Job Heterogeneity and Aggregate Labor Market Fluctuations☆

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Abstract

Individual data show that workers who suffer job displacement experience large and persistent earnings losses. Concurrently, aggregate data reveal large and sluggish movements of worker flows over the business cycle. This paper provides a link between, and an explanation for these phenomena with a search and matching model that features idiosyncratic productivity (demand) and match-quality. Poor match-quality among first jobs implies large earnings losses upon job loss and large fluctuations in unemployment due to a responsive job destruction margin. A significant job ladder, consistent with empirical wage dispersion, delivers a protracted earnings recovery and provides ample scope for the propagation of vacancies and unemployment.

Keywords: Match-quality, amplification, propagation, displacement

JEL: D83, E24, J63, J64

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1. Introduction

Individual data reveal that workers who experience job displacement (e.g. layoff) face surprisingly large and persistent earnings losses. For example, Davis and von Wachter (2011) (henceforth DV) find that at the time of displacement real earnings fall around 30 percent, and even twenty years after displacement, annual earnings are 10-20 percent below pre-displacement earnings. At the same time, aggregate data based on the United States show large and sluggish movements of the aggregate job-finding rate, as well as the vacancy posting and unemployment rates, in response to aggregate labor productivity shocks ("amplification" and "propagation").[1] This paper proposes a connection between, and a possible explanation for these phenomena with a model that simultaneously delivers the earnings profile of displaced workers, and significant amplification and propagation of aggregate productivity shocks.

The model features two-dimensional heterogeneity with idiosyncratic productivity and match-quality in the form of a job ladder. The job ladder captures the idea that workers suit some jobs better than other jobs, and it takes time for workers to find the jobs for which they are well suited. The "length" of the ladder is consistent with observed wage dispersion. Together with relatively poor quality matches among first jobs, the job ladder implies large earnings declines upon job loss. The idea that unemployed workers accept low match-quality also leaves many jobs susceptible to destruction during a downturn, amplifying aggregate productivity shocks.

In addition to the substantial job ladder, the model endogenously generates serially correlated displacements, and therefore the slow earnings recovery post-displacement observed in live data. Serially correlated displacements slow down workers' climb up the job ladder because each displacement event triggers a series of displacements. The model captures the following intuition: compared to their job prior to displacement, workers might not be as well matched in their first job coming out of unemployment. This poor fit results in tentative new employment relationships and small downward movements in productivity can terminate these relationships. This serial correlation coincides with empirical work by Stevens (1997) who finds that multiple additional job losses explain much of the persistence and

[1] For a rough progression of research on amplification see Merz (1995), Andolfatto (1996), den Haan et al. (2000a) and Shimer (2005). See Fujita and Ramey (2007) for a discussion of propagation.

magnitude in lowered earnings after displacement. The model's protracted earnings recovery also implies that the joint distribution of idiosyncratic productivity and match-quality exhibits slow-moving behavior. Since the job-finding rate is a function of this distribution, it displays marked persistence following a movement in aggregate productivity.

The first contribution of this paper is to provide a quantitative framework that delivers the earnings and employment experiences of displaced workers. The following analysis constructs a model and chooses parameters to fit the earnings time-path of displaced workers and the observed serial correlation in displacements. In a recent paper, Davis and von Wachter (2011) point out that comparable models cannot deliver these prominent empirical facts. In the current paper I also investigate several alternative models that all fall short of matching the data. In addition to successfully capturing these aspects of the data, the present model also matches several moments of the data that it was not calibrated to target. The calibrated job ladder delivers realistic wage dispersion as documented by Hornstein et al. (2011), and this goes a long way towards explaining the success of the proposed framework. Since all agents in the model are ex-ante homogeneous and the model is stationary, misfortune or "bad luck" can account for all the earnings losses associated with displacement. The model also matches the empirical decomposition of earnings losses into reduced wages and employment, empirical establishment-level fluctuations in total factor productivity, the volatility of earnings within matches, and the pattern of employment-to-employment (E-E) transitions after displacement.

The second contribution of this paper is to derive the implications of the model for the aggregate labor market, and in this sense provide a link between the outcomes of displaced workers and aggregate dynamics. Quantitatively, the model delivers significant amplification of aggregate productivity shocks, matching empirically observed fluctuations in employment-to-unemployment (E-U) transitions, the aggregate unemployment rate, and labor market tightness. The model undershoots the observed volatility of vacancies, although makes much progress when compared to the basic Mortensen-Pissarides (MP) model, as well as a similar model with on-the-job search that has only one-dimensional job heterogeneity. The model simultaneously delivers significant propagation. Unemployment takes around four years to complete 75 percent of its adjustment to a permanent reduction in aggregate productivity. Vacancies, and therefore the job-finding rate, take even longer to adjust to the same shock. Average match-quality initially exhibits a cleansing effect, as low quality matches are destroyed at the beginning of the downturn, and then a sullying effect as job-finding probabilities are permanently lower, and it takes workers longer to climb the job ladder.

As a result of significant vacancy amplification and on-the-job search, the model delivers the Beveridge curve: the negative co-movement between unemployment and vacancies observed in the U.S. labor market. In the standard MP model, a separation shock increases unemployment which induces vacancy creation, delivering a positive relationship between the two aggregate statistics. The present model overcomes this weakness by allowing firms to contact employed workers. As the E-U rate rises, firms still want to post more vacancies due to a higher job-filling rate. However, in the current setup, firms also face lower productivity of employed workers when aggregate productivity falls, and since these workers make up the majority of the potential applicant pool, this serves to reduce vacancies. The model also features a procyclical job-finding rate for both the unemployed and the employed, consistent with observation.

This paper is not the first to take seriously the evidence on the earnings losses of displaced workers. Den Haan et al. (2000b) and Pries (2004) both contribute to this literature, but omit on-the-job search, which is at the heart of the current paper. Low et al. (2010) report the implications of a similar model for the cost of displacement, noting that these losses are relatively small and short lived. This underscores one of the key contributions of the present paper, which is its ability to match the magnitude and persistence of displaced worker earnings losses.[2] Finally, Davis and von Wachter (2011) show that a standard MP model, and a more sophisticated model found in Burgess and Turon (2010) (henceforth BT), cannot explain observed displaced worker earnings losses. The present model builds on the BT model by incorporating

[2]Since the present work and Low et al. (2010) have different emphasis, and the models differ substantially, with distinct calibration approaches, it is difficult to pinpoint why the two approaches imply different costs of displacement. One potential reason is that the model presented here might feature more serial correlation in displacements than the model in Low et al. (2010) because the match-quality coming out of unemployment is likely to be significantly smaller relative to the average match-quality among the employed than the initial match-quality in Low et al. (2010).

serially correlated displacements and a job ladder that is consistent with realistic wage dispersion.

The model presented in this paper also improves upon previous applications of the MP model to aggregate labor market dynamics. Aside from the calibration of Hagedorn and Manovskii (2008), which arguably has an unrealistically large value of leisure, the basic MP model fails to deliver sufficient amplification of aggregate productivity shocks.[3] If one feeds in realistic aggregate labor productivity shocks into a basic MP model, the fluctuations in the vacancy and unemployment rates are smaller than what one observes in the data. As far as propagation, in the standard search and matching model, the vacancy-unemployment ratio is a jump variable, making the job-finding rate a jump variable, not matching the empirical evidence. As productivity fluctuates, the job-finding rate adjusts immediately and exhibits no further movement.

The rest of this paper is organized as follows. Section 2 presents the steady-state version of the model, which features exogenous contact rates. Section 3 extends the model and endogenizes the contact rates. Section 4 elaborates on calibration. Section 5 presents steady-state features of the baseline model with match-quality, and Section 6 presents transition dynamics of key endogenous variables in response to an aggregate productivity shock. Section 7 presents a simpler version of the baseline model with only one state variable, but still featuring a job ladder and E-E transitions. The simpler model delivers very little amplification and propagation, consistent with work by Bils et al. (2011). Section 8 summarizes.

2. Model with Fixed Contact Rates

2.1. Model Introduction

The work on search and matching by Mortensen, Diamond and Pissarides provides the foundation for this paper. Two quantities characterize every match: the quality of the match and idiosyncratic productivity (demand). The framework incorporates endogenous privately efficient separations, which means that worker and firm act to maximize their joint value, as well as exogenous separations. In this

[3]See, for example, Shimer (2005).

model all unemployed workers are identical and workers are endowed with linear utility (risk-neutrality). In this section I develop the model with constant aggregate productivity and with exogenous contact rates.

2.2. Setup

Workers look for jobs and firms post vacancies to attract workers. Unemployed workers receive utility from leisure and encounter vacancies at an exogenous probability f_U. Employed workers receive a flow payment w and produce a flow output. Employed workers participate in on-the-job search and contact vacancies at a different probability f_E.[4] All employer-employee matches are characterized by two state variables: match-quality denoted by y, and idiosyncratic productivity (demand) denoted by x. The product of x and y $(x \cdot y)$ provides the flow output of the match. When an unemployed worker contacts a firm, the match draws an initial, deterministic match-quality, y_0. Match-quality remains constant within a job. Setting match-quality to y_0 in new matches implies that there exists a set of entry-level positions. This coincides with what Doeringer and Piore (1971), and more recently Martins et al. (2010), call "port-of-entry" jobs; jobs into which employers are consistently observed to hire new workers. On a more technical note, with variation in initial y, unemployed individuals reject offers. The data do not help us distinguish unemployed individuals who have rejected low offers and those without any offers. In the model all unemployment is frictional.

All initial idiosyncratic productivities (demands) are fixed at a deterministic value, x_0, and then exhibit persistence within a match evolving according to $F_x(x'|x)$. Setting x to x_0 in all new matches follows Mortensen and Pissarides (1994). On-the-job search results in offers to the employed with match-quality drawn from $y \sim F_y(\tilde{y})$. This induces a job-ladder which agents climb over time. This can be interpreted as finding more suitable jobs within the same firm (promotions) or simply learning specific skills and moving onto jobs that are better suited for the worker. In this sense, y captures the acquisition of firm specific human capital. Resetting y to y_0 in all new matches from unemployment captures the idea that workers lose all firm-specific

[4] The differing job contact probabilities on and off the job may result from differing levels of search intensity exhibited by the employed and the unemployed. The model presented here abstracts from the reason behind this difference.

human capital during unemployment.

The idiosyncratic component delivers endogenous flows into unemployment; when the realization of the idiosyncratic random variable is low enough, the worker and the firm decide to part ways. Involuntary endogenous separations on either side of the market do not occur in this model. The model does incorporate exogenous separations, however.

2.3. Timing of Events within a Period

Within each period, events among unemployed workers unfold according to the following timing. At the outset of a period firms post vacancies to recruit unemployed workers, and workers look for jobs. When workers contact open vacancies the worker and firm consummate the match. New matches wait until next period to produce, where δ denotes the discount factor. For established employment relationships the timing for workers and firms is as follows. First, firm and worker bargain over the wage. Second, production occurs and the firm pays the worker. Third, the exogenous separation shock occurs with probability p_s. Fourth, the idiosyncratic component, x, undergoes a shock. Finally, workers receive outside offers with probability f_E. If an employed worker receives a favorable outside offer, he moves to the poaching firm. If an employed worker receives no outside offer, the firm and the employee decide to preserve the match or separate.

2.4. Bargaining

At the beginning of each period, every worker-firm pair bargains over the wage that the firm pays the worker for production. This model features a linear surplus sharing rule, so that the worker (firm) receives a fraction, β $(1 - \beta)$ of the total match surplus. If an employed worker receives a favorable outside offer, he moves to the poaching firm, and renegotiates his wage using unemployment as his outside option.[5] If an employed worker receives an outside offer that does not induce a switch,

[5]Nagypal (2007) also uses this convenience in an on-the-job search model. In the setup of Postel-Vinay and Robin (2002) workers can use the surplus at their previous firm as an outside option. That setup includes no idiosyncratic productivity so that all wage changes within a firm result from outside offers. Including idiosyncratic productivity into this type of model gives the efficient rigid wage model presented in Appendix A. Also, Shimer (2006) points out that with on-the-job search the simple surplus splitting rule may not be Pareto efficient. Given that the efficient rigid wage

the worker cannot use that outside offer to negotiate with his current employer. Appendix A outlines a model with efficient rigid wages, similar to a framework found in MacLeod and Malcomson (1993).[6] In that model, workers can use their current offer to bargain with an outside firm, and they can use outside offers to raise their wage at the current firm. This alternative model delivers very similar results to the model that features the simple surplus sharing rule (see Appendix B). In order to remain consistent with previous work, the benchmark model in this paper implements the standard surplus sharing protocol.

2.5. Intuition for the Model with Fixed Contact Rates

The model delivers a slow recovery in earnings post-displacement for three reasons. First, immediately post-displacement the calibrated model suggests that workers take jobs with lower match-qualities compared to their pre-displacement jobs. Second, in conjunction with a low match-quality among first jobs, the job ladder introduces persistence in earnings; it takes time for employed workers to find good quality matches. Third, low post-displacement match-qualities mean that newly created jobs are likely close to the job destruction threshold. This makes it more likely that these matches will be destroyed, resulting in multiple displacements. This serial unemployment dovetails with empirical work by Stevens (1997) who finds that multiple job losses explain some of the persistence of earnings losses.

model in Appendix A delivers qualitatively similar results, I suspect that amending this model's bargaining structure will not yield substantially different conclusions.

[6]The appendices are provided for the reader's interest and are not necessary in the published version of this paper.

2.6. Bellman Equations

This subsection deals with the formal recursive equations of the model. The value of work satisfies the following equation:

$$\begin{aligned} W(x,y) &= w + \delta(1-f_E)(1-p_s)\int \underbrace{\max\{U, W(x',y)\}}_{\substack{\text{Match continues or}\\ \text{terminates}}} dF_x(x'|x) + \underbrace{\delta p_s U}_{\substack{\text{Exogenous}\\ \text{separation}}} \\ &+ \delta f_E(1-p_s)\int\int \underbrace{\max\{U, W(x',y), W(x_0,\tilde{y})\}}_{\substack{\text{Worker moves to unemployment,}\\ \text{stays at current firm,}\\ \text{or goes to new firm}}} dF_x(x'|x)dF_y(\tilde{y}) \end{aligned} \tag{1}$$

The first term on the right hand side is the flow payoff from working, which is the current wage: w. The second term on the right hand side corresponds to the event of no outside job offer. Since the productivity shock arrives every period, this term captures what happens when the productivity changes. If $W(x',y) > U$ there is positive surplus, and the worker and firm bargain over the new wage. If $W(x',y) < U$ the relationship is no longer viable. The employment partnership comes to an end. The third term on the right hand side captures exogenous separation, in which case the worker flows into unemployment and receives U.

The fourth term on the right hand side corresponds to the worker contacting an outside firm. The worker leaves the current employment relationship only if the match value of the new match exceeds the value at the current firm. In this case, the worker chooses between two options: unemployment and working at the new firm. In the latter case, the worker bargains with the outside firm using unemployment as his outside option. In the event that the match value at the current firm exceeds both the value of unemployment and the match value at the outside firm, the worker remains at the current firm receiving value $W(x',y)$. If the value of unemployment exceeds the worker's value at the current firm and at the outside firm, the worker moves to unemployment receiving continuation value U.

The value of a filled job to the employer satisfies the following equation:

$$
\begin{aligned}
J(x,y) = xyz - w + \delta(1-f_E)(1-p_s)\int \underbrace{\max\{0, J(x',y)\}}_{\substack{\text{Match continues or}\\ \text{terminates}}} dF_x(x'|x) \\
+ \delta f_E(1-p_s)\int\int \underbrace{\mathbb{I}\{J(x',y) \geq J(x_0,\tilde{y})\}}_{\substack{\text{Worker turns down}\\ \text{poaching firm}}} \underbrace{\max\{0, J(x',y)\}}_{\substack{\text{Match continues or}\\ \text{terminates}}} dF_x(x'|x)dF_y(\tilde{y})
\end{aligned}
\tag{2}
$$

The first term on the right is the flow payoff from a filled job, the output xyz, less the wage paid to the worker for production w. The second term on the right corresponds to the event of no outside job offer, and no exogenous separation shock. It is completely analogous to the value of work.

The third term on the right hand side corresponds to the worker contacting an outside firm. If the worker stays at the current firm, the expression is the same as if no outside offer was made. If the worker leaves the current employment relationship, the current firm's continuation value equals zero.

The value of unemployment satisfies:

$$
U = b + \delta(1-f_U)U + \delta f_U \underbrace{\max\{U, U + \beta[W(x_0,y_0) + J(x_0,y_0) - U]\}}_{\text{Match consummates or not}} \tag{3}
$$

where f_U is the probability that an unemployed worker contacts a vacancy. The first term captures the flow payoff from unemployment: b. The second term corresponds to no job offer, so the worker remains unemployed. The third term corresponds to a job offer. In this case the worker chooses between working at the contacting firm and unemployment. The payoff from working at the firm is the outside option, U, plus β times the surplus.

2.7. Solving the Model

The expressions in the previous sections can be summarized in one central functional equation: the surplus from a match, $S(x,y)$. Appendix C provides the details of this derivation and the numerical approach taken to find an approximation of

$S(x,y)$. Here I simply present the result:

$$
\begin{aligned}
S(x,y) = xyz &+ \delta \underbrace{(1-f_E)}_{\substack{\text{No outside}\\ \text{offer}}} \underbrace{(1-p_s)}_{\substack{\text{No separation}\\ \text{shock}}} \int \max\{0, \underbrace{S(x',y)}_{\substack{\text{Match}\\ \text{continues}}}\} dF_x(x'|x) \\
&+ \delta f_E (1-p_s) \int\int \Bigg[\underbrace{\mathbb{I}\{S(x',y) \geq S(x_0,\tilde{y})\}}_{\substack{\text{Worker turns down}\\ \text{poaching firm}}} \underbrace{\max\{0, S(x',y)\}}_{\substack{\text{Match continues or}\\ \text{terminates}}} \\
&+ \underbrace{\mathbb{I}\{S(x',y) < S(x_0,\tilde{y})\}}_{\substack{\text{Worker leaves}\\ \text{current firm}}} \underbrace{\max\{0, \beta S(x_0,\tilde{y})\}}_{\substack{\text{Match at new firm}\\ \text{or unemployment}}} \Bigg] dF_x(x'|x) dF_y(\tilde{y}) \\
&- \underbrace{[b + \delta f_U \beta \max\{0, S(x_0,y_0)\}]}_{\text{Worker's outside option}}
\end{aligned}
\tag{4}
$$

The first part of the right hand side is the flow payoff from a match, xyz. The second piece captures the event of no outside job offer, no exogenous separation shock and the continuation surplus of the match. In this case, the match either comes to an end or the match continues with the new idiosyncratic productivity. The third piece captures the event of the worker receiving an outside offer and potentially moving to the poaching firm. When the worker moves to the poaching firm he uses unemployment as a threat point, and then the current firm has zero continuation value and the worker's continuation value is $\beta S(x_0, \tilde{y})$. The final piece is the outside option of an employed worker: he forgoes the value of unemployment, b, and the possibility of finding a new job with surplus $S(x_0, y_0)$ and receiving β of this surplus. Notice that equation (4) is a functional equation in only $S(x, y)$, and the surplus sharing rule pins down the equilibrium wage equation as a function of (x, y).

3. Endogenizing the Contact Rates

This section outlines how to couch the model of Section 2 in a general equilibrium framework, including an aggregate matching function and optimal vacancy posting by firms. These additions allow the firm's decisions to affect the number of matches every period. Suppose that two matching functions determine the number of contacts that occur between unemployed and employed workers and firms in the economy every period.[7] Let v denote the number of vacancies in the economy. As with the model that has no match-quality, I assume Cobb-Douglas matching functions so that the number of contacts equals:

$$m_i(s,v) = m_0^i v^\alpha s^{1-\alpha}, i \in \{U, E\} \tag{5}$$

where s denotes the measure of searchers and m_0^i is the matching efficiency. In the current framework both unemployed and employed agents search and therefore $s = 1$ and the matching function satisfies:[8]

$$m_i(1,v) = m_0^i v^\alpha, i \in \{U, E\} \tag{6}$$

The aggregate meeting rate is:

$$f_i(v) = \frac{m_i(1,v)}{1} = m_i(1,v) = m_0^i v^\alpha, i \in \{U, E\} \tag{7}$$

and the vacancy filling rate is:

$$q_i(v) = \frac{m_i(1,v)}{v} = \frac{f_i(v)}{v} = m_0^i v^{\alpha-1}, i \in \{U, E\} \tag{8}$$

With this addition to the model, I can determine the contact rates given the number of vacancies in the economy.

I still need to determine how many vacancies firms open in equilibrium. To determine the equilibrium vacancy rate I introduce the vacancy creation condition. This condition represents the costs and benefits from opening a vacancy for an individual firm. There exists a flow cost, c, to maintaining an open vacancy. The benefit from posting a vacancy has two parts: the payoff from meeting an unemployed worker, $\mathbb{E}_u$, and the payoff from meeting an employed worker, $\mathbb{E}_e$. The payoff to meeting an unemployed worker is simply the portion of surplus, $1-\beta$, that the firm receives at combination (x_0, y_0). The payoff from meeting an employed worker depends on

[7] This is a reduced form way of incorporating search intensity into the model. It is necessary that the contact rates for the unemployed and the employed differ because $f_U \neq f_E$.

[8] See Mortensen and Nagypal (2005) for a similar set up.

whether the poaching firm successfully attracts the worker and the poaching firm's payoff from this new employment relationship. The probability of poaching a worker, in turn, depends on the distribution of (x, y) among employed workers, which I denote by $\pi(x, y)$, with associated cumulative density function $\Pi(x, y)$.[9] Hence, the vacancy creation condition can be written as:

$$V = -c + \delta m_0^U v^{\alpha-1} u \mathbb{E}_u + \delta m_0^E v^{\alpha-1}(1-u)\mathbb{E}_e \tag{9}$$

where u is the unemployment rate, $\mathbb{E}_u = \max\{0, (1-\beta)S(x_0, y_0)\}$ and $\mathbb{E}_e = \int\int\int\int \mathbb{I}\{S(x_0, \tilde{y}) > S(x', y)\} \max\{0, (1-\beta)S(x_0, \tilde{y})\} dF_x(x'|x) dF_y(\tilde{y}) d\Pi(x, y)$.

I assume that in equilibrium $V = 0$ due to free entry into vacancies so that equation (9) implies that the flow cost of opening a vacancy must equal the expected benefit from maintaining that open vacancy. If we know the expected benefit from posting a vacancy, this equation pins down the equilibrium vacancy rate. Notice that the expected payoff from meeting an employed worker enters the optimal decision of the firm. This presents an important deviation from the MP model that features no on-the-job search. In this context, searching while employed has consequences for aggregate dynamics.

4. Calibration Strategy

This section discusses the processes of state variables and calibration.

4.1. Processes for Idiosyncratic Productivity (x), Match-Quality (y) and Aggregate Productivity (z)

The model period length is one month. Idiosyncratic productivity starts out at a fixed and deterministic level x_0 in all matches, and then within the match follows a log AR(1) process:

$$\ln x' = \rho_x \ln x + \epsilon_x' \tag{10}$$

[9] $\pi(x, y)$ is in fact the distribution of (x, y) conditioned on employment so that it sums to one. When finding this distribution I iterate on the unconditional distribution, $\tilde{\pi}(x, y)$ so that together the unemployment rate and $\tilde{\pi}(x, y)$ sum to one. The two functions are related by the simple equation: $\frac{\tilde{\pi}(x,y)}{1-u} = \pi(x, y)$.

where $\epsilon'_x \sim \mathcal{N}(0, \sigma^2_{\epsilon_x})$. This process captures the intuition that productivity at the match level, or demand for the match's output, exhibits some persistence. Match-quality follows the process:

$$\ln y' = \begin{cases} \ln y_0 & \text{for jobs out of unemployment (U} \to \text{E)} \\ \ln y & \text{if no job change} \\ \epsilon'_y & \text{if changes jobs (E} \to \text{E)} \end{cases} \tag{11}$$

where $\epsilon'_y \sim \mathcal{N}(0, \sigma^2_{\epsilon_y})$. Hence, match-quality remains constant within a job, and is log-normally distributed when a worker meets a new firm. In the first job coming out of unemployment, match-quality is set to y_0.

I consider the following log AR(1) process for aggregate productivity:

$$\ln z' = \rho_z \ln z + \epsilon'_z \tag{12}$$

where $\epsilon'_z \sim \mathcal{N}(0, \sigma^2_{\epsilon_z})$.

4.2. Calibration Methodology

Given the optimal decisions of workers and firms, the model generates simulated data at a monthly frequency. In particular, I simulate 6,000 agents for 480 months (40 years). To remove the effects of initial conditions, I simulate the model for 1280 months and then discard the first 800 months of the sample. This simulation provides a time-path of wages and annual earnings, as well as an employment history.

I calibrate the parameters of the model using simulated method of moments, except for the aggregate parameters; namely the parameters associated with the aggregate productivity process, ρ_z and σ_{ϵ_z}, the flow cost of posting a vacancy, c, and the elasticity of the matching function with respect to vacancies, α. These aggregate parameters are chosen externally, which I describe in Section 4.3. The simulated method of moments procedure minimizes the distance between the summary statistics of the simulated data and the summary statistics of real data. Specifically, if θ represents the vector of structural parameters, $\hat{g}$ represents the moments of the actual data, and $g(\theta)$ represents the moments of simulated data, then the simulated minimum distance estimator is defined as:[10]

$$\hat{\theta} = \arg\min_{\theta} \mathcal{L}(\theta) = \arg\min_{\theta} [g(\theta) - \hat{g}]' W [g(\theta) - \hat{g}] \tag{13}$$

Here $g(\theta)$ represents a non-linear transformation of the structural parameters by the model and a transformation of the simulated data to achieve moments that match observed moments.

The optimization is implemented using the software package MATLAB, and KNITRO, a state-of-the-art solver, respected in the optimization community (see, for example, Byrd et al., 1999).

4.3. Calibration

This section presents the key moments of the data and discusses the calibration strategy. Table 1 summarizes the baseline parameters and the targeted empirical moments. Table 2 displays the simulated moments at the calibrated parameter values and shows that the model matches well the calibration targets.

As far as the aggregate parameters, I calibrate $\rho_z = 0.983$ and $\sigma_{\epsilon_z} = 0.005$. This calibration of the monthly productivity process yields an HP detrended series of logged labor productivity using a 10^5 smoothing parameter that has monthly persistence 0.92 and standard deviation 0.017, very close to the empirical counterpart in the U.S.: 0.88 and 0.02 respectively. I choose the elasticity of the matching function with respect to vacancies, α, to be 0.524 which is taken from Mortensen and Nagypal (2005) and captures the empirical elasticity of the job-finding rate with respect to vacancies. Finally, I choose c to normalize the vacancy rate in steady-state to one.

The idiosyncratic component (x) affects the level and persistence of displacements. The variance of ϵ_x is set to match the displacement probability observed in the data in the year following the first displacement. If x displays no variation ($\sigma_{\epsilon_x} = 0$), then the model features exogenous separations only and there is no serial correlation in displacements, which means that the probability of displacement in the year following displacement is the same as in all other years. With initial match-quality coming out of unemployment fixed to some y_0, the separation rate is likely

[10] The weighting matrix, W, is chosen so as to target percent deviations; namely the weight equals $\frac{1}{\hat{g}_i^2}$ for moment i.

to be higher in the year following displacement when σ_{ϵ_x} is higher.

Conditional on targeting the displacement probability in the year following the first displacement, ρ_x targets the persistence in this displacement probability. Fixing match-quality at some y_0 along with time-varying idiosyncratic productivity, delivers some serial correlation in displacements as individuals low on the job ladder experience a higher probability of separation than those further up the job ladder. Conditional on a relatively high x_0, as turns out to be the case, higher ρ_x will serve to mitigate serially correlated displacements as individuals experience high idiosyncratic productivity for longer.[11]

PSID data provide a way to measure serial correlation in displacements. The PSID began in 1968 with an interview of 5,000 families, and follows any new families formed from the original group of families. I follow Polsky (1999) closely with my empirical approach to calculating job switches. Anticipating the E-E transition analysis later in this paper, I use the 1976-1997 waves of the PSID study. I drop the years prior to 1975 because the job history data for these years are poor (Brown and Light, 1992), and I omit the years following 1997 because of the biennial surveys. I include an individual in the sample if they appeared as a household head for three consecutive years from their first year as household head in the survey. In the data, job displacements are determined from a question that asks respondents with low levels of current job tenure "What happened to that employer (job)?" (the individual's previous job). The two categories of responses used to identify displacements are "plant closed/employer moved" and "laid off/fired." Using this data, in the year following their first displacement, workers' probability of experiencing another displacement increases by around 25 percentage points. The effect of the first displacement displays serial correlation with a 0.63 annual persistence parameter.

The starting idiosyncratic productivity for unemployed individuals, x_0, is targeted to generate the employment-to-unemployment (E-U) transition probability found in the United States gross flows data. As x_0 rises, unemployment becomes more appealing because the first job coming out of unemployment has higher productivity. This induces a larger fraction of the employed to flow into unemployment every

[11] A relatively high x_0 implies that productivity within the match trends down over time. This is consistent with Mortensen and Pissarides (1994) and Hall (1999).

period. Elsby et al. (2010) find the monthly layoff inflow rate is around 1.5 percent (table 9 in their paper). Since most displacements represent no fault termination or layoff from employment, using the layoff inflow rate is appropriate.

The model provides a convenient way of calibrating the standard deviation of match-quality: the on-impact dip in earnings resulting from displacement. Increasing the dispersion in y implies that agents on average move further up the job ladder, and have more earnings to lose, when they experience a displacement. This increases the on-impact dip in earnings resulting from displacements. In a very similar model to the one presented here, Low et al. (2010) estimate the standard deviation of match-quality at 0.23.

I target the time-path of displaced worker earnings in this analysis.[12] This shows that there exists a model and a set of parameter values that delivers something close to the observed earnings experience of displaced workers, which is an accomplishment in and of itself given the recent article by DV highlighting the inability of standard search models to capture this fact. Appendix D provides further implications of this calibrated model for a range of un-targeted outcomes.

The observed E-E transition rate is targeted using f_E. Raising the number of contacts employed workers have with outside firms raises the probability that workers experience E-E switches. Intuitively, this implies that E-E flows in the model are monotonically increasing in f_E. Fallick and Fleischman (2004) use data from the basic monthly Current Population Survey (CPS) from January 1994 to December 2003. They find that an average of 2.6 percent of employed persons change employers each month.

The contact probability for the unemployed, f_U, is determined by targeting the aggregate job-finding probability. Increasing the job-contact rate means that unemployed workers experience more frequent contacts and since workers accept all first

[12] In practice I target four points of this time-path: the initial point (six years before displacement), the point in the year just before displacement, the trough, and the point 20 years after displacement. The difference between the point in the year just before displacement and the initial point is referred to as the 'pre-displacement rise in earnings' in Table 2. The difference between the trough of this time-path and the point 20 years after displacement is referred to as the 'recovery of displacement earnings' in Table 2. Since there are eight internally calibrated parameters (that is, not counting the externally set aggregate parameters), and 10 micro moments, the model is over-identified.

offers in this model, the unemployment-to-employment probability rises. Following Shimer (2005) this analysis targets a monthly job-finding rate of 45 percent.

The exogenous separation rate (p_s) targets a slight increase in earnings prior to displacement, as in DV. With only exogenous separations ($p_s = 0.015$) in the model there would be no movement in average earnings prior to displacement. Alternatively, if all the displacements in the model were endogenous ($p_s = 0$) then earnings tend to vary more prior to displacement.

The value of leisure, b, is chosen to target the value found in Hall and Milgrom (2008): 0.71 of average productivity of labor (APL). The value found in Hall and Milgrom (2008) serves as a benchmark as we know that the value of leisure has important implications for aggregate volatility in this context.[13]

The starting match-quality is normalized to the expected value of the stochastic process for y, which is close to one. At the solution, this means that y_0 falls around two standard deviations below the average match-quality among employed workers. The bargaining power of the worker, β, is set to 0.5, realistic adjustments of which I have found to be immaterial. Finally, δ targets a five percent annual interest rate.

5. Steady-State Equilibrium

5.1. Individual Earnings and Employment

To compare the simulated and observed data, the simulated monthly wage information is aggregated into annual earnings data and the following equation is estimated, which is equivalent to equation (1) in DV:

$$e_{it}^d = \alpha^d + \sum_{k=-6}^{20} D_{it}^k \delta_k^d + u_{it}^d \tag{14}$$

where the superscript d denotes the displacement year, the outcome variable e_{it}^d is annual earnings of individual i in year t, α^d represents a constant, D_{it}^k are dummy variables equal to one in the worker's kth year before or after his displacement and zero otherwise, and the error u_{it}^d represents random factors. Note that $k = 1$ denotes the displacement year and $k = 0$ denotes the final year of positive earnings from

[13] Table 2 shows that the model cannot quite hit the Hall and Milgrom (2008) target.

the pre-displacement employer. I omit individual and time fixed effects because the model of this paper does not feature individual or time variation. Although DV estimate this equation separately for each displacement year d, in the model presented in this paper all years are identical, so d is fixed at an arbitrary year.

As in DV, I impose a tenure restriction on the sample. In particular, the worker must have positive earnings from the employer in question in $d-3$, $d-2$, and $d-1$. This could mean as little as 14 months of tenure at the time of displacement. Furthermore, a worker "separates" from an employer in year d when he has earnings from the employer in $d-1$ but not in d and the worker experiences a separation into unemployment in year $d-1$. Conditioning on job loss is important because a worker may not have earnings from his previous employer in year d because of an E-E transition. These workers are not included in the treatment or the control groups. This resembles the treatment group used by DV as they omit so-called non-mass-layoff separators from the control group. I cannot impose the same "mass layoff" definition as DV because the model features one-worker firms.

For year d, the treatment group includes those workers displaced in year d, $d+1$ and $d+2$. Including workers from three years serves to smooth the estimated earnings effects of job displacement from year to year. The control group includes individuals with the same tenure requirement who do not experience a displacement in year d, $d+1$, and $d+2$. For the control group, $D_{it}^k = 0$ for all t so that the dummy variables reflect the change in earnings relative to this control group. The tenure restriction implies that most individuals in the treatment group separate from their employer via an exogenous separation. Nonetheless, endogenous separations play a key role in explaining serial correlation in displacements.

Figure 1 presents a comparison between the results from the baseline model and the results from DV, together with results from the standard MP model. The outcome is very encouraging, with the baseline model delivering an earnings trajectory that closely resembles the empirical counterpart. The search model outlined in this paper can account for the time-path of displaced worker earnings.

On impact the model predicts the losses in annual earnings well: around 30 percent. Additionally, the model captures the movements in earnings post-displacement very well. For the first 10-15 years of the recovery the model provides a remarkable fit. The model cannot deliver the plateauing, and even declining, earnings time-path after 15 years observed in the data. The model features ex-ante homogeneous agents and a steady state wage distribution, which imply that eventually the earnings of displaced workers will recover. Nevertheless, after 20 years the model implies earnings losses similar to those found in the data.[14]

Loss in match-quality results in the on-impact dip in earnings, as workers fall from higher rungs of the job ladder, to a low job rung in their first job out of unemployment. Earnings fall slightly in the year following displacement because some workers lose their jobs late in the '0' year and so have a substantial amount of earnings in the year of job loss. Since it takes unemployed workers time to find jobs, and $y_0 < \mathbb{E}[y|match]$ so that first jobs pay very little, in the year immediately following job loss workers may actually experience a small dip in earnings. This additional loss in earnings is also attributable to using observations from years d, $d+1$ and $d+2$, which serves to smooth out the effects of displacement.

The slow recovery in earnings represents the slow move up the job ladder for recently displaced workers, which in turn manifests serially correlated displacements. Agents experience serially correlated displacements because match-quality remains low in first jobs and therefore only small movements in idiosyncratic productivity cause further displacements.

Figure 2 compares the percentage-point change in the displacement probability (from the average displacement probability) for the model and the PSID after the first displacement.[15] The line implied by the model incorporates the PSID survey algorithm. In other words, I look at individuals every 12 months and, if their tenure is less than 12 months, and their most recent job ended in an unemployment spell, I classify them as displaced. I divide the number of displacements every year by the number of employed individuals last year to obtain the model implied displacement

[14] The time-path of earnings from the simulated data is not smooth due to the limited number of agents. Adding more agents to the simulation would smooth out this time-series. Also, Davis and von Wachter (2011) do not present results that do not distinguish between expansions and recessions so a direct comparison to the model is not possible. Since times of expansion are much more prevalent than times of recession, most displacements occur during times of expansion. Thus, I suspect that results averaged over expansions and recessions would appear close to the 'expansion' estimates.

[15] The figures only document probabilities up to 10 years following displacement due to small sample sizes beyond this horizon in the PSID.

probability.

The model displays serial correlation in displacements which quantitatively matches the evidence from the PSID. The model delivers the initial spike in displacement probability, and delivers slightly more persistence in displacements than we observe in the data, with the first displacement effect not quite subsiding after 10 years.

Appendix D presents additional, un-targeted, moments of the data that the model speaks to. The model performs well in explaining wage related moments, the decomposition of earnings losses into reduced wages and employment, and movements in total factor productivity (TFP).

5.2. Steady-State Distribution of Match-Quality

In order to understand the aggregate fluctuations it is instructive to discuss the steady-state distribution of idiosyncratic productivity and match-quality. The algorithm for solving for the steady-state distribution in the model with match-quality is presented in Appendix E.1.1.

For illustration, define $y^R_{avg[x]}$ as:

$$S(avg[x], y^R_{avg[x]}) = 0 \tag{15}$$

That is, $y^R_{avg[x]}$ is the reservation match-quality when x is at its average value in equilibrium.

Figure 3 shows the steady-state distribution of y, holding productivity at $avg[x]$, in the model with match-quality with the baseline calibration.[16] The figure shows that there exists a spike at the bottom of the distribution at $y = y_0$. This makes clear that y_0 is quite low in the distribution of y. It also shows that there exist a large fraction of employment relationships near the destruction threshold. Small movements in aggregate productivity will erase a large fraction of employment relationships that have the starting match-quality. This will have a large impact on the E-U probability in transition and therefore the unemployment rate. In this sense the model with match-quality can alleviate some of the tensions of the model without match-quality. Introducing a second state variable that is particularly low among new matches means that many employment relationships are susceptible to

[16] This is similar to Figure 5 in Bils et al. (2011).

destruction with a reduction in aggregate productivity, which will result in increased amplification. At the same time, new matches are still viable. With one-dimensional heterogeneity the starting value cannot be erased because then no new matches would form.

The model will deliver significant propagation due to the slow-moving nature of the joint distribution of idiosyncratic productivity (demand) and match-quality. Since the firm's vacancy posting decision depends on this slow-moving object, the job-finding rate is likely to be persistent following an aggregate downturn. The amount of aggregate propagation is closely linked to the amount of persistence at the individual level. Since workers face very persistent earnings losses post-displacement because they have to climb the job ladder, this implies a slow-moving aggregate distribution of match-quality after a reduction in aggregate productivity. Furthermore, serial correlation in displacements will show up as persistent unemployment at the aggregate.

Hornstein et al. (2011) show that when the standard MP model is calibrated to match observed wage dispersion, the value of leisure needed to deliver observed U-E flows is very low. This is because large wage dispersion implies a large optional value to remaining unemployed, so only very low values of leisure can induce workers to take jobs.[17] The model presented here is able to hit the wage dispersion documented by Hornstein et al. (2011) without resorting to a very low value of leisure because the initial match-quality does not vary. In this version of the model the initial match-quality is fixed at y_0. This implies that there exists a large mass of jobs at the destruction frontier that results in large movements in unemployment in response to aggregate productivity shocks. The initial match-quality does not need to be fixed for this effect, but the amplification properties do require that unemployed workers draw match-quality from a distribution that has higher mass at low-quality matches than the distribution faced by employed workers.

6. Aggregate Fluctuations

This section presents the business cycle movements of the baseline model. This is not a trivial extension because the equilibrium vacancy rate, determined by equa-

[17] Hornstein et al. (2011) show that E-E transitions mitigate this necessity although not fully.

tion (9), depends on the distribution of idiosyncratic productivity and match-quality among employed workers, $\Pi(x,y)$. Furthermore, today's vacancy rate depends on tomorrow's distribution, $\Pi'(x,y)$, because tomorrow's surplus depends on tomorrow's vacancy rate, v'. Hence firms need to forecast the entire distribution, $\Pi(x,y)$, to make optimal decisions.

6.1. Computational Strategy

The algorithm for computing the perfect foresight transition is detailed in Appendix E.1.3, and is similar to the technique used for the model without match-quality presented in Section 7. The procedure implements the Krusell and Smith (1998) algorithm in the current context.

Forecasting the entire $\Pi(x,y)$ is infeasible so I assume that the agents only use certain moments of the endogenous $\Pi(x,y)$ distribution when making vacancy posting decisions, and I postulate a guess for how these moments determine equilibrium v. I assume the moment of the distribution is the average x among the employed (denoted $X = \int x\pi_x(x)dx$ where $\pi_x(x) = \int \pi(x,y)dy$ is the marginal distribution of x) and I conjecture a log-linear transition equation:

$$\ln X' = \chi_0 + \chi_X \ln X + \chi_z \ln z' \tag{16}$$

The firm then uses this forecast for the aggregate state to determine tomorrow's vacancy rate via the equation:

$$\ln v' = v_0 + v_X \ln X' + v_z \ln z' \tag{17}$$

The goal is to find the parameters $\{\chi_0, \chi_X, \chi_z\}$ and $\{v_0, v_X, v_z\}$ that accurately forecast aggregate variables. Given an arbitrary sequence of aggregate productivity $\{z\}_{t=1}^{T}$, I simulate the economy and estimate the coefficients using ordinary least squares with the simulated data. Once I have coefficients that yield sufficiently accurate forecasts, I use the simulated data to compute the elasticities of the respective series with respect to labor productivity. I also use the coefficients to perform impulse response functions.

6.2. Results

Table 3 presents the elasticities of aggregate variables with respect to output per worker in the model with match-quality. In this table, I also present the baseline results from Shimer (2005) for comparison.[18] The model delivers a significant amount of amplification in the E-U rate and unemployment. The volatility of the E-U rate in the model exceeds the volatility of the E-U rate in the data, although the model hits the volatility of unemployment almost exactly. This is largely due to the mass of recently hired unemployed workers who start at the bottom of the job ladder and are therefore susceptible to downward movements in aggregate productivity. The model delivers larger fluctuations in the vacancy rate and the job-finding rate than both the MP model and the model without match-quality of Section 7. Nevertheless, the model only explains about 60 percent of the empirical volatility of the vacancy rate, and only 35 percent of the empirical volatility of the job-finding rate.

Figure 4 depicts the outcomes of key aggregate variables in response to a permanent, unexpected, one percent reduction in aggregate productivity. That is, aggregate productivity falls unexpectedly by one percent and remains at that level forever, although agents continue to have expectations about aggregate productivity consistent with equation (12). The model displays a sharp rise in E-U separation in the wake of a recession that then falls slightly and continues to rise thereafter to its new steady state value. The rise in the E-U rate results from a discrete mass of jobs becoming unprofitable and being destroyed immediately. The new steady state of the E-U probability is above the original steady state due to a lower aggregate productivity, which means more jobs are fragile. The response of the E-U rate is very large, reflecting that many employment relationships are near the destruction threshold due to the low match-quality of new hires. As a result of the large rise in the E-U rate, unemployment also reacts strongly. The response of the E-U rate in this model stands in sharp contrast to the response in the basic MP model. In that model, separations are exogenous so that the E-U rate does not deviate at all from its initial value when aggregate productivity falls.

The model with match-quality also exhibits a slight cleansing effect as low-quality

[18]These results are based on a numerical replication at monthly frequency performed by the author.

matches are destroyed. Due to lower aggregate productivity, firms post fewer vacancies which results in lower job-finding rates. This causes average match-quality to fall as agents make their way up the job ladder at a reduced rate. This is the central point of a related paper, Barlevy (2002). The rise in unemployment, and the large expected payoff from meeting an unemployed worker, dampen the negative effect of lower productivity on vacancies.

Due to the initial reduction in vacancies, the model also delivers a falling E-E rate during a recession, which is what one observes in the data.[19] It takes years for the E-E rate to recover, and eventually it reaches a level that is higher than its original value because of a higher E-U rate into unemployment, which, in equilibrium, implies more individuals grouping at the bottom of the job ladder. Workers coming out of unemployment initially transition quickly up the ladder as they encounter many favorable outside offers so they tend to raise the average E-E rate in the economy. Moreover, due to a lower average match-quality more workers are available for poaching, thereby raising the average E-E rate.

The economy delivers significant propagation of aggregate productivity shocks. It takes unemployment around five years to complete 80 percent of its adjustment to the one-time change in aggregate productivity. After five years, vacancies, and therefore the job-finding rate, have only completed half of their transition to their new values. This significant propagation is due to the slow moving nature of the distribution of idiosyncratic productivity and match-quality among the employed, and is more in line with empirical observation.[20] This propagation of the job-finding rate is a marked improvement over the standard MP model, which features a job-finding rate that adjusts instantaneously to aggregate productivity shocks. Figure 4 plots the response of vacancies and unemployment in the standard MP model. As mentioned in Section 1, the MP model delivers almost no propagation: both unemployment and vacancies adjust to their new level within a year. The figure also reinforces that the model with match-quality delivers more amplification than the standard MP model, especially in the unemployment rate.

Aside from the cyclicality of worker flows, Shimer (2005) also notes that with

[19] See, for example, Nagypal (2008).

[20] See, for example, Fujita and Ramey (2007).

countercyclical separations, the baseline MP model fails to deliver the observed procyclicality of vacancies. This implies that the model fails to deliver the observed negative relationship between vacancies and unemployment, called the Beveridge curve. The model with match-quality presented here, due to procyclical vacancies, delivers a negative relationship between vacancies and unemployment. The model overcomes the weakness of the MP model by allowing firms to contact employed workers. As in the standard MP model, as the E-U rate rises, firms want to post more vacancies due to a higher job-filling rate. However, in the current setup, firms also face lower productivity of employed workers when aggregate productivity falls, and since these workers make up the majority of the potential applicant pool, this serves to reduce vacancies. Estimating a linear regression with de-trended vacancies as the dependent variable and de-trended unemployment as the independent variable yields a coefficient of -0.42. The empirical counterpart, using data from the Job Openings and Labor Turnover Survey (JOLTS), is -1.12.[21] Hence, the model delivers the qualitative relationship between vacancies and unemployment, but falls short quantitatively. This is because vacancies exhibit too little volatility in the baseline model compared to the data.

These impulse responses show that the baseline economy behaves remarkably like the actual economy. At the start of a recession there is a spike in the E-U rate, while the job-finding rate for the employed and the unemployed falls, just as observed in empirical worker flows in the United States data (see, for example, Elsby et al., 2009). The model delivers significant propagation of aggregate shocks and matches the observed negative relationship between vacancies and unemployment.

7. A Model without Match-Quality

To gain intuition for the aggregate dynamics of the model with match-quality, this section presents a simpler version of that model without match-quality. The framework still incorporates search and matching, and is in large part the same as the model with match-quality, aside from one exception: the model presented here

[21]This analysis uses quarterly-averaged data from JOLTS and the Current Population Survey from 2001Q1 to 2007Q4. Job openings are used as a fraction of the labor force and regressed on the unemployment rate after logging and HP de-trending both time-series.

features no match quality, denoted by y previously. The output of every match is a linear function of the idiosyncratic productivity, denoted by x, and distributed according to $F(x'|x)$. Aggregate productivity, z, still affects all matches. The model still features E-E transitions and a job ladder.[22] There are substantive differences between this model and the model with match-quality. The addition of a second state variable in the model with match-quality allows the economy to lose many productive matches in a downturn, while still allowing new matches to survive. In the model without match-quality, one cannot shed relationships at the starting productivity level because then no new employment relationships would form. Moreover, the model with match-quality features two-dimensional heterogeneity which leaves room for more propagation.

This model can be characterized by a series of Bellman equations. The value of work to the employee, $W(x)$, satisfies:

$$\begin{aligned} W(x) = w + \delta(1-f_E)(1-p_s)\int \max\{U, W(x')\}dF(x'|x) + \delta p_s U \\ + \delta f_E(1-p_s)\int\int \max\{U, W(x'), W(\tilde{x})\}dF(x'|x)dG(\tilde{x}) \end{aligned} \tag{18}$$

The intuition here is straightforward. The flow payoff to a job equals the wage, w. In the future, given no outside offer, and no exogenous separation shock, idiosyncratic productivity changes according to distribution $F(x'|x)$ and, depending on the level of this future shock, the worker decides whether to remain at the current firm, or flow into unemployment and look for an alternative match. With an outside offer, and no separation shock, the worker decides whether to remain at the current firm or to move to the new firm, with unemployment always remaining as an option. The right hand side also includes the possibility of an exogenous separation shock occurring with probability p_s.

The value of unemployment satisfies:

$$U = b + \delta(1-f_U)U + \delta f_U W(x_0) \tag{19}$$

[22] In other words this is a simplification of the model with match-quality with $y_0 = 1$ and $\sigma_{\epsilon_y} = 0$.

The unemployed receive flow payoff b. They either receive an offer and take it or remain unemployed. Notice that all jobs start at the same level of idiosyncratic productivity, x_0, which is set to the mean value of the unconditional distribution, denoted by $\bar{x}$. The calibration presented later guarantees that $W(x_0) > U$, so that the worker prefers employment to unemployment at the starting productivity level.

The value of a filled job to the firm, $J(x)$, satisfies:

$$\begin{aligned} J(x) = z \cdot x - w + \delta(1 - f_E)(1 - p_s) \int \max\{0, J(x')\} dF(x'|x) \\ + \delta f_E(1 - p_s) \int \int \mathbb{I}\{J(x') \geq J(\tilde{x})\} \max\{0, J(x')\} dF(x'|x) dG(\tilde{x}) \end{aligned} \tag{20}$$

where z denotes aggregate productivity. The payoff to the firm includes the output, $z \cdot x$, less the wage paid to the worker, w. In the next period, given no outside offer and no separation shock, the firm decides to continue with the match or to let the worker go and open a vacancy, depending on the level of the idiosyncratic shock. In equilibrium vacancies are assumed to have value zero, which is guaranteed by a free-entry condition into vacancy posting. With an outside offer, the employer does not leave only if the value of the current job exceeds the value of the outside job.

These value functions can be summarized by one equation, the surplus from a match:

$$\begin{aligned} S(x) &= W(x) - U + J(x) \\ &= z \cdot x + \delta(1 - f_E)(1 - p_s) \int \max\{0, S(x')\} dF_x(x'|x) \\ &+ \delta f_E(1 - p_s) \int \int \Big[\mathbb{I}\{S(x') \geq S(\tilde{x})\} \max\{0, S(x')\} \\ &+ \mathbb{I}\{S(\tilde{x}) > S(x')\} \max\{0, \beta S(\tilde{x})\} \Big] dF(x'|x) dG(\tilde{x}) \\ &- [b + \delta f_U \beta S(x_0)] \end{aligned} \tag{21}$$

which is the flow payoff from the match and the continuation value of the match, less the worker's outside option that includes a flow payoff from unemployment and the chance of finding a new job with idiosyncratic productivity at x_0.

In the calibration below, it turns out that $S(x)$ is increasing in x, so define the reservation cutoff as the level of productivity that makes worker and firm indifferent between maintaining the current match and terminating the current employment relationship:

$$S(x_R) = 0 \tag{22}$$

Wage bargaining follows the standard Nash bargaining protocol so that:

$$w = \arg\max_{w} [W(x) - U]^{\frac{1}{2}} [J(x) - V^s]^{\frac{1}{2}} \tag{23}$$

This implies that worker and the firm split surplus evenly.

The number of new meetings between the unemployed and vacancies is determined by an aggregate matching function, exactly as in the model with match-quality:

$$m_i(1, v) = m_0^i v^\alpha, i \in \{U, E\} \tag{24}$$

where v is the number of vacancies. The aggregate meeting rate is:

$$f_i(v) = \frac{m(1, v)}{1} = m(1, v) = m_0^i v^\alpha, i \in \{U, E\} \tag{25}$$

and the vacancy filling rate is:

$$q_i(v) = \frac{m(1, v)}{v} = \frac{f_i(v)}{v} = m_0^i v^{\alpha-1}, i \in \{U, E\} \tag{26}$$

In order to determine the number of vacancies in equilibrium, we need the value of posting a vacancy:

$$\begin{aligned} V^s &= -c + \delta q_U(v) u (1 - \beta) S(x_0) \\ &+ \delta q_E(v)(1 - u) \int \int \int \mathbb{I}\{S(\tilde{x}) > S(x')\} \max\{0, (1 - \beta) S(\tilde{x})\} dF(x'|x) dG(\tilde{x}) d\Pi(x) \\ &= -c + \delta q_U(v) u \mathbb{E}_u^s + \delta q_E(v)(1 - u) \mathbb{E}_e^s \end{aligned} \tag{27}$$

where c is the flow cost of maintaining a vacancy, $q_i(v)$ is the job-filling probability, $\mathbb{E}_u^s$ and $\mathbb{E}_e^s$ denote the expected payoff to meeting an unemployed and employed worker respectively, and the s superscript denotes the simple model. $\Pi(x)$ is the distribution of idiosyncratic productivity among employed workers and is a slow-moving, endogenous object. Notice that this pins down v uniquely:

$$v = \left(\frac{\delta m_0^U u \mathbb{E}_u + \delta m_0^E (1-u)\mathbb{E}_e}{c} \right)^{\frac{1}{1-\alpha}} \tag{28}$$

7.1. Calibration of the Model without Match-Quality

Given the optimal decisions of workers and firms, the model generates simulated data at a monthly frequency. In particular, I simulate 6,000 agents for 480 months (40 years). To remove the effects of initial conditions, I simulate the model for 980 months and then discard the first 500 months of the sample. This simulation provides a time-path of wages and annual earnings, as well as an employment history.

The model period length is one month. In what follows, idiosyncratic productivity starts out at a fixed and deterministic level x_0 in all matches, and then within the match follows a log AR(1) process:

$$\ln x' = \rho_x \ln x + \epsilon_x' \tag{29}$$

where $\epsilon_x' \sim \mathcal{N}(0, \sigma_{\epsilon_x}^2)$. This process captures the intuition that productivity at the match level, or demand for the match's output, exhibits some persistence.

As in the model with match-quality I consider the following log AR(1) process for aggregate productivity:

$$\ln z' = \rho_z \ln z + \epsilon_z' \tag{30}$$

where $\epsilon_z' \sim \mathcal{N}(0, \sigma_{\epsilon_z}^2)$.

Aggregate parameters are set as in the model with match-quality. Namely, $\rho_z = 0.983$ and $\sigma_{\epsilon_z} = 0.005$, yielding, after logging and HP filtering, a 0.92 autocorrelation of aggregate labor productivity, with a standard deviaion of 0.017. c is set to normalize $v = 1$ in steady state.

The calibration follows Hornstein et al. (2011) and Bils et al. (2011) very closely, although I additionally target the average E-E rate. Table 4 shows the baseline parameter values for the model without match-quality. I take the annualized interest rate to be five percent. The key targeted outcomes are the average rates of unemployment and separations, and the average E-E rate. Following Bils et al. (2011) I target an average unemployment rate of six percent, and a monthly separation of two percent that is consistent with work using the Survey of Income and Program Participation. The two percent separation rate and the six percent unemployment rate pin down the steady-state monthly job-finding rate at 31 percent. I target a monthly E-E rate of 2.6 percent, consistent with evidence in Fallick and Fleischman (2004).

The vacancy posting cost c is chosen to normalize the steady-state vacancy level to one. The matching technologies are Cobb-Douglas; $m_U(1, v) = 0.31v^{\alpha}$; $m_E(1, v) = 0.096v^{\alpha}$ which hit the steady-state finding rates. The matching power parameter α is set to 0.5. In the model without match-quatliy I fix the persistence of idiosyncratic productivity at 0.97, the value in Bils et al. (2011), to match highly persistent individual wage earnings. I choose the standard deviation of idiosyncratic productivity to match an observed mean-min wage ratio of 1.75, as documented by Hornstein et al. (2011). I choose the value of leisure, b, to match the two percent separation probability.[23]

Notice that in order to match the duration of unemployment and the mean-min wage ratio, the model requires a low value of leisure. Hornstein et al. (2011) point out the same phenomenon. Their main message is that the wage dispersion delivered by search models is constrained by preference parameters and the observed size of the transition rates of workers. The intuition is that if we observe very large U-E rates this must mean that the wage offer distribution is not very dispersed, otherwise workers would be willing to wait longer for a potentially better offer. With the particular set up outlined here, the U-E probability is calibrated to 31 percent and the E-U probability at two percent. In order to simultaneously target a large mean-min wage ratio (1.75) and the large U-E flows, the model requires the value of leisure to be around 20 percent of average labor productivity among the employed. This low value of leisure insures that, despite the large wage dispersion, the model can match the observed U-E probability.

[23] An alternative calibration involves fixing the value of leisure at 0.4, the value found in Shimer (2005) and choosing σ_{ϵ_x} to match the two percent separation probability. This calibration falls far short of the observed mean-min wage ratio, but conveys the same message: the standard search model cannot hit the time-path of earnings around displacement. Although raising the value of leisure helps this model's amplification properties it would severely undermine the model's ability to deliver observed wage dispersion.

7.2. Steady State of the Model without Match-Quality

This section outlines some steady-state features of the model without match-quality. The focus of the analysis is on the ability of the model to match observed cross sectional wage facts and the earnings losses of displaced workers. The distribution of idiosyncratic productivity is largely responsible for the model's earnings implications, and this distribution is described in detail.

Table 5 presents some steady-state moments of the simulated data, and Figure 1 presents the earnings losses of displaced workers in this model, along with the empirical losses from Davis and von Wachter (2011). The figure imposes at least three years of positive earnings from the same employer, and then a separation that moves the worker into unemployment. The earnings losses are relative to a non-displaced control group with the same three year tenure requirement as the displaced treatment group, and are plotted as a fraction of pre-displacement earnings of the treatment group.

The calibration can match the targeted moments exactly, hitting the unemployment rate, the separation rate and the job-finding rates. The calibration delivers significant wage dispersion, obtaining an observed mean-min wage ratio of 1.75, and around 37 percent cross-sectional variation in wages. Figure 1 plots the earnings losses of displaced workers associated with this calibration. The on-impact dip in earnings is around 20 percent, a little lower than the observed losses, and the recovery in earnings is far too quick. With this calibration, earnings, relative to a control group, recover within three years. The pre-displacement dip in earnings occurs because of endogenous separations and persistence in the idiosyncratic productivity process. Conditional on displacement in year d, agents' earnings are falling in the years before d as idiosyncratic productivity begins to fall. In contrast, based on no separation in year d, the average idiosyncratic productivity of the control group begins to rise before year d. The basic MP model delivers very similar earnings dynamics, except that all separations are exogenous so that this time-path does not exhibit a pre-displacement dip. The recovery is slightly faster with earnings catching up with the non-displaced in two years. In the MP model the only reason for earnings losses is unemployment because all agents earn the same wage when employed.

One key to understanding wage dispersion in this calibration, and for understanding the aggregate implications for job destruction, is the underlying steady-state distribution of idiosyncratic productivity. The numerical approach for obtaining this steady-state distribution is detailed in Appendix E.2.1. Figure 5 presents the steady-state distribution above the economy's reservation productivity x_R. The distribution peaks at the starting idiosyncratic productivity x_0, as there exists a mass of unemployed workers entering employment at this productivity level.

This figure highlights that the model without match-quality features a disperse idiosyncratic productivity distribution, with very little mass towards the reservation productivity. This means that this economy features realistic wage and wage growth dispersion. However, this calibration exhibits very small fluctuations in unemployment and vacancies because downward aggregate productivity movements affect few matches.

In addition to the job destruction margin, the job creation margin plays an important role in aggregate dynamics. As noted before, the value of leisure is very small in the model without match-quality. This allows the model to simultaneously hit the job-finding rate, and the large wage-dispersion observed in the data. A small value of leisure implies that the surplus from employment relationships is very large. Hagedorn and Manovskii (2008) show that the volatility of labor market tightness is very closely related to the value of leisure and the size of accounting profits. This calibration features large accounting profits which means that the vacancy posting incentives in this economy will be muted. The job creation margin also features prominently in the aggregate fluctuations, which I point out in the next section.

7.3. Aggregate Fluctuations in the Model without Match-Quality

This section presents the responses of the calibrated economy to aggregate productivity shocks. As with the model with match-quality, the procedure implements the Krusell and Smith (1998) algorithm in the current context, and I refer the reader to Appendix E.2.3 for the computational algorithm and Section 6.1 for an outline of the method used.

Table 3 presents the elasticities of aggregate variables with respect to output per worker in the model without match-quality. The model fails to deliver sufficient amplification of aggregate shocks. This is most starkly visible in the elasticities of the E-U rate and unemployment, which fall far short of the observed values. The model does, however, manage to deliver a counter-cyclical E-U rate and a negative relationship between vacancies and unemployment as observed in the data. As in the model with match-quality, this is because firms can contact employed workers.

Figure 6 portrays the results for the model without match-quality in response to this aggregate productivity reduction. On impact the returns to posting a vacancy fall immediately as aggregate productivity falls, and hence vacancies jump down on impact. As a result of the reduced aggregate productivity, the model displays a sharp rise in E-U separations that then falls, and continues to rise thereafter to its new value. The rise in the E-U rate results from a discrete mass of jobs becoming unprofitable and being destroyed immediately. With the new aggregate productivity, the E-U probability is above the original value due to more fragile jobs. The model also displays a very small cleansing effect as low productivity jobs are destroyed. This is not obvious since the values plotted are end-of-period values and, although low match-quality jobs have been destroyed, the E-E probability is also immediately lower, and hence employed workers have fewer outside opportunities to move up to. It seems that the cleansing effect dominates initially. Due to lower aggregate productivity, firms post fewer vacancies which results in lower job-finding probabilities. This causes average match-quality to fall as agents make their way up the job ladder at a reduced rate. As in the model with match-quality, this relates to Barlevy (2002). The E-E probability falls on impact due to the lowered vacancy rate, however as average match-quality falls in the economy, and more workers are available for poaching, the E-E probability rises slightly towards its new value.

The model without match-quality, however, delivers very little propagation compared to the model with match-quality. Notice that vacancies effectively jump down to their new value on impact, while the E-U probability and unemployment complete most of their transition in around one year. The E-E probability is slightly more sluggish, completing its transition after about two years, and the same holds true for the average match-quality in the economy. Hence, although the simpler model does perform better than the baseline MP model, quantitatively it falls short of matching the data as well as the model with match-quality.

8. Summary and Discussion

This paper investigates the aggregate labor market fluctuations associated with a model that features both idiosyncratic productivity and match-quality. Closing the model involves introducing aggregate matching functions for the unemployed and the employed, and introducing the optimal vacancy creation condition for the firm. A simpler version of the model, with only one state variable delivers far less amplification and propagation of aggregate productivity shocks. With idiosyncratic shocks that deliver the correct mean-min wage ratio there exist very few matches at the destruction threshold which implies that aggregate productivity shocks have very little bite. In addition to little job destruction, this calibration of the model implies large accounting profits which means that vacancies do not respond sufficiently to lower aggregate productivity.

This tension is mitigated in the model with match-quality. This is not obvious since the model with match-quality resembles the model without match-quality, but features additional volatility due to variation in match-quality. Since variation in idiosyncratic productivity was the reason for a lack of amplification of aggregate productivity shocks in the simpler model, it may seem that adding additional volatility via match-quality would exacerbate the amplification problem in the model with match-quality. Despite this additional volatility, the model with match-quality delivers significant unemployment amplification because new matches begin with low match-quality which implies that there exist many relationships at the reservation frontier. Relationships with this starting match-quality can be destroyed in the model with match-quality because relationships are characterized by two state variables: idiosyncratic productivity and match-quality. In the model without match-quality eliminating employment relationships with the starting productivity was not a possibility because then no new matches would form. In the model with match-quality aggregate productivity shocks have large effects on unemployment that are quantitatively consistent with the observed facts.

The model with match-quality also delivers significant propagation of aggregate productivity shocks. After a one percent unexpected reduction in aggregate productivity, unemployment takes around five years to complete 80 percent of its transition. Vacancies, and therefore the job-finding rate, take even longer to respond to the same shock. The propagation stems from the slow-moving distribution of idiosyncratic productivity and match-quality among employed workers. A downturn induces separations for workers who are low on the job ladder. As workers climb back up the job ladder after unemployment, this induces changes in the vacancy-posting incentives of firms.

The individual experience of workers has important roles for aggregate dynamics. First, the model with match-quality delivers the large and persistent earnings losses of displaced workers. This results from a slow climb up the job ladder after separation. These protracted earnings dynamics imply a slow evolution of the match-quality distribution resulting in significant propagation of aggregate productivity shocks. Second, serial correlation in displacements at the individual level contributes to persistently high aggregate unemployment following a recession.

As a whole, the model with match-quality performs remarkably well. In the wake of recession there is a spike in the E-U rate, and the job-finding rate for the employed and the unemployed falls. The model delivers the Beveridge curve; the observed negative relationship between vacancies and unemployment, as well observed wage dispersion and the earnings time-path of displaced workers. The model features significant amplification and propagation of aggregate productivity shocks.

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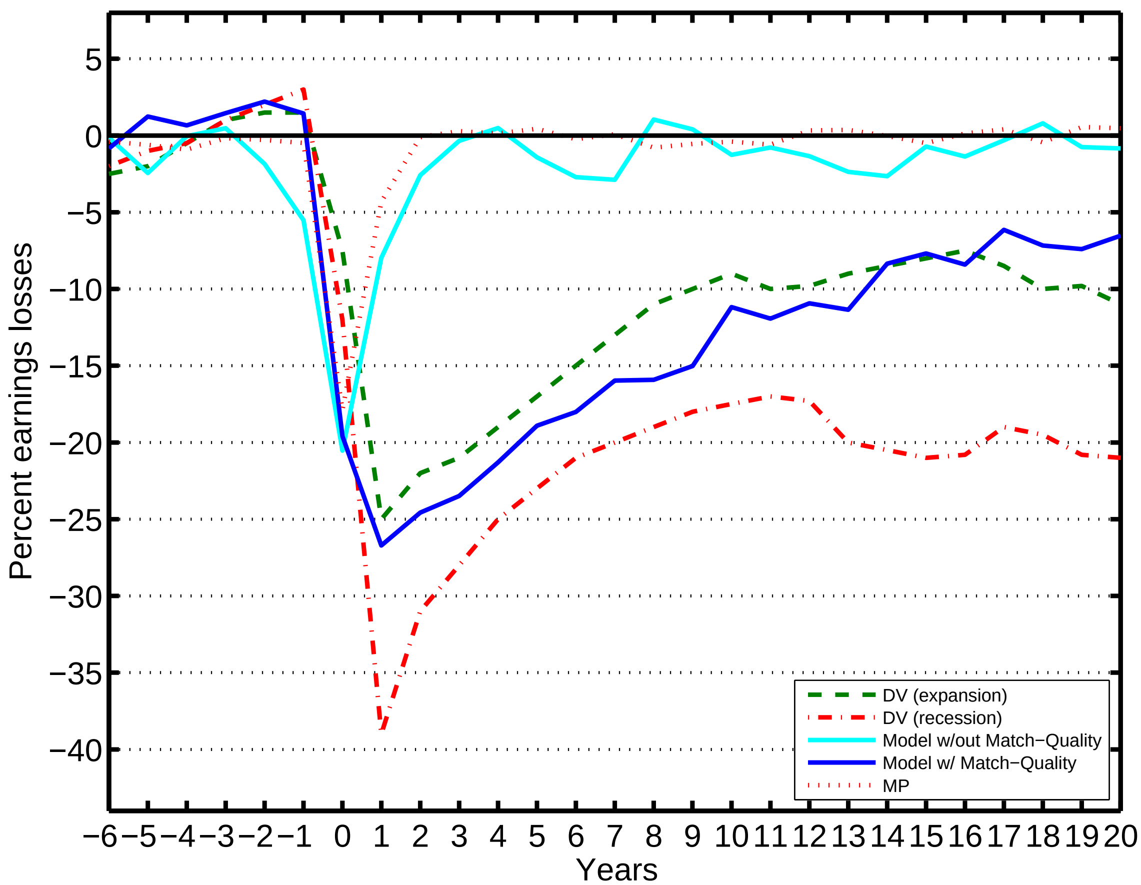

Figure 1: Annual Earnings Losses: Models vs. Data

Note: On impact and for the first 10-15 years of the recovery the model provides a remarkable fit. These are the estimated coefficients δ_k from equation (14), as a fraction of average pre-displacement earnings of the treatment group in the four years prior to displacement. Includes the results from DV and the results from the model. The earnings losses are relative to a non-displaced control group with the same three year tenure requirement as the displaced treatment group. For a definition of displacement and the tenure requirement see the text.

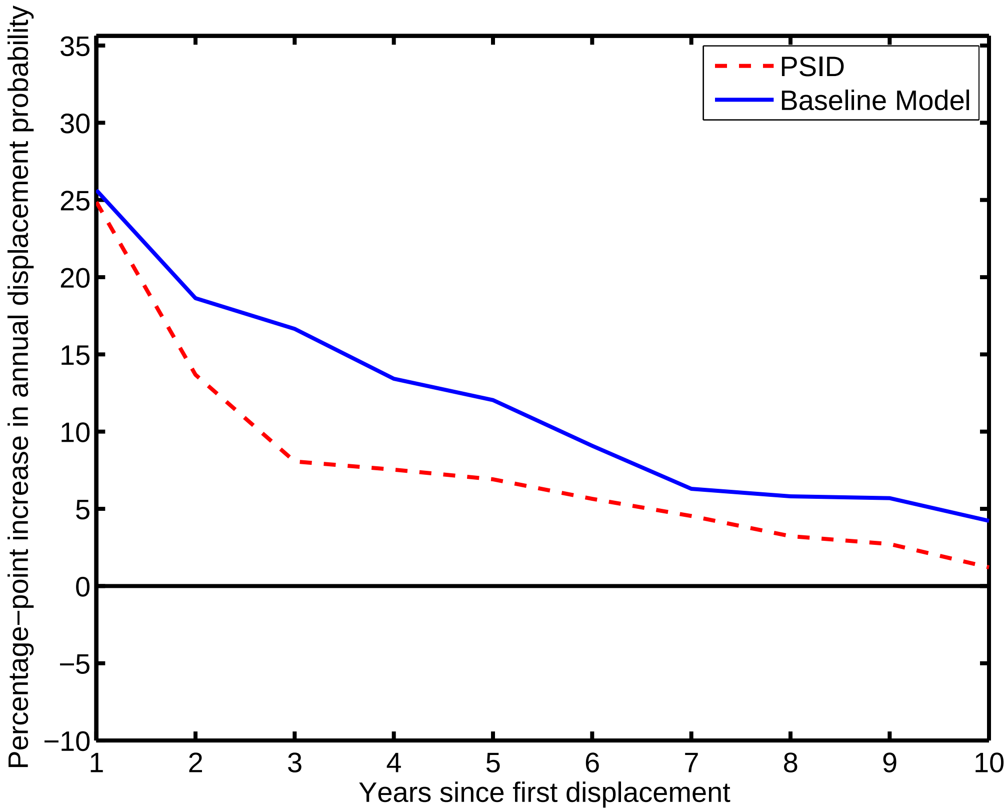

Figure 2: Level Increase in Annual Displacement Probability over Average Displacement Probability: Model vs. PSID

Note: The model endogenously generates the observed serial correlation in displacements. To obtain the line for the PSID, by year since first displacement, take the number of individuals reporting a displacement and divide by the number of employed individuals in the previous year. Perform precisely the same calculation with the simulated data. This includes replicating the PSID survey and classifying someone as displaced if they have less than one year of tenure at the time of the interview, and their *most recent* job ended in a displacement. In the first year after displacement there are around 850 displacements in the PSID. This number falls to around 100 after 10 years. The average displacement probability during this period in the PSID is around nine percent, which is significantly higher than results from Davis and von Wachter (2011) (around 3.5 percent annual displacement probability). This is not surprising. Davis and von Wachter (2011) focus on male employees 50 years or younger with at least three years of prior job tenure. My analysis makes no such restrictions. The implied annual layoff probability using the monthly probability of 1.5 percent is around 16 percent. This is more in line with the number from the PSID, but the annual PSID survey misses short spells of employment between surveys and makes recall bias more pronounced, which are likely to bias the displacement probability downwards. Imposing the PSID survey algorithm makes a significant difference to the results. An alternative would involve taking the monthly E-U probabilities post-displacement in the simulated data, enlarging them to annual rates, and calculating the 12-month average. Taking the difference between this quantity and the annualized monthly E-U probability, with the baseline calibration, results in a 70 percentage-point increase in the E-U rate in the first year after displacement. This displays an annual persistence of 0.9. Hence, the simulated data imply that following the PSID algorithm and analyzing only the most recent job misses many E-U transitions in the period right after a displacement.

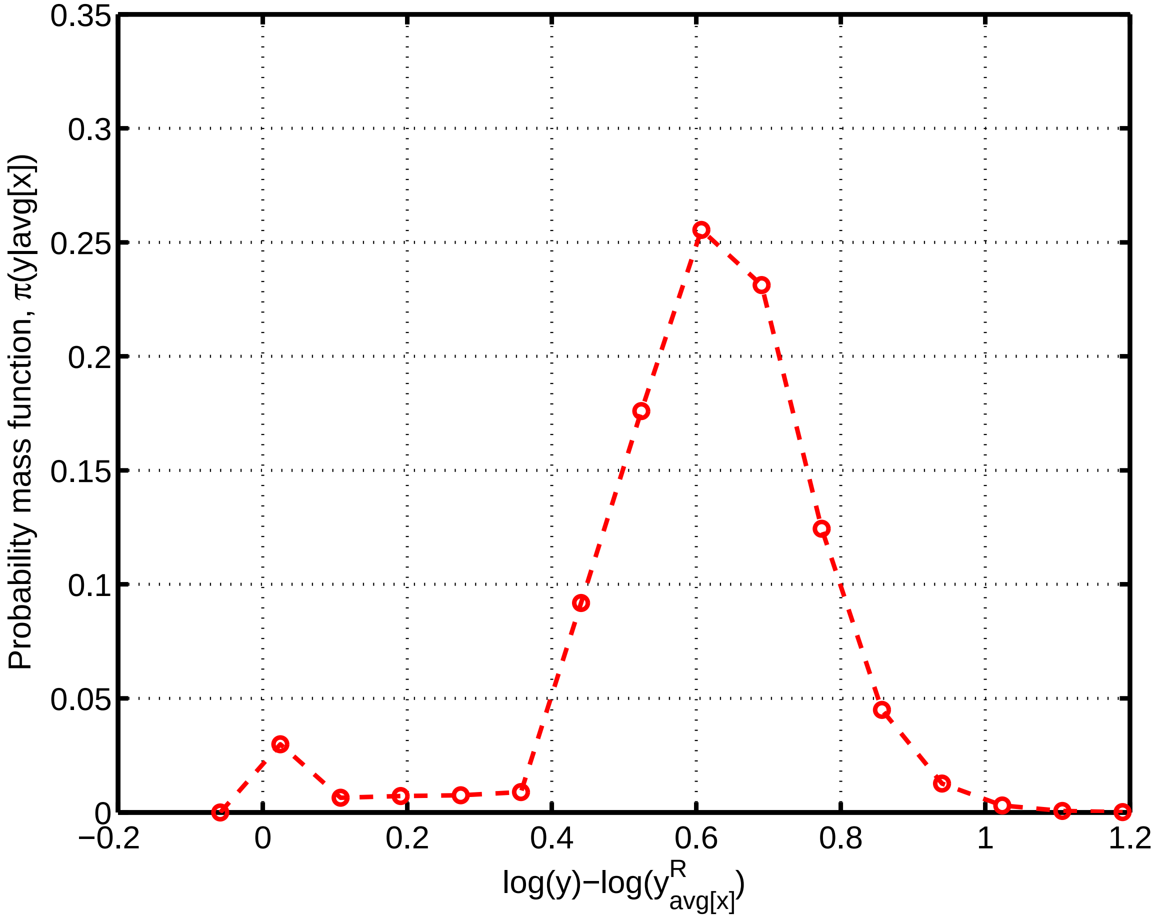

Figure 3: Distribution of Match-Quality (y)

Note: Distribution of y above $y^R_{avg[x]}$, holding x at $avg[x]$ in the full model with baseline calibration. For a definition of $y^R_{avg[x]}$ see equation (15) in the text. Since this is a conditional probabilities, it sums to one.

MODEL WITH MATCH-QUALITY

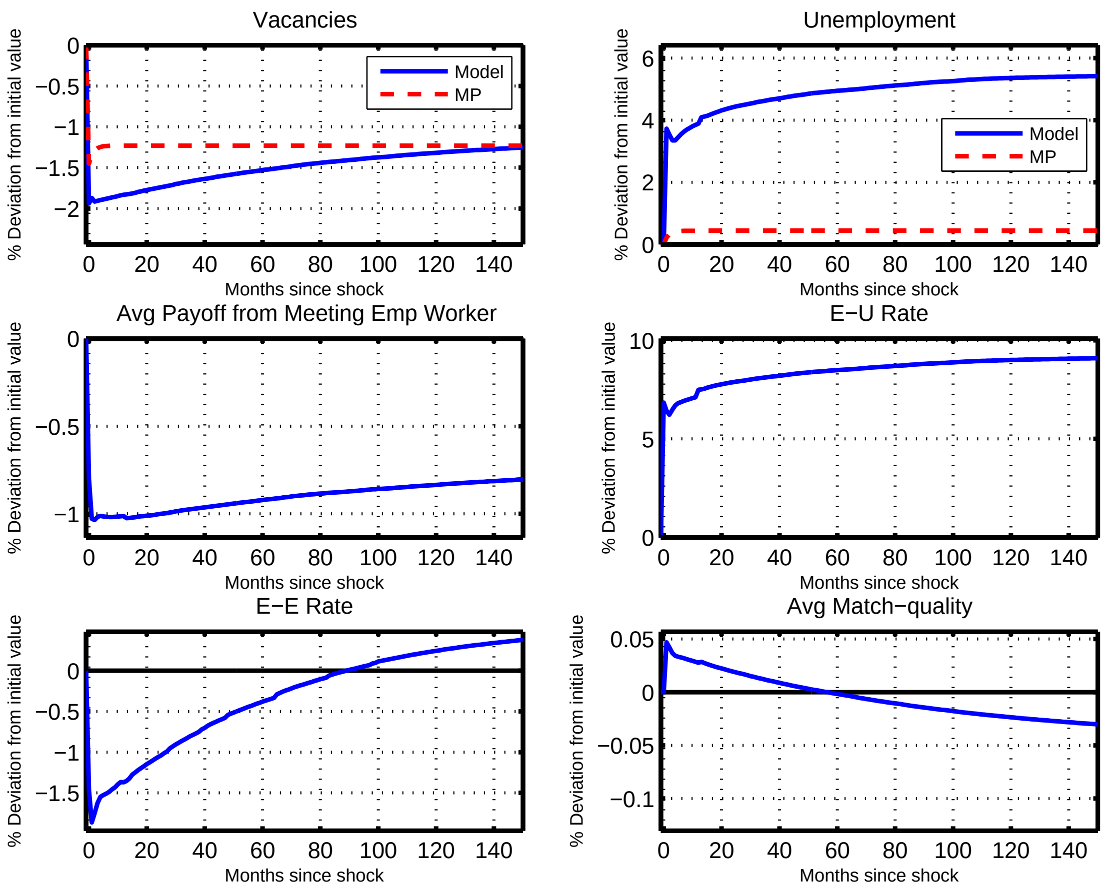

Figure 4: Impulse Response to a 1% Permanent, Unexpected Decrease in Aggregate Productivity: Model with Match-Quality

Note: The model with match-quality delivers significant propagation of aggregate productivity shocks. The permanent reduction in aggregate productivity occurs at time period 0. Where applicable I have included the impulse response functions from the basic MP model.

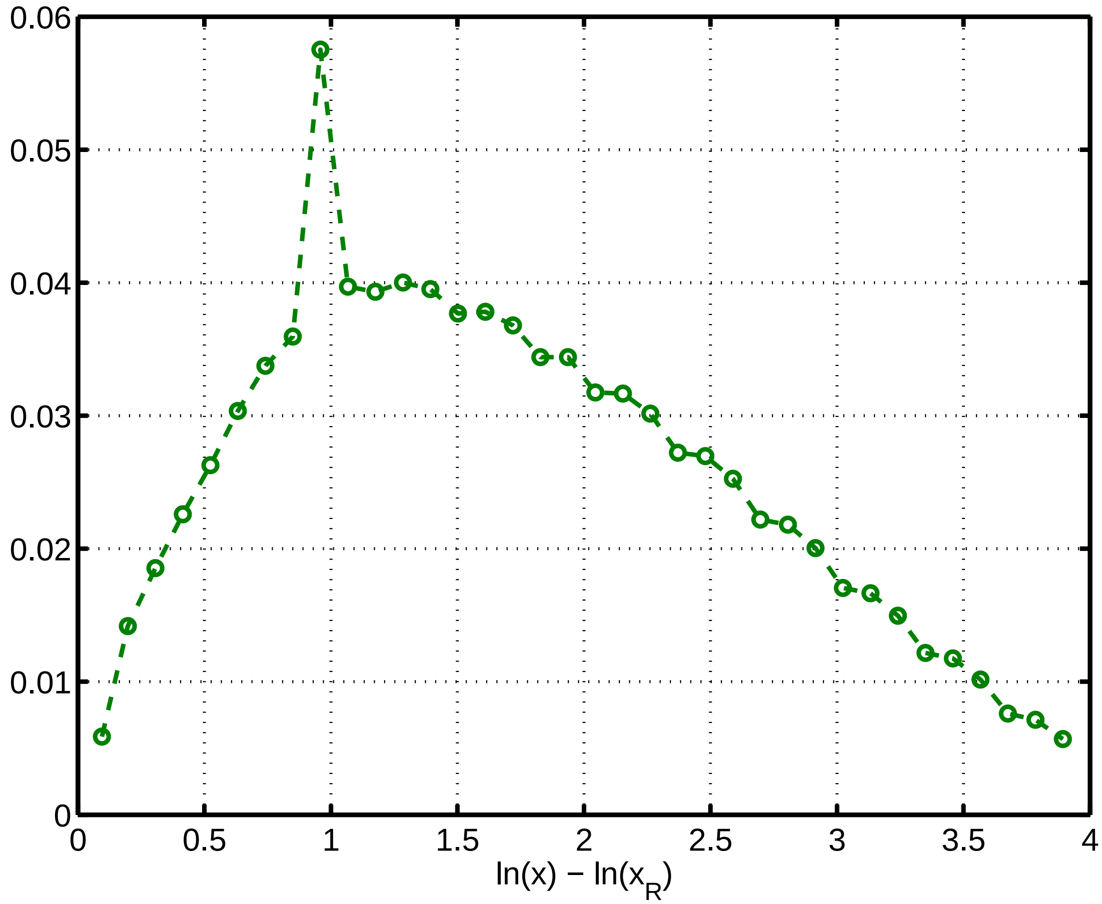

Figure 5: Distribution of Idiosyncratic Productivity (x): Model without Match-Quality

Note: The distribution is very spread out with few matches near the destruction threshold. The figure is not perfectly smooth because the x grid for outside offers is coarser than the overall x grid, thereby making some nodes have artificially more weight. This is inconsequential for the main results.

MODEL WITHOUT MATCH-QUALITY

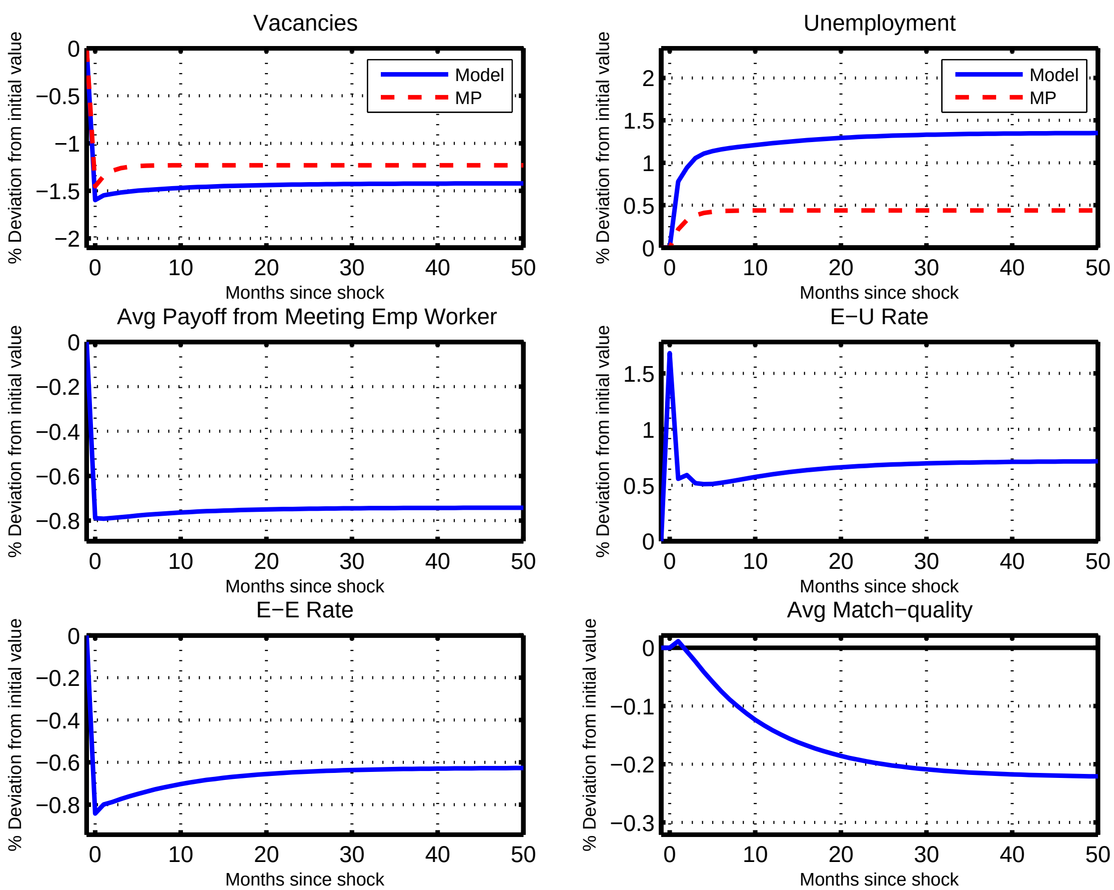

Figure 6: Impulse Response to a 1% Permanent, Unexpected Decrease in Aggregate Productivity: Model without Match-Quality

Note: The model without match-quality features some propagation, although most variables jump almost immediately to their new values. The permanent reduction in aggregate productivity occurs at time period 0. Where applicable I have included the impulse response functions from the basic MP model.

Table 1: Calibrated Parameters for Model with Match-Quality

Parameter (θ)	**Meaning**	**Calibrated Value** ($\hat{\theta}$)	**Main Source of Identification**
ρ_z	Agg prod persistence	0.983	Persistence of Y/L
σ_{ϵ_z}	Std. dev. of agg productivity	0.005	Std. dev of Y/L
α	Matching elasticity	0.524	Mortensen and Nagypal (2005)
c	Flow vacancy cost	0.1184	$v = 1$ in steady state
r	Real interest rate	0.0041	Annual interest rate = 0.05
y_0	Match-quality in first jobs	$\mathbb{E}[y] \approx 1$	Normalization
ρ_x	Productivity persistence	0.43	Persistence of displacements
σ_{ϵ_x}	Std. dev. of productivity	0.24	Post-disp. increase in disp. prob.
σ_{ϵ_y}	Std. dev. of match-quality	0.23	On-impact dip of annual earnings
f_E	Contact probability (E)	0.26	E-E flow probability
f_U	Contact probability (U)	0.45	U-E flow probability
b	Value of leisure	1.19	Hall and Milgrom (2008)
x_0	Starting productivity	$0.58\times \max[x]$	E-U flow probability
p_s	Exo separation probability	0.0014	Pre-displacement earnings

Note: Calibrated parameters of the model at monthly frequency. 'Reason' refers to empirical estimates found in the literature. The citations and values of these empirical moments appear chiefly in Table 2. 'APL' stands for Average Productivity of Labor.

Table 2: Calibration Targets

Moments in the data	**Data** ($\hat{g}$)	**Model** ($g(\hat{\theta})$)
Persistence of agg labor prod	0.02 (M)	0.017 (M)
Std. dev. of agg labor prod	0.88 (M)	0.92 (M)
Persistence of displacement probability	Author: 0.63 (A)	0.80 (A)
Initial spike in displacement probability	Author: 25pp	26pp
Recovery of displacement earnings	Davis and von Wachter (2011): ~20%	21%
On-impact dip of annual earnings	Davis and von Wachter (2011): ~30%	27%
Employer-to-employer flows	Fallick and Fleischman (2004): 0.026	0.023
Job-finding rate	Shimer (2005): 0.45	0.45
Value of leisure	Hall and Milgrom (2008): $0.71\times APL$	$0.62\times APL$
Employment-to-unemployment flows	Elsby et al. (2010): 0.015	0.015
Pre-displacement rise in earnings	Davis and von Wachter (2011): ~3%	2%

Note: The model matches the empirical targets very well. The middle column presents the value of the moment in the data and the citation. The column on the right presents the value of the equivalent moment in the model at the calibrated parameter values. The parenthetical (A) denotes annual frequency moments, and the paranthetical (M) denotes monthly frequency. 'APL' stands for Average Productivity of Labor. 'pp' stands for percentage points.

Table 3: Elasticity With Respect To Output Per Worker

Outcome	**MP (Shimer)**	**Model w/out Match-Quality**	**Model w/ Match-Quality**	**Data**
Job-finding prob, f_U	0.47	0.66	0.91	2.65
E-U rate	0	-1.28	-2.94	-1.89
Vacancies, v	1.24	1.32	1.73	2.91
Unemployment, u	-0.41	-1.75	-3.56	-3.53

Note: Empirical counterparts are taken from Elsby and Michaels (2013). These represent the elasticities with respect to output per worker. In particular, following Mortensen and Nagypal (2007), these elasticities are computed by regressing the log deviations from trend of the respective series on the log deviation from trend of output per worker. To calculate the trend, I follow Shimer (2005) and use a Hodrick-Prescott filter with smoothing parameter 10^5. After HP detrending, the productivity series used in the simulation exhibits an autocorrelation of 0.92 and a standard deviation of 0.017. These are very close to US quarterly data: 0.88 and 0.02 respectively. The table also includes elasticities from a replication of Shimer (2005).

Table 4: Parameter Values for Model without Match-Quality

Parameter	**Meaning**	**Value**
ρ_z	Agg prod persistence	0.983
σ_{ϵ_z}	Std. dev. of agg productivity	0.005
α	Matching technology $m_i(1, v) = m_0^i v^\alpha$	0.5
c	Vacancy posting cost	2.32
r	Real interest rate	0.0041
ρ_x	Productivity persistence	0.97
σ_{ϵ_x}	Std. dev. of innovation to $\ln x$	0.325
f_U	Job-finding prob for ue	0.31
f_E	Job-finding prob for emp	0.096
b	Value of leisure	2.15 (0.21 $\times APL$)

Note: Calibrated parameters of the model without match-quality at monthly frequency.

Table 5: Steady-State Features of Model without Match-Quality

Statistic	**Value**
Unemployment prob	0.06
Separation prob	0.02
Job-finding prob for ue	0.31
E-E flows	0.026
Standard deviation of $\ln w$	0.37
Mean-min wage ratio	1.75

Note: See Table 4 for parameter values for the calibration.