized human capital: before entering the marriage market, females invest in household human capital and males in market human. The second prong shifts the focus from wives to mothers. The claim is that even without specialized premarital investments in household human capital, mothers, because of their ability to breast-feed infants, would slide down a slippery slope toward specialization in household production. That is, even if fathers and mothers were initially equally productive in home and market, the ability of mothers to breast-feed leads to an equilibrium in which the efficient pattern of specialization is gendered with mothers specializing in the household and fathers in the market. In this paper I focus on the core specialization claim, not on the gendering claim.

market and the other spouse works exclusively in the household). With weak specialization, one spouse may work in both sectors, so there are three possible patterns of weak specialization:

(1) both spouses work in the market and only one works in the household;

(2) both spouses work in the household and only one works in the market; and

(3) each spouse allocates time to only one sector (i.e., strong specialization).

The interpretation and analysis of the specialization claim is sensitive to whether we recognize more than one household activity. To focus on sector specialization, I assume that there is only one household production activity.[5]

Four assumptions play critical roles in the analysis of sector specialization. Three of these are about household technology (perfect substitutes, additivity, and constant returns to scale), and the fourth is about preferences (i.e., the absence of "process preferences"). I begin with household technology.

With perfect substitutes, an "efficiency factor" converts the time input of the wife into units comparable with the time input of the husband. Hence, the marginal rate of substitution of the husband's time for the wife's time is constant. Becker uses the perfect substitutes assumption to motivate his discussion of specialization and, although none of his specialization theorems explicitly assumes perfect substitutes, the surrounding

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[5] This assumption is consistent with the discussion of specialization in Chapter 2 of the *Treatise* which recognizes only one household production activity. I relax this assumption in section 6. The discussion of household production in Chapter 1 of the *Treatise*, like the classic discussion in Becker (1965), recognizes m household production activities.

discussion and the proofs of several of the specialization theorems rely on perfect substitutes.[6]

The perfect substitutes assumption is problematic for two reasons. First, because it is implausible, the perfect substitutes assumption severely limits the applicability of the specialization claim.[7] Second, because perfect substitutes imply specialization, the perfect substitutes assumption makes human capital irrelevant to the specialization claim and makes redundant the explicitly stated hypotheses of the specialization theorems in the *Treatise*. More precisely, in the absence of process preferences, if spouses' time inputs are perfect substitutes, then efficiency implies specialization. Full stop. That is, with perfect substitutes, additional assumptions (other than the absence of process preferences) are not needed to reach the specialization conclusion. Because I analyze perfect substitutes in detail in Pollak (2012), I discuss them only briefly in this paper.

"Additivity" is the second critical assumption. If the household technology is additive and both spouses engage in household production, then the total output they produce is the sum of the outputs they could produce separately.[8] Additivity implies that the output of each spouse is independent of the time the other spouse allocates to household production. For some nonadditive household technologies (e.g., the Cobb-Douglas), time inputs by both spouses are required to produce positive output. But even if time inputs by both spouses are not essential, bilateral household production (i.e., both spouses allocate time to the household sector) may be efficient. The additivity assumption

[6] In Pollak (2012) I erroneously claimed that although Becker (1991) uses the perfect substitutes case to motivate his discussion of specialization, "his specialization theorems do not assume that spouses' time inputs are perfect substitutes." This is flat-out wrong. Most of Becker's specialization theorems assume perfect substitutes and the interpretation of the remaining theorems depend on the perfect substitutes assumption.

[7] Lundberg (2008) and in Pollak (2012) elaborate this point.

rules out a wide range of technologies, including the Cobb-Douglas. I prove that with nonadditivity, efficiency may require bilateral household production and, for some wage rates, nonspecialization. I also show that, in the absence of process preferences, if the household technology is additive and exhibits constant returns to scale, then efficiency requires specialization.

"Nondecreasing returns to scale" is the third critical assumption. I show that with decreasing returns to scale efficiency may require bilateral household production and, for some wage rates, nonspecialization. I argue that decreasing returns are plausible if individuals' productivities decline as spouses become tired or bored with an activity (such as childcare, perhaps).[9]

The fourth critical assumption is the absence of "process preferences." With process preferences, individuals care how they spend their time. When there are two or more household activities, process preferences may take the form of a preference for cooking rather than cleaning. When there is only one household activity, process preferences allow for the possibility that individuals enjoy working in the market more than they enjoy working in the household, or vice versa. Even when spouses' time inputs are perfect substitutes in household production, if both spouses have sufficiently strong preferences for allocating time to both sectors, then Pareto efficiency will require that both spouses allocate time to both sectors.[10]

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[8] This informal definition implicitly assumes that output is produced without nonlabor inputs. The formal definition, given in section 3, is more complicated because of the need to deal with nonlabor inputs.
[9] The assumption that productivity declines as individuals become tired or bored is distinct from process preferences, although both productivity and preference effects can operate simultaneously.
[10] As Folbre (2004) points out, economists generally interpret Pareto efficiency in a way that ignores outcomes for children, except to the extent that these outcomes enter into their parents' preferences. A broader notion of Pareto efficiency would take account of the preferences, interests, or well-being of children.

Becker's analysis of specialization overemphasizes the role of human capital. The specialization conclusion follows directly from the assumption that spouses' time inputs are perfect substitutes.[11] Although human capital is unnecessary for the specialization conclusion when spouses' time inputs are perfect substitution, recognition that human capital plays a role in household production leads in two fruitful directions. First, when there are two kinds of human capital, household and market, we can investigate *human capital* specialization as well as *time* specialization. Second, in a dynamic setting human capital strengthens the incentives for time specialization. With perfect substitutes or with additivity and constant returns to scale, strengthened incentives for specialization are redundant. But when the household technology does not necessarily lead to specialization, human capital investments may provide incentives that tip the balance in favor of specialization. The mere presence of human capital, however, does not automatically lead to specialization. The strength of the effect of human capital on wages and on productivity in the household determines whether human capital can tip the balance in favor of specialization.

The paper proceeds as follows. Section 2 discusses the meaning of specialization, process preferences, and perfect substitutes. I define additivity in section 3 and, in section 4, prove that, in the absence of process preferences, additivity and constant returns to scale imply the specialization conclusion. Section 5 argues that decreasing returns are plausible and shows that, even with additivity, the specialization conclusion need not hold when household technology exhibits decreasing returns. Section 6 considers multiple activities in the household sector and "activity specialization." I show that even

[11] It also follows directly from the assumption that the household technology is additive and exhibits constant returns to scale. In both of these cases, process preferences can invalidate the specialization conclusion.

with perfect substitutes, when there are m household activities, the *sector* specialization claim fails; it is easy to construct examples in which one spouse allocates time to m* household activities, the other spouse allocates time to the remaining m-m* activities, and both spouses allocate time to the market. Section 7 discusses human capital. I argue that, in a dynamic setting, human capital can tip the balance in favor of specialization, but that the strength of human capital effects on wages and on productivity in the household is crucial. Section 8 is a brief conclusion. In the appendix I discuss the specialization theorems from the *Treatise on the Family*.

2. Specialization, Process Preferences, Perfect Substitutes

Time allocation in married couple households depends on three elements: preferences, constraints, and the household's "governance structure."[12] Preferences, in this context, means the preferences of both spouses. The constraints reflects the wage rates of both spouses, the prices of market goods, the household technology, and spouses' individual technologies. Individuals' technologies include the technologies to which they would have access if they were to leave the marriage. The later are important because in virtually all models individuals' technologies if they were to leave the marriage determine the spouses' outside options and, in some models, also determine bargaining power. The "governance structure" determines the mapping from preferences and constraints into allocations of goods, commodities, and time. Examples of governance structures include Becker's altruist model and cooperative Nash bargaining. Chiappori's (1988, 1992)

[12] I ignore information structure which, as Randy Wright pointed out when he discussed a very early version of this paper, is an additional basic element.

"collective model" can be interpreted as a reduced form corresponding to any model with a single-valued, Pareto-efficient solution.

In Pollak (2012) I discuss these three elements of the time allocation model as well as the meaning of specialization. Except in special cases, conclusions about time allocation (e.g., specialization) depend on all three elements -- preferences, constraints, and the governance structure -- and cannot be inferred from the constraints or a subset of the constraints (e.g., wage rates and household technology).

The distinction between goods and commodities is central to the household production model. Becker (1965) wrote: "Households will be assumed to combine time and market goods to produce more basic commodities that directly enter their utility functions." I begin by introducing notation and terminology. I denote the household production function for the commodity z by $g[t_h,t_w,y]$, where t_h and t_w denote the time inputs of the husband and wife into the production of z; y denotes the market goods used to produce z.[13] For a commodity produced within the household, either both spouses allocate time to its production ("bilateral production") or only one spouse allocates time to its production ("unilateral production"). Bilateral production and unilateral production are properties of the spouses' time allocation and, except in special cases, they depend on preferences, constraints, and the governance structure. If a commodity is produced unilaterally, the relevant domain of the household production function consists of the values at which $t_h = 0$ or $t_w = 0$.[14] Thus, $g[t_h,0, y_h]$ and $g[0,t_w, y_w]$ are the unilateral production functions.

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[13] My notation here differs from that in the *Treatise* and also from that in Pollak (2012).

[14] The claim that unilateral production is efficient requires comparing alternatives involving unilateral production with alternatives involving bilateral production.

"Essentiality" assumptions formalize the notion that positive output requires positive time inputs from one or both spouses.

Strong Essentiality Assumption:

$$g[0,t_w, y_w] = g[t_h,0, y_h] = 0 \quad \text{for all } \{t_h,t_w,y_h,y_w\}.$$[15]

That is, positive output requires positive time inputs from both spouses, as in the Cobb-Douglas household production function.

***Weak Essentiality Assumption*:**

$$g[0,0,y] = 0 \text{ for all } y.$$

That is, positive output requires positive time inputs from at least one spouse. Throughout this paper, I assume that the household production function satisfies weak essentiality and, sometimes, that it also satisfies strong essentiality.

One or both spouses may work in the market, earning money to purchase market goods. I denote the time that each spouse allocates to work by $\{T_h, T_w\}$ and the time that each spouse allocates to the market sector by $\{t_{h0}, t_{w0}\}$ so

$$t_h + t_{h0} = T_h$$

and

$$t_w + t_{w0} = T_w.$$

In my examples, I generally assume $T_h,= T_w$ which is consistent with the finding of Burda, Hamermesh, and Weil (forthcoming) that "in rich non-Catholic countries, men and women average about the same amount of total work." They also find that this pattern of "isowork" holds, on average, for married couples in rich non-Catholic countries. I assume that there is

[15] Equivalently, if $t_h t_w = 0$, then $g[t_h,t_w,y] = 0$ for all y.

only a single market good and normalize its price to 1.[16] The quantity of the market good is given by

$$x = w_h t_{h0} + w_w t_{w0} + x^* = w_h(T_h - t_h) + w_w(T_w - t_w) + x^*$$

where $\{w_h, w_w\}$ are the spouses' wage rates and x^* is nonlabor income.[17]

Market goods play two roles. They are (or may be) inputs into the production of commodities, and they are (or may be) arguments of the spouses' utility functions. I allow for the possibility that market goods enter the spouses' utility functions directly, unmediated by household time.[18]

To define "specialization," it is best to begin with its opposite, "nonspecialization." With two sectors, home and market, we say there is nonspecialization if and only if both spouses allocate time to both sectors. Specialization is anything else (i.e., any time allocation in which both spouses do not allocate time to both sectors). This definition of specialization is consistent with standard usage in the economics of the family and analogous to usage familiar in international economics.

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[16] Unless the relative prices of market goods vary, we lose nothing by ignoring the multiplicity of market goods and restricting our attention to a single, aggregate market good. As I argue in section 6, we do lose something by ignoring the multiplicity of commodities.

[17] Becker (1965, 1991) emphasizes the role of nonlabor inputs (e.g., calories, nutrients, sleep) as well as human capital as determinants of market wage rates and earnings. I ignore these effects because they do not affect the validity of the specialization claim.

[18] Although I have not done so, it is sometimes convenient to treat the market sector as if it were another household activity, while recognizing that the "technology" for "producing" the market good has a different structure than most household production activities. The usual assumption that spouses' wage rates are constants (i.e., independent of the time inputs of the spouses to the market sector) implies that the marginal product of labor in market work (i.e., the wage rate) is constant. Hence, under the usual assumption that individuals face market wage rates that are independent of the number of hours they allocate to market work, the implied "production function" for the market good is linear. Progressive taxes destroy this linear relationship.

I have assumed that the market good enters the spouses' utility functions directly. If we insist that the arguments of the utility functions are "commodities," then we could introduce an additional household production activity with the property that $g[t_h, t_w, y] = y$ for all $\{t_h, t_w, y\}$. This would, of course, violate the weak essentiality assumption. As Gershuny (2000) points out, Becker (1965) emphasized that consuming market goods takes time, an insight that has been largely eclipsed by the subsequent emphasis on household production rather than consumption technology.

To summarize:

1. Nonspecialization: Both spouses allocate time to both sectors.
2. Specialization: At least one spouse allocates time to only one sector.[19]
3. Strong Specialization: Each spouse allocates time to only one sector.

Strong specialization includes not only the case in which spouses allocate time to different sectors (e.g., husbands work in the market and only in the market; wives work in the household and only in the household), but also to the case in which both spouses allocate all of their time to the same sector (e.g. both spouses allocate all of their time to the household). I call this case "superstrong specialization."[20]

Nonspecialization implies bilateral household production, but specialization opens up the possibility of unilateral household production. With specialization, there are three cases:

(i) if both spouses work in the market, then only one spouse works in the household and, hence, the only portions of the household production functions we observe are the unilateral production functions;

(ii) if both spouses work in the household, then only one spouse works in the market; in this case, we have bilateral household production;

(iii) each spouse allocates time to only one sector (i.e., strong specialization).

[19] "Specialization" might be called "weak specialization."

[20] Superstrong specialization may at first seen pathological because we usually apply the household production model to working-age couples in which at least one spouse works in the market and at least one spouse works in the household. Retired couples in which both spouses work in the household are obvious examples of superstrong specialization. Unemployed couples in which both spouses work in the household may also exemplify superstrong specialization provided neither spouse engages in job search (assuming that job search counts as market work).

The validity of the specialization claim depends on assuming away process preferences or restricting them so that they strengthen rather than weaken the incentives to specialize. In the context of one-person households, Pollak and Wachter (1975, p. 256) emphasize that the allocation of time may depend on the direct utility associated with time spent in an activity: “time spent in many production activities is a *direct* source of utility as well as an input into a commodity."[21] Juster and Stafford (1991) call these "psychic benefits" or "process benefits." "Process preferences" is a better term because it more easily accommodates negative effects ("disbenefits") as well as positive effects. The usual assumption in the new home economics is the absence of process preferences. Without process preferences, individuals care only about the nominal outputs of home production (a clean house; a home-cooked meal) but not about how they spend their time (cleaning; cooking). With two sectors, market and household, the absence of process preferences implies that market work and household work are perfect substitutes in both spouses' utility functions. Spouses care about total work, $\{T_h, T_w\}$, but are indifferent between an hour of market work and an hour of household work; in terms of (dis)utility, "work is work."[22] [23] For the remainder of this paper, I assume the absence of process preferences.

None of Becker's specialization theorems explicitly impose the perfect substitutes assumption, but he makes it clear in the surrounding text that he assumes perfect substitutes; this is confirmed by his use of the perfect substitutes assumption in several proofs. Formally, perfect substitutes imply a household production function of the form

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[21] Pollak and Wachter show that process preferences imply joint production and that process preferences require us to use production sets rather than production functions to represent the household technology. Except in section 6 where I discuss multiple household activities and economies of scope, I assume that household technology does not involve joint production.

[22] Perfect substitutes in the spouses’ utility functions is quite different from perfect substitutes in the household production function. Hereafter, perfect substitutes refers to the household production function.

$$g[t_h,t_w,y] = G[t_h + \alpha(y)t_w,y]$$

where the "efficiency factor," $\alpha(y)$, converts the time input of the wife into units comparable with the time input of the husband. In Pollak (2012) I analyze in detail the implications of the perfect substitutes assumption and argue that the assumption that spouses' time inputs are perfect substitutes is implausible. In the absence of process preferences, if spouses' time inputs are perfect substitutes then, with no further assumptions about technology (e.g., about additivity, returns to scale, or human capital), efficiency implies specialization.

3. Additivity

The additivity assumption postulates that total output is the sum of the outputs the spouses could produce unilaterally when nonlabor inputs are allocated between them so as to maximize output. [24]

Formally,

***Additivity Assumption*: ** The household technology is of the form

$$g[t_h, t_w,y] = \max \{g^h[t_h,0,y_h] + g^w[0,t_w,y_w]\}$$

$$\text{subject to } y_h + y_w \leq y.$$

With additivity, bilateral production implies that spouses produce "side-by-side," each using his or her unilateral technology. When there are no nonlabor inputs, the additivity assumption simplifies to

$$g[t_h, t_w] = g[t_h, 0] + g[0, t_w].$$

[23] In labor economics, interest in process preferences and non-pecuniary benefits goes back to Adam Smith.

In Pollak (2012) I show that additivity and perfect substitutes are compatible only in a narrow class of cases.

The additivity assumption requires scrupulously maintaining the distinction between the household production function and the spouses' unilateral production functions. For example, the Cobb-Douglas household production function is given by

$$g[t_h, t_w, y] = A t_h^{\beta h} t_w^{\beta w} y^{\gamma}.$$

The unilateral production functions corresponding to the Cobb-Douglas household production function are given by

$$g[t_h, 0, y_h] = 0$$

and

$$g[0, t_w, y_w] = 0.$$

That is, the unilateral production functions corresponding to the Cobb-Douglas household production function produce 0 output -- not a surprise, because the Cobb-Douglas household production function yields 0 output unless both spouses' time inputs are positive.

With additivity, the spouses' unilateral production functions contain all the information required to construct the household production function. That is, with additivity the spouses' unilateral production functions are a sufficient statistic for the

[24] In Pollak (2012) I incorrectly claim that Becker's analysis of specialization "implicitly assumes that household technology satisfies the additivity assumption."

household production function. For example, if we assume that the household production function is additive, and if we begin with Cobb-Douglas *unilateral* production functions

$$g^h[t_h,0,y_h] = A_h \, t_h^{\delta h} \, y_h^{\varepsilon h}$$

and

$$g^w[0,t_w,y_w] = A_w \, t_w^{\delta w} \, y_w^{\varepsilon w}$$

then the household production function is given by

$$g[t_h, t_w,y] = \max \{ A_h \, t_h^{\delta h} \, y_h^{\varepsilon h} + A_w \, t_w^{\delta w} \, y_w^{\varepsilon w} \}$$

subject to

$$y_h + y_w \leq y.$$

The implied household production function is not Cobb-Douglas and does not exhibit strong essentiality.

Additivity is a useful special case for household production for two reasons. First, because additivity is tractable it provides a ready source of transparent examples and counterexamples. Second, the additive case provides an alternative to Becker's interpretation of the assumption that spouses are "intrinsically identical." Becker interprets "intrinsically identical" to mean that spouses' time inputs are perfect substitutes in household production. The additive case in which spouses have identical unilateral production functions provides an alternative instantiation of "intrinsically identical."

Although additivity may have been plausible for international trade in the 18th and early 19th centuries, it is implausible for households.[25] For households, we want to leave open the possibility that bilateral production yields output greater than the sum of the outputs the spouses could produce unilaterally. For example, spouses might produce greater output if they were able to divide household production into component tasks, mirroring within the household the division of labor that Adam Smith observed in the pin factory.

Without additivity, efficiency may require nonspecialization. The Cobb-Douglas household production function provides a simple example. Suppose that the household technology is given by

$$g[t_h, t_w, y] = A t_h^{\beta h} t_w^{\beta w} y^{\gamma}.$$

where $\gamma = 0$ and $\beta_h = \beta_w = 1/2$. That is, the household commodity is produced by time alone and the spouses are equally productive. Suppose that $T_h = T_w = 1$, so that each spouse has one unit of time to be allocated between household production (t_h, t_w) and market work $(t_{h0}, t_{w0}) = (1 - t_h, 1 - t_w\}$. With no nonlabor income and the price of the market good normalized to 1, the market good is given by

$$x = w_h t_{h0} + w_w t_{w0} = w_h(1 - t_h) + w_w(1 - t_w).$$

[25] Additivity is a standard assumption in international economics. The Ricardian model of comparative advantage begins with each country's unilateral production function for each good (e.g., cloth; wine). The world's production function is the sum over all countries of these unilateral production functions. This assumes that all factors other than labor inputs are mobile; this assumption is not as restrictive as it at first appears because nonmobile factors can be incorporated in the unilateral production functions.

Suppose the spouses' wage rates are $w_h = w_w = 1$, and consider the symmetric nonspecialized time allocation $t_h = t_w = 1/4$. This time allocation implies z = 1/4 and x = 3/2, so the vector (z,x) = (1/4, 3/2) is feasible. But the vector (z,x) = (1/4, 3/2) cannot be produced with specialization, contrary to any general claim that efficiency requires specialization. When $w_h = w_w = 1$, both spouses must allocate time to the market to satisfy $w_h t_{h0} + w_w t_{w0} = w_h(1- t_h) + w_w(1- t_w) = 3/2$. And both spouses must allocate time to household production to produce z = 1/4.

This counterexample to the general specialization claim is not a razor's edge case. It is easy to see that efficiency requires nonspecialization as we vary the parameters (β_h , β_w, w_h, w_w) where $\beta_w = 1 - \beta_h$ in a neighborhood of (β_h , β_w, w_h, w_w) = (1/2, 1/2, 1, 1).

4. A New Specialization Theorem

In this section I prove a new specialization theorem.

Theorem: In the absence of process preferences, if the household technology is additive and exhibits constant returns to scale, then efficiency implies specialization.[26]

This establishes that the class of technologies for which efficiency implies specialization includes more than perfect substitutes.

Proof: Suppose, on the contrary, that nonspecialization is efficient. Efficient allocations maximize the output of the household commodity, subject to appropriate constraints (see below). Hence, the program

[26] For the remainder of this paper, I assume the absence of process preferences.

$$M = M[t_h,y_h,t_w,y_w] = \max \{g^h[t_h,0,y_h] + g^w[0,t_w,y_w]\}$$

subject to the constraint

$$y_h + y_w + x^{**} \leq w_h(T_h - t_h) + w_w(T_w - t_w) + x^*$$

has an interior solution -- that is, a soluton satisfying

$$0 < t_h < T_h \text{ and } 0 < t_w < T_w .$$

The term x^{**} is the required output of the market good that is consumed directly. Thus, $x^{**} = 0$ corresponds to the case in which the market good does not enter the spouses' utility functions, but serves only as an input into the production of the home produced commodity.

From the first order conditions

$$\frac{\dfrac{\partial g^h[t_h,0,y_h]}{\partial t_h}}{\dfrac{\partial g^h[t_h,0,y_h]}{\partial y_h}} = w_h$$

Because $g^h[t_h,0,y_h]$ is homogeneous of degree 1, the marginal rate of substitution is homogeneous of degree 0. Hence, for values of $\{t_h,y_h\}$ satisfying the first order conditions we have

$$y_h = \mu^h(w_h)t_h .$$

By an analogous argument

$$y_w = \mu^w(w_w)t_w .$$

Substituting for y_h and y_w in the constraint yields

$$w_h(T_h - t_h) + w_w(T_w - t_w) = \mu^h(w_h)t_h + \mu^w(w_w)t_w + x^* - x^{**}.$$

Because the constraint is linear in $\{t_h,t_w\}$, we can solve it for t_w as a linear function of t_h.

Substituting for y_h and y_w in the maximand yields

$$M = M^*[t_h,w_h,t_w,w_w] = \max \{g^h[t_h,0,\mu^h(w_h)t_h] + g^w[0,t_w,\mu^w(w_w)t_w]\}.$$

Because the unilateral production functions are homogeneous of degree 1, this becomes

$$M = M^*[t_h,w_h,t_w,w_w] = \max \{ t_h\, g^h[1,0,\mu^h(w_h)] + t_w\, g^w[0,1,\mu^w(w_w)]\}.$$

That is, the maximand is a linear function of $\{t_h,t_w\}$.

Because the constraint implies that t_w is a linear function of t_h, we can eliminate t_w from the maximand and write it as a linear function of t_h. Hence, the program has a corner solution (i.e., either $t_h = 0$ or $t_h = T_h$). This implies specialization, contrary to our initial assumption of nonspecialization. ■

5. Returns to Scale

Increasing returns and decreasing returns raise distinct issues. In section 5a I discuss Becker's claim that, with perfect substitutes and increasing returns, efficiency implies strong specialization. I show that this claim, unlike Becker's other specialization claims, does not hold when each spouse's stocks of human capital are fixed; I defer until section 7 the discussion of whether the claim holds when households optimally adjust spouses' stocks of human capital. In section 5b I show that the specialization theorem of section 4 (i.e., with additivity and constant returns, efficiency implies specialization) ceases to hold when we replace constant returns with decreasing returns. That is, with additivity and decreasing returns, efficiency may require nonspecialization. I also argue that decreasing returns is sometimes plausible.

5a. Increasing Returns to Scale and Strong Specialization

In this subsection I construct a transparent counterexample to the claim that with perfect substitutes and increasing returns to scale, efficiency implies strong specialization. Regardless of whether the household technology exhibits decreasing, constant, or increasing returns to scale and regardless of whether households optimally adjust spouses' stocks of human capital or whether each spouse's stocks of human capital are fixed, with perfect substitutes efficiency implies specialization.[27] When each spouse's stocks of human capital are fixed, the further claim that with perfect substitutes and increasing returns, efficiency implies strong specialization is false.[28] When households optimally adjust spouses' stocks of human capital, the validity of the strong specialization claim is sensitive to assumptions about the strength of human capital effects on wage rates and on productivity in the household. In this section I discuss returns to scale when each spouse's stocks of human capital are fixed, deferring until section 7 the discussion of the case in which households optimally adjust spouses' stocks of human capital.

With strong specialization there are 4 possible patterns of time allocation, (t_h, t_w), and, corresponding to each, a consumption vector, (z,x). Two of these four patterns of time allocation correspond to super-strong specialization (i.e., both spouses allocate all of their

[27] Any model in which human capital plays no explicit role can be reinterpreted as one in which each spouse's stocks of human capital are fixed.
[28] This is not Becker's claim. Becker is concerned with the case in which households optimally adjust spouses' stocks of human capital.

time to the same sector).[29] If market goods must be paid for with current market earnings, super-strong specialization implies that the corresponding consumption vectors are of the form (0,x) or (z,0).[30] That is, either the household commodity or the market good is not produced and, hence, not consumed. The two remaining patterns of time allocation correspond to consumption vectors in which both the household commodity and the market good are produced and consumed.

To proceed further, I introduce an assumption that allows me to discuss time allocation and Pareto-efficient consumption patterns without becoming bogged down in extraneous issues involving spouses' preferences and the household governance structure. I avoid these issues by assuming that spouses have identical preferences and that both the household commodity and the market good are household public goods. The focus on this special case is legitimate because I am constructing a counterexample to the claim that with perfect substitutes and increasing returns, efficiency implies strong specialization.[31] If the spouses have Cobb-Douglas preferences then any consumption vector with positive consumption of both the market good and the household commodity is preferred to every consumption vector with a 0.[32] Hence, for such preferences super-strong specialization is Pareto inefficient.

Having ruled out super-strong specialization, I turn to the two remaining cases. One of these corresponds to the time allocation in which the husband allocates time only to the

[29] Recall that I defined strong specialization to include superstrong specialization, the case in which both spouses allocate all of their time to the same sector.

[30] The assumption that market goods must be paid for out of current earnings rules out couples in which both spouses are fully retired.

[31] An alternative way to avoid spouses' preferences, distribution, and bargaining is to assume that the spouses have identical homothetic preferences, so that the optimal consumption vector for the couple is independent of distributional weights and bargaining power.

[32] This is true for all CES preferences with an elasticity of substitution between the Cobb-Douglas and fixed coefficient cases.

market and the wife allocates time only to the household, ($t_h = 0$, $t_w = 1$). The other corresponds to the time allocation in which the wife allocates time only to the market and the husband allocates time only to the household, ($t_h = 1$, $t_w = 0$). One of these two patterns may dominate the other in the sense that the consumption vector implied by one may dominate the consumption vector implied by the other. When this is the case, efficiency implies that we can disregard the time allocation corresponding to the dominated consumption vector.[33]

Suppose that both of the time allocations corresponding to strong specialization with strictly positive levels of z and x imply less of the household commodity and more of the market good than the spouses prefer.[34] Under these assumptions, a pattern of time allocation in which both spouses allocate time to household production and only one allocates time to the market may yield a Pareto-superior consumption vector, in this example, a consumption vector that provides more of the household commodity and less of the market good.

Unlike the other specialization conclusions, the strong specialization conclusion of Theorem 2.4 does not hold when each spouse's stocks of human capital are fixed at arbitrary levels. With strong specialization, the scope for reallocating time is severely limited because only 4 patterns of time allocation are consistent with strong specialization, and two of these imply 0 consumption of either the household commodity or the market good. Thus, the burden of adjustment must fall on human capital.

When households can optimally adjust spouses' stocks of human capital, the implications for specialization depend on the strength of human capital effects, on wage

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[33] Dominance arises if, for example, the husband's wage rate is greater than the wife's, the wife's productivity in the household is greater than the husband's, and $T_h = T_w$. In this case, strong specialization with the wife allocating time only to the market and the husband allocating time only to the household is inefficient.

rates, and on productivity in the household. I postpone until section 7 the discussion of the strength of human capital effects.

5b. Decreasing Returns to Scale and Specialization[35]

In this subsection I show that, even with additivity, decreasing returns can lead to nonspecialization. That is, the specialization theorem of section 4 ceases to hold when constant returns is replaced by decreasing returns. I then argue that decreasing returns are plausible. Specifically, if individuals become tired or bored as they devote more time to household production, and if fatigue or boredom causes them to become less productive, then the unilateral production functions and the household production function are likely to exhibit decreasing returns to scale.[36]

To illustrate decreasing returns to scale, suppose that output is produced by time alone, and that the unilateral production functions are of the form

$$g[t_h,0] = A_h\,(t_h)^{\delta h} \quad \text{and } g[0, t_w] = A_w\,(t_w)^{\delta w}.$$

Decreasing returns corresponds to the case in which the exponents δh and δw are less than 1. I assume $\delta h = \delta w = 1/2$.

If there is no nonlabor income ($x^* = 0$), in the market sector we have

$$x = w_h t_{h0} + w_w t_{w0} = w_h(T_h - t_h) + w_w(T_w - t_w).$$

Assuming $T_h = T_w = 1$, this becomes

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[34] The assumptions that spouses have identical preferences and that the household commodity and the market good are household public goods implies that spouses always agree about which allocation is best. Hence, the governance structure is irrelevant because the spouses never disagree.

[35] For the remainder of this section I assume additivity.

[36] Even if productivity is undiminished, a Pareto efficient household may allocate less time to activities with which individuals become tired or bored. The disutility effects of fatigue and boredom require

$x = w_h(1- t_h) + w_w(1 - t_w)$.

Suppose the spouses' wage rates are $w_h = w_w = 1$, and consider the symmetric nonspecialized time allocation $t_h = t_w = 1/4$. It is straightforward to calculate that $(z, x) = (1, 3/2)$.

This output cannot be produced with specialization. It is easy to verify that it cannot be produced with strong specialization. There are two cases of weak specialization to consider: (1) both spouses allocate time to the market and only one allocates time to home production and (2) both spouses allocate time to home production and only one allocates time to the market.

(1) Suppose only one spouse (for definiteness, the wife) allocates time to home production. Then to produce $z = 1$, she must allocate all of her time to home production. But when all remaining time (i.e., in this example, all of the husband's time) is allocated to the market sector, he cannot earn enough to purchase $x = 3/2$. Instead we have $x = w_h t_{h0} = 1 < 3/2$.

(2) Now suppose only one spouse (for definiteness, the husband) allocates time to the market. Even if the husband allocates all of his time to the market sector, he cannot earn enough to purchase $x = 3/2$. Instead we have $x = w_h t_{h0} = 1 < 3/2$. Because the example is symmetric, the same is true if we reverse the roles of husband and wife.

In this example, efficiency requires nonspecialization (i.e., both spouses must allocate time to both sectors). This nonspecialization example is not a razor's edge case: efficiency requires nonspecialization as the parameters $(\delta_h, \delta_w, w_h, w_w)$ vary in a neighborhood of $(\delta_h, \delta_w, w_h, w_w) = (1/2, 1/2, 1, 1)$. That is, even with additivity, if both

recognizing process preferences -- that is, time allocated to an activity is itself an argument of individuals' utility functions.

spouses' unilateral production functions exhibit decreasing returns, then efficiency may *require* nonspecialization.[37]

Decreasing returns to scale are plausible. The effect of fatigue or boredom on productivity is well documented and provides the primary rationale for regulating the working hours of airline pilots, air-traffic controllers, and truck drivers.[38] When output is produced by time alone, the negative productivity effects of fatigue and boredom imply that increases in hours worked yield less than proportional increases in output. When output requires both time and nonlabor inputs, the implications for returns to scale depend on how nonlabor inputs enter the production function. The leading case, however, is one in which fatigue or boredom imply decreasing returns. Two examples illustrate the possibilities and confirm that decreasing returns is the leading case. (1) Suppose that the household technology exhibits constant returns when time is measured in efficiency units and that, as individuals grow tired or bored, each additional hour produces fewer and fewer efficiency units. For definiteness, suppose that time in efficiency units is related to hours by t^{σ} where $0 < \sigma < 1$; hence, if time is substitutable for nonlabor inputs when time inputs are measured in hours, then the production function exhibits decreasing returns. (2) Now suppose that the household technology exhibits increasing returns when time inputs are measured in efficiency units. In this case, whether the production function exhibits decreasing, constant, or increasing returns when time inputs are

[37] A progressive tax on individuals' earnings is formally similar to decreasing returns to scale and can also lead to efficient nonspecialization even when the household technology exhibits constant returns. But with constant returns in the household technology, a progressive tax on joint earnings (i.e., the sum of individuals' earnings) cannot lead to efficient nonspecialization because joint earnings make spouses' time inputs perfect substitutes in the "production" of the market good.

[38] Iglehart (2010) argues that the evidence supporting the claim that limiting the hours of medical interns and residents increases patient safety is very weak. Worker health and safety provide a secondary rationale for limiting work hours (e.g., exhausted interns and residents are more likely to stick themselves with needles, exposing them to blood-borne infectious diseases).

measured in hours depends on the relative strength of the efficiency units effect and the increasing returns effect as well as on the substitutability of time for nonlabor inputs. The efficiency units effect may be offset by nonlabor inputs becoming more productive as their use increases.[39]

6. Multiple Household Production Activities

With multiple household production activities, the sector specialization claim may fail even if spouses' time inputs are perfect substitutes in every household production activity. Suppose the household sector consists of m distinct activities, $m > 1$.[40] It is easy to construct examples in which both spouses allocate time to the market, one spouse allocates time to m* household activities, and the other spouse allocates time to the remaining m-m* activities, where $m^* \neq 0$ and $m\text{-}m^* \neq 0$.[41] Because the market is the only "activity" to which both spouses allocate time, this pattern of time allocation exhibits specialization. But it does not exhibit *sector* specialization because both spouses allocate time to the household sector and both spouses allocate time to the market sector.

Because sector specialization is defined in terms of spouses' time allocation, the analysis of sector specialization does not require an aggregate measure of the output of the household sector. I avoid the issue of what constitutes an activity by treating household production activities as a primitive.

[39] The Cobb-Douglas provides a transparent example, but I omit the details.

[40] I assume that the household operates all m activities at positive levels and ignore the prior issue of which activities operate at positive levels and which at zero levels.

[41] Lundberg (2008) points out that, with multiple household activities, if spouses' time inputs are perfect substitutes, then efficiency requires activity specialization.

The definitions of "bilateral" and "unilateral" extend to multiple household activities in the obvious way. The household sector is bilateral if both spouses allocate time to it, and unilateral if only one spouse allocates time to it. The household sector is unilateral only when all household activities are unilateral and all are performed by the same spouse. These definitions imply that with multiple household activities, the household sector is itself bilateral if one or more household activities is bilateral, or if some household activities are carried out unilaterally by the husband and others unilaterally by the wife. Subdividing a bilateral activity (e.g., "cooking") into two unilateral activities (e.g., "cooking indoors" and "cooking outdoors") has no effect on whether the household sector itself is bilateral or unilateral.[42]

Efficiency may require unilateral production in the household sector for two distinct reasons. First, if the spouses are equally productive in each household activity but their wage rates differ, then efficiency may require the lower-wage spouse to perform all household activities. This conclusion continues to hold when spouses' productivities are similar but not identical provided spouses' wage rates are sufficiently different. Second, if economies of scope knit together all household activities, then efficiency may require the same spouse to perform all of them.

Economies of scope are a property of the technology for producing two or more commodities and arise from complementarities among activities.[43] Economies of scope can

[42] Indexes of "activity specialization" depend on specifying what constitutes an activity and, hence, may be affected by subdividing activities. See Bonke, Deding, Lausten, and Stratton (2008) for a rare discussion of activity specialization.

[43] Thus economies of scope involve joint production. In the context of a multiproduct firm, the cost of producing the output vector (z_1, z_2) is less than the sum of the costs of producing $(z_1, 0)$ and $(0,z_2)$. Panzar and Willig (1981) provide a formal cost-function definition of economies of scope and discuss the relationship between economies of scope and multiproduct firms. The standard assumption that household technology can be represented by separate production functions for each commodity precludes joint production and, hence, economies of scope. In a one person household, let $C(z_1, z_2, y)$ denote the time required to produce the

arise in single-person as well as in multiple-person households. Additivity, on the other hand, can arise only in multiple person households but can arise when there is only one household commodity.[44]

Economies of scope provide a technology-based explanation of why, with many household activities, efficiency may dictate that the same spouse perform a suite of linked activities. A number of researchers, including Becker, allude to economies of scope in household production, although without necessarily using the term. Usually the context is child care. Becker (1991) writes: "...a mother can more readily feed and watch her older children while she produces additional children than while she engages in most other activities. This complementarity between bearing and rearing children has been important because, until the last century, practically all women spent most of their prime adult lives with children" (p. 38). Fafchamps and Quisumbling (2008) make a similar point, referring explicitly to economies of scope: "One common example of economies of scope is child care and house-based chores: many chores can be completed while at the same time attending to a child" (p. 3198). Hadfield (1993), criticizing Becker's analysis of the gendering of specialization, writes "...nor is there an analysis of how women's self-evident advantages in childbearing extend (presumably through economies of scope and complementarity) to create advantages in the full range of childcare household activities" (97).[45]

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commodity vector (z_1, z_2) where y is the vector of nonlabor inputs. Then economics of scope imply $C(z_1, z_2, y) < \min \{C(z_1, 0, y_1) + C(0, z_2, y_2)\}$ subject to $y_1 + y_2 \leq y$.

[44] The definition of additivity in section 3 assumes no joint production.

[45] Unlike Hadfield, my concern is with specialization itself, not with the gendering of specialization.

7. The Roles of Human Capital

Although Becker's argument that "efficiency implies specialization" *appears* to depend on households optimally adjusting spouses' stocks of human capital, it actually does not. More specifically, specialization follows directly from perfect substitutes, Becker's default assumption. The specialization conclusion holds regardless of whether households optimally adjust spouses' stocks of human capital; it holds when each spouse's stocks of human capital are fixed, regardless of the level at which they are fixed.[46]

Human capital or, more precisely, the ability of households to adjust spouses' stocks of human capital, presents two new issues. First, when there are two or more types of human capital, we can investigate "human capital specialization" as well as time specialization. Second, as Becker argues informally, the ability of households to adjust spouses' stocks of human capital increases the incentives for specialization.

The definition of human capital specialization is analogous to the definition of time specialization. For example, with two types of human capital, "nonspecialization" is the case in which both spouses invest in both types of human capital. If time specialization is efficient and human capital is sector specific (i.e., market and household rather than, for example, cognitive and noncognitive), then human capital specialization is also efficient: in an efficient household, a spouse who allocates time to only one sector invests only in human capital that is specific to that sector.

For technologies that imply specialization when each spouse's stocks of human capital are fixed at arbitrary levels (e.g., perfect substitutes; additivity and constant returns to

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[46] As I showed in section 5, Becker's claim that perfect substitutes and increasing returns to scale imply strong specialization (Theorem 2.4) does not hold when each spouse's stocks of human capital are fixed at arbitrary levels. The strength of human capital effects determine whether it holds when households optimally adjust spouses' stocks of human capital.

scale) human capital specialization is a consequence of time specialization, not its cause. But for household technologies that do not imply time specialization when each spouse's stocks of human capital are fixed at arbitrary levels, human capital specialization is both a consequence and a cause of time specialization: specialized time allocation and specialized investment in sector-specific human capital go hand-in-hand. They are simultaneously determined and mutually reinforcing.

For technologies that do not imply specialization when each spouse's stocks of human capital are fixed at arbitrary levels, the strength of human capital effects plays a crucial role in the analysis of specialization. Although Becker does not discuss the strength of human capital effects, his functional form assumptions imply that these effects are strong. If human capital has only weak effects on wage rates and on productivity in the household, then all human capital vectors may imply the same pattern of specialization or nonspecialization. That is, the mere presence of human capital does not automatically transform technologies for which nonspecialization is efficient into technologies for which specialization is efficient.[47] The household's ability to optimally adjust even a single type of human capital can provide incentives for sector specialization. For example, suppose that each spouse's household human capital is fixed but the household can adjust each spouse's stock of market human capital. In this case, market human capital, through its effect on wage rates, may provide sufficient incentives for specialization.[48] Economic theory alone cannot establish whether specialization is efficient, but it can identify the modeling assumptions and parameter values that determine whether specialization is efficient.

[47] When stocks of human capital are variable, it is convenient to imagine a household technology corresponding to each human capital vector.

[48] The situation is much the same if there is only one type of human capital that has, for example, a greater effect on wage rates than on productivity in the household.

Whether these modeling assumptions hold and whether the parameter values lie within the critical range that corresponds to specialization is an empirical question.

To say more about how human capital affects the incentives for specialization requires specifying the relationship between human capital, wage rates, and productivity in the household. Following Becker, I assume two types of human capital. But unlike Becker, who assumes sector-specific human capital (i.e., market human capital which affects only wage rates; household human capital which affects only productivity in the household), I begin by allowing both types of human capital to affect both wage rates and productivity in the household. This would be the case, for example, if one type of human capital corresponds to cognitive and the other to noncognitive skills, or if one type corresponds to verbal and the other to mathematical skills. I denote the two types of human capital vector by H^a and H^b, the husband's human capital vector by (H^{ah}, H^{bh}), and the wife's by (H^{aw}, H^{bw}).

How does human capital enter the household production function? With sector-specific human capital, the simplest assumption is that human capital is "time augmenting" in the sense that the time that each spouse allocates to a sector is multiplied by a function of that spouse's sector-specific human capital.[49] I generalize this beyond the case in which human capital is sector specific by introducing sector-specific aggregator functions that convert the spouses' human capital vectors into indexes that multiply the spouses' time inputs.[50] I denote the functions that aggregate the husband's human capital by $\psi^h = \psi^h[H^{ah}, H^{bh}]$ and $\psi^{h0} = \psi^{h0}[H^{ah}, H^{bh}]$, and those that aggregate the wife's human capital by $\psi^w = \psi^w[H^{aw}, H^{bw}]$ and $\psi^{w0} = \psi^{w0}[H^{ah}, H^{bh}]$. These assumptions allow us to measure the time

[49] This is Becker's assumption.

[50] A more general approach to incorporating human capital into the household production function is to allow some or all of the production function parameters to depend on the human capital vectors of both spouses, $\{H^{ah}$,

inputs that each spouse allocates to each sector in efficiency units: for the husband, $\{\psi^{h}[H^{ah}, H^{bh}]t_h, \psi^{h0}[H^{ah}, H^{bh}]t_{h0}\}$ and for the wife $\{\psi^{w}[H^{aw}, H^{bw}]t_w, \psi^{w0}[H^{aw}, H^{bw}]t_{w0}\}$.

Using this parametric approach, we can formalize both the substitutability of one type of human capital for the other in each sector and the strength of human capital effects. Substitutability determines the extent to which a particular type of human capital is associated with a particular sector. Sector-specific human capital is the extreme case in which neither type of human capital can substitute for the other.[51]

The strength of human capital effects depends on the range of the four aggregator functions. For example, suppose that one of the aggregator functions is bounded below by φ- and above by φ+. This assumption does not limit *amount* of human capital, but if the interval [φ-, φ+] is small, then the *effect* of human capital (e.g., on wage rates and on productivity in the household) is also small.

Given the spouses' market wage rates, suppose efficiency implies bilateral household production for all admissible values of φ (i.e., $\varphi- \leq \varphi \leq \varphi+$). This implies that, given market wage rates, the effect of human capital on productivity in the household is small, perhaps sufficiently small that bilateral household production is efficient for all relevant wage rates and household technologies.

This formulation generalizes Becker's in two respects. First, Becker assumes that human capital is sector specific. In my notation, sector-specific human capital corresponds to the case in which

$\{\psi^{h}[H^{ah}]t_h, \psi^{h0}[H^{bh}]t_{h0}\}$

$H^{bh}, H^{aw}, H^{bw}\}$. An even more general approach treats human capital as an argument of the household production function: $g[t_h, t_w, y, H^{ah}, H^{bh}, H^{aw}, H^{bw}]$.

[51] In a dynamic setting with uncertainty, there may be incentives to invest in "general" rather than sector specific human capital.

and

$\{\psi^{w}[H^{aw}]t_{w}, \psi^{w0}[H^{bw}]t_{w0}\}$.

Second, Becker assumes that the aggregator functions are of the form

$$\psi^{h}[H^{ah}] = H^{ah}$$

$$\psi^{h0}[H^{bh}] = H^{bh}$$

$$\psi^{w}[H^{aw}] = H^{aw}$$

$$\psi^{w0}H^{bw}] = H^{bw}.$$

This functional form assumption, combined with the (time) essentiality assumption of section 2, implies that specialized household human capital is also essential for household production. This functional form assumption is not a harmless normalization but a strong substantive assumption about the role of human capital in household production. It maximizes and therefore almost certainly exaggerates the strength of the effects of human capital.

We have little empirical evidence about the importance of specialized human capital in household sector. We know that many people rely on their children and grandchildren for computer support. We also know anecdotes about elderly widowers who don't know how to cook and elderly widows who don't know how to balance a checkbook. I cannot resist a Winston Churchill anecdote: "At one point ... Clementine [Winston Churchill's wife] decides that her husband can't stay at Chartwell [their country house] for the weekend as all the servants are away. 'I shall cook for myself. I can boil an egg. I've seen it done,' Churchill retorts."[52]

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[52] Quoted by D. J. Taylor in a review of two books on Churchill in *Times Literary Supplement*, 14 November 2011; the quotation is from Cita Stelzer, *Dinner with Churchill: The Prime Minister's Tabletop Diplomacy*, Short Books, 2011).

Anecdotes aside, the importance of activity-specific or sector-specific human capital in household production is an open empirical question. Market wage rates in occupations that involve household production skills (e.g., cleaning, child care) may provide some evidence. In the labor market the returns to experience in these occupations are generally low. Becker's current assessment of the importance of sector-specific human capital may differ from the view he expressed in the *Treatise*. Becker and Murphy (2007) write: "However, returns to education and other training could still be greater in households [than in the market] if persons investing in such human capital acquired general skills that were particularly useful at household tasks. This is likely for investments in education since education improves a person's skills at processing information, preparing for future events, and managing multiple tasks. These skills are especially important in the modern household because these households perform many complicated tasks that must be coordinated" (p. 33). While the specialization argument in the *Treatise* presupposes sector-specific human capital, Becker and Murphy emphasize the importance of general rather than sector-specific human capital.[53]

To summarize: human capital can increase the incentives for specialization, but whether these incentives actually lead to specialization depends on the strength of human capital effects.

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[53] Becker's assumption that human capital is sector specific is an expositional devise: "I have assumed that each type of human capital raises efficiency at only a single activity, but we do not need to hold to this limitation" (p. 36).

8. Conclusion

Where does this leave us? For some household technologies efficiency implies specialization regardless of spouses' preferences and regardless of the household governance structure. Becker showed that if spouses' time inputs are perfect substitutes, then efficiency implies specialization. I have shown that if the household technology is additive and exhibits constant returns to scale, then efficiency implies specialization.[54] Except in special cases, however, whether efficiency implies specialization depends not only on the household technology but also on the spouses' preferences and the household governance structure.

Even when specialization is efficient, couples may fail to specialize. For example, inefficiency may arise if spouses are unwilling or unable to make binding intertemporal commitments. Becker makes this point in the *Treatise on the Family*, interpreting marriage and divorce laws as societies' attempts to provide the assurance needed to support efficient specialization and investment in human capital.[55].

Without binding agreements, specialization can have strong distributional consequences. More specifically, if distribution within marriage depends on bargaining in marriage and if bargaining power depends on wages or earnings, then equality in marriage requires that both spouses work in the market. But if both spouses work in the market and if efficiency requires specialization, then efficiency requires that one spouse do all the housework. Under these assumptions, equity and efficiency are incompatible.[56] For those of

[54] Both of these special cases assume the absence of process preferences and assume two sectors, household and market. Neither of them depends on households optimally adjusting spouses' stocks of human capital; the specialization conclusion holds when spouse's stocks of human capital are fixed at arbitrary levels.
[55] Lundberg and Pollak (2003) develop and analyze a two-period model in which spouses' inability to make binding intertemporal commitments can lead to dynamic inefficiency in the context of the "two earner couple location problem." Lundberg (2008) analyzes dynamic inefficiency in a two-period model in which the failure to accumulate market human capital in the first period disadvantages a spouse in second period bargaining.
[56] We can avoid this unpleasant conclusion by assuming, as Becker does in Chapter 4 of the *Treatise*, that prospective spouses make binding agreements in the marriage market that determine distribution in marriage.

us concerned with equity as well as efficiency and who think that bargaining takes place within marriage, it is good news that efficiency need not imply specialization.

Lundberg and Pollak (2009) assume that prospective spouses do not make binding agreements in the marriage market. They propose and analyze a model in which the marriage market determines who marries and who marries whom, and in which distribution in marriage is determined by bargaining in marriage.

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Appendix: Becker's Specialization Theorems

In this appendix I discuss the five specialization theorems from Chapter 2 of the *Treatise on the Family*. The question is: how do these theorems rule out cases in which efficiency requires nonspecialization? The answer is the perfect substitutes assumption.

Before stating the specialization theorems formally, Becker emphasizes that his discussion assumes perfect substitutes: "A major assumption of the present section [Specialization in Households] is that at the beginning everyone is identical; differences in efficiency are not determined by biological or other intrinsic differences....Since all persons are assumed to be intrinsically identical, they supply the same kind of time to the household and market sectors. Therefore, the *effective* time of different members would be perfect substitutes even if they accumulate different amounts of household capital..." (p. 32; italics in original).[57]

Theorem 2.1 is about time specialization and Theorem 2.2 about human capital specialization. The formal statements of the theorems do not mention perfect substitutes and they impose assumptions that become redundant when the perfect substitutes assumption is added to the hypothesis.

Because these two theorems have identical hypotheses (i.e., "different comparative advantages"), I state both theorems before discussing them.

Theorem 2.1 "If all members of an efficient household have different comparative advantages, no more than one member would allocate time to both the market and household sectors. Everyone with a greater comparative advantage in the market than

[57] I do not discuss whether the assumption that all persons are "intrinsically identical" implies that spouses' time inputs are perfect substitutes. Instead, I proceed as if Becker assumes perfect substitutes.

this member's would specialize completely in the market, and everyone with a greater comparative advantage in the household would specialize completely there" (p. 33).

Theorem 2.2 "If all members of a household have different comparative advantages, no more than one member would invest in both market and household capital. Members specializing in the market sector would invest only in market capital, and members specializing in the household sector would invest only in household capital" (p. 34).

If we include perfect substitutes in the hypothesis of Theorem 2.1, then the time specialization conclusion holds even if we remove "different comparative advantages" from its hypothesis. That is, efficiency and perfect substitutes imply specialization and "different comparative advantages" becomes redundant.[58] Notice that the specialization conclusion of Theorem 2.1 holds even when each spouse's stocks of human capital are fixed at arbitrary levels.

If households optimally adjust spouses' stocks of human capital, then time specialization implies human capital specialization, so Theorem 2.2 follows from Theorem 2.1. As Becker writes: "...members specializing entirely in the market sector have strong incentives to invest in market capital (H^1) and no incentive to invest in household capital (H^2). Similarly, members specializing in the household sector have strong incentives to invest in H^2 and no incentive to invest in H^1" (p. 34).

Theorem 2.1 holds even if we do not include perfect substitutes in its hypothesis: different comparative advantages imply specialization. But the interpretation of the theorem as implying that specialization is pervasive depends on the implicit assumption that the normal case is "different comparative advantages" and that "equal comparative advantages" is an unlikely coincidence. Without perfect substitutes or some other strong assumption, however, efficiency requires equal comparative advantages. If we do not include perfect substitutes or some other strong assumption in the hypothesis, then "different comparative advantages" would be an unlikely coincidence.

To prove that the two theorems hold without assuming perfect substitutes, compare the definition of comparative advantage with the first order conditions for production efficiency. Before stating Theorem 2.1, Becker defines comparative advantage: “The comparative advantage of a [household] member can be defined by the relation between the ratio of his marginal products in the market and household sectors, and the ratios of other members" (p. 33). That is, equal comparative advantages means

$$\frac{\dfrac{\partial g[t_h,t_w,y]}{\partial t_h}}{w_h} = \frac{\dfrac{\partial g[t_h,t_w,y]}{\partial t_w}}{w_w}$$

The first order conditions for production efficiency arise from maximizing output

$$g[t_h,t_w,y]$$

subject to the constraint

$$y_h + y_w + x^{**} \leq w_h(T_h - t_h) + w_w(T_w - t_w) + x^{*}.$$

If this maximization problem has an interior solution, then the first order conditions are

$$\frac{\partial g[t_h,t_w,y]}{\partial t_h} = \lambda w_h$$

[58] This assumes the absence of process preferences. It also requires carving out an exception for the case in which both specialization and nonspecialization are efficient (e.g., spouses have identical wages rates and are equally productive in the household.

and

$$\frac{\partial g[t_h, t_w, y]}{\partial t_w} = \lambda w_w$$

Eliminating λ and rearranging, yields the equal comparative advantage condition. In effect, the theorem says: *If we don't have an interior solution (i.e., a solution in which both spouses allocate time to both sectors), then we have a corner solution (i.e., a solution in which at least one spouse does not allocate time to both sectors).* This paraphrase of Theorem 2.1 is not a criticism: theorems, after all, are tautologies. But the interpretation of Theorem 2.1 as implying that specialization is pervasive depends on imposing perfect substitutes or some other strong assumption.

Theorem 2.3 "At most one member of an efficient household would invest in both market and household capital and would allocate time to both sectors" (p. 34).

Theorem 2.3 depends on the perfect substitutes assumption. Indeed, unless we reinterpret Theorem 2.3 to include perfect substitutes as an hypothesis, it would have no hypothesis at all. Becker's proof of Theorem 2.3 depends on perfect substitutes and also appears to depend on assuming that the household optimally adjusts spouses' stocks of market and household human capital. In fact, however, the conclusion follows directly from the perfect substitutes assumption and holds even when each spouse's stocks of human capital are held fixed at arbitrary levels.

Theorem 2.4 makes a claim about strong specialization:

Theorem 2.4 "If commodity production functions have constant or increasing returns to scale, *all* members of efficient households would specialize completely in the market or household sectors and would invest only in market or household capital" (p. 35; italics in original).

As I showed in section 2.5a, the conclusion of Theorem 2.4 does not hold when each spouse's stocks of human capital are held fixed at arbitrary levels.[59] But here, as elsewhere, Becker assumes that the household optimally adjusts spouses' stocks of human capital. As I argue in section 2.7, the analysis of specialization when the household optimally adjusts spouses' stocks of human capital requires assumptions about the strength of human capital effects on wage rates and on productivity in the household.

Theorem 2.5 addresses the case in which the number of household members exceeds the number of commodities.

Theorem 2.5 "All but possibly one member of households with more members than independent commodities would completely specialize their investments and time to the market or to a particular commodity. Moreover, with constant or increasing returns to scale, *all* members of efficient households must be completely specialized." (p. 36; italics in original).

The first sentence of the theorem is about weak specialization and the second about strong specialization. If we restrict our attention to the case in which the household consists of two members and there is a single home produced commodity, the first sentence becomes Theorem 2.3 and the second sentence Theorem 2.4. Thus, the action in the theorem requires considering household with more than two individuals, a topic beyond the scope of this paper.

[59] The hypothesis of Theorem 2.4 is unusual: in economics the usual assumption is that production functions are concave, and concavity implies constant or *decreasing* returns to scale.