Marriage Market and Intra-Household Allocation : Evolution of preferences and transfers in the UK from 1991 to 2008*

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Abstract

Whereas the within household wage gap has fallen during the last thirty years, labor supply of married women has not increased as much as traditional microeconomic theory would have predicted. To model intra-household allocation of time and consumption, we need to add marital sorting and bargaining effects to the standard income and substitution effects. The household is indeed a several-member collectivity where individuals do not pool their resources but bargain over them. Collective models have proved that opportunities of spouses outside marriage can influence the intra-household balance of power and ultimately the final allocation of resources. The knowledge of who marries whom allows then a better understanding of the distribution of revenues and labor supplies. In this paper, I model the intra-household allocation of resources jointly with the formation and separation of couples in a dynamic search and matching framework. Using the British Household Panel Survey (BHPS), I estimate the matching preferences of individuals over different characteristics such as wages and family values. Taking domestic production into account, I identify the within household transfers and show that bargaining effects reduce labor supply of married women by 2 hours a week and increase married men labor supply by 1 hour a week.

KEYWORDS : Search and Matching models, Multidimensional Matching, Collective models, Marriage market, Household Labor Supplies, Structural Estimation

JEL classification : C78, D13, J12, J22

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1 Introduction

Whereas the within household wage gap has fallen during the last thirty years, labor supply of women has not increased as much as traditional macroeconomic theory would have predicted (Knowles,[22], 2013). Classic macroeconomic theory considers the household as a unit and neglects bargaining effects within the household. The two individuals of the household are supposed to pool their income and maximize a neoclassical household utility function subject to the household's budget constraint. However, many empirical studies show that the income pooling hypothesis is rejected by the data[1]: couples do not pool their resources. The pooling assumption at the aggregate level leads to the underestimation of income inequalities among individuals (Lise and Seitz ([23], 2011) and to a bias in the estimation of labor supply trends (Knowles,[22], 2013). Collective models[2] assume that the household members bargain over their resources and identify the sharing rule from observed labor supplies of couples. Although these models repeatedly show evidence that the within household sharing rule varies with the outside options of individuals (Chiappori, Fortin and Lacroix, [9], 2002), they consider couples as given and can't make predictions on the impact of a taxation reform on the sharing rule. Such a reform may influence marital sorting through divorce and couples' formation (Francesconi et al.,[18], 2009, Bitler et al. [2], 2004). This calls for a model which could explain both the formation and separation of couples and the intra-household allocation of couples which is what I do in this paper. I jointly model the marriage market and resource sharing within the household. Using the British Household Panel Survey (BHPS), I observe wages, working hours, domestic work an marital history of each household member from 1991 to 2008. I recover the matching patterns and the preferences of men and women for leisure, consumption and domestic production. Then I identify within-household transfers and their impact on aggregate labor supply of men and women.

Modeling the marriage market requires the identification of mating preferences over different characteristics. In this paper, matching patterns are recovered from observed joint distributions of characteristics among couples and among singles. Similar strategies are used in models with perfect information as in Choo and Siow ([14], 2006), Chiappori, Salanié and Weiss ([13], 2013) and Galichon and Salanié ([19], 2013). These models are static and consider a competitive stable equilibrium, which is not realistic on the marriage market. Searching for a partner takes time. I use a search framework to model frictions. Identification of matching patterns is obtained with the steady-state assumption of search models as in Shimer and Smith ([27], 2000) and Wong ([29],2003). Very few matching models aim at modeling preferences for consumption and leisure to find the impact of transfers on economic outcomes. The work of Jacquemet and Robin ([20], 2013) is the first attempt to link heterogeneity in marriage formation and intra-household allocation. The present paper builds on their framework and includes a collective structure of labor

[1] Bourguignon et al. ([3],1994)

[2] These models developed by Chiappori ([7],1988) assume that the household members bargain over their resources and make Pareto-optimal agreement.

supply in a search and matching model of marriage. In the collective structure that is used, individuals are egoistic and enjoy their own leisure and consumption. Single individuals earn labour income whereas married individuals earn labour income and may also benefit from a transfer from their spouse. A search model is used to model the matching process: single individuals meet randomly and decide whether they marry. They evaluate the match anticipating what will be the surplus generated by the match and how it will be split. I assume that individuals bargain à la Nash to choose an optimal sharing rule. Either the surplus is high enough and both want to match, or it is not and both prefer to stay single. If they match, they first split the surplus and then choose separately their consumption and leisure according to their new budget constraint. My paper extends the paper of Jacquemet and Robin (2013) in three ways. First, individuals directly enjoy the consumption of a domestically produced public good in addition to leisure and consumption. Domestic production is crucial in analyzing household behavior. Omitting household production leads to a significant bias in the estimation of the sharing rule (Couprie, [15],2007). When two people decide to live together, their purchasing power increases due to economies of scale (sharing the rent, the electricity) and due to increasing returns to scale in domestic production (cleaning, meal preparation, or caring for children)[3]. Individuals may also enjoy the produced public good which can be raising children or eating a home-made meal. Public good production depends on three different inputs: the time spent in housework by each partner, the characteristics of each partner and some time-varying unobserved characteristics of the match. This unobserved heterogeneity leads to my second contribution, the separation of couples is endogenised. Some shocks can hit the unobserved characteristics and lower the value of the match. In that case, the match breaks up. Only couples with high enough complementarity in observed characteristics will last. Finally, as one household member's value on the marriage market has an impact on the sharing rule, all characteristics which are important in couple formation must have an impact on the sharing rule and then on labor supply. I extend the setting to multidimensional matching and I allow people to choose their partners for different characteristics such as wage and family values[4]. Only a few papers consider multidimensional matching. Some build a marriageability index (Wong, [29], 2003, Chiappori, Oreffice, Quintana-Domeque [11], 2012), others match on two characteristics, one of which is a binary characteristic (Chiappori et al. [10], 2013). Recent work of Galichon and Salanié 2013, [19] and Galichon and Dupuy, 2013 [17] uncover matching preferences over many different characteristics. In this paper, people match on two continuous characteristics. I find a positive assortative matching in wages and in family values. I show that if total surplus increases in wages of both members, complementari-

[3] Many studies have attempted to estimate the additional revenue generated by living in couples (Browning et al., 2006, [5], Couprie, [15], 2007). In collective models with domestic production, you can still decentralize the intra-household allocation process (Chiappori, 1997, [8]). First, individuals decide on the level of domestic production they want. Then they define a conditional sharing rule that is how they will share the rest of the total income conditional on the chosen level of domestic production.

[4] I use an index representing family values. The higher this index the more conservative the individual is about family and gender roles. This index expresses opinion about divorce, the importance of marriage in raising children, the fact that the man should be the head of the household etc.

ties in characteristics can be higher for same wage couples. Similarly, people with conservative family values are more attractive on the marriage market, particularly women. Women with conservative family values get higher shares of the surplus.

This model fits well the observed working hours of men and women. I identify the impact of transfers on hours worked and show that it is significant. When women get transfers from their male partner, they work less than if they didn't get any transfer through standard income effects. On the opposite, their male partner works more to compensate their loss of revenue. Taking domestic production into account, I identify the within household transfers and show that they reduce labor supply of married women by 2 hours a week and increase married men labor supply by 1 hour a week. I compute the evolution of sharing rules and matching preferences for each year from 1991 to 2008[5]. I show that welfare of women has increased and within household inequalities have decreased. Finally, this model allows me to simulate the equilibrium which would result from any initial distribution of characteristics among men and women (the number of couples, the number of single men and women and their characteristics, the resulting transfers and the resulting labor supplies). I present some simulation exercises where I simulate the equilibrium obtained with a change in wage distribution of men or women or the impact of a subsidy given to all low-wage single women. The ultimate goal of this model is to simulate the impact of a family taxation reform on within household allocations and labor supplies. I explain how the model could be extended to include taxation on the one hand and children on the other hand, or both.

The model is described in section 2 and the data in section 3. Section 4 presents the estimation strategy and section 5 the results. Simulation and equilibrium conditions are computed in section 6 and direct extensions of the model are proposed in section 7. Section 8 concludes.

2 Model

I consider two different populations of agents that are likely to match : a population of males and a population of females (labeled m and f). In this paper, a match is a two-people household[6]. Each population is composed of agents heterogenous with respect to several characteristics. These characteristics define the type of the agent. They are exogenous and do not vary overtime. Let's assume that a type is composed of K continuous characteristics. Then each agent is defined by the population he belongs to (m or f) but also by his individual type noted $i \in \mathbb{R}^K$ for men and $j \in \mathbb{R}^K$ for women[7]. I will first define how individuals value consumption, leisure and domestic production through their utility function and their budget constraint. Then, I will describe how the marriage market works, how do people meet and decide whether they match.

[5]These years are interesting in Great-Britain as many family taxation reforms took place with the introduction of the WFTC in 1999 and the introduction of the Working Tax Credit and the Children Tax Credit in 2003.

[6]For convenience, I use indifferently the terms match, marriage or couple

[7]The agent type will at least contain his wage. Then w_i will denote the wage of an agent of type i

2.1 Utilities and household program

2.1.1 Utilities for singles

Individuals' utilities depend on their consumption, leisure and also on a domestic good produced with domestic work. I specify the following Stone-Geary utility function[8] rescaled and translated for a single man of type i

$$U_{mi}(d, C, L) = (d - D_{0m})^{\kappa_m}(C - \mathfrak{C}_{0mi})^{\alpha_{mi}}(L - \mathfrak{L}_{0mi})^{1-\alpha_{mi}}.$$

C denotes the consumption expenditure of the individual, L his leisure time and d his time spent doing housework. $\mathfrak{C}_{0mi}$ represents the minimum level of consumption required to live and $\mathfrak{L}_{0mi}$ represents a minimum amount of time to spend in leisure. α_{mi} represents the individual preference for consumption with respect to leisure. The rescaling function α_{mi} and the translating functions $\mathfrak{C}_{0mi}$, $\mathfrak{L}_{0mi}$ reflect heterogeneity of preferences among agents and depend on their type and their population. D_{0m} represents a minimum of domestic work required for a single man (minimum of time to spend in meal preparation, cleaning etc.) and κ_m is a preference parameter for domestic production. Preference parameters for domestic production are also gender specific but contrarily to consumption and leisure parameters, they are not varying with the type of the agent. The individual utility is defined similarly for women. The utility of a single a woman of type j is

$$U_{fj}(d, C, L) = (d - D_{0f})^{\kappa_f}(C - \mathfrak{C}_{0fj})^{\alpha_{fj}}(L - \mathfrak{L}_{0fj})^{1-\alpha_{fj}}.$$

The singles maximize their utility under their budget constraint which is for a single man of type i

$$w_i L + C + w_i d \leq w_i T,$$

with w_i the hourly wage of the single man of type i and T the total time endowment of individuals. I assume there is no non-labor income. The resulting indirect utility for a single man of type i and a single woman of type j are[9] :

[8] This specification allows to obtain a Linear Expenditure System

[9] The expressions for domestic work, leisure and consumption are

$$d = D_{0m} + \frac{\kappa_m}{(1+\kappa_m)w_i}(w_i T - \mathfrak{C}_{0mi} - w_i \mathfrak{L}_{0mi} - w_i D_{0m})$$
$$L = \mathfrak{L}_{0mi} + \frac{1-\alpha_{mi}}{(1+\kappa_m)w_i}(w_m T - \mathfrak{C}_{0mi} - w_i \mathfrak{L}_{0mi} - w_i D_{0m})$$
$$C = \mathfrak{C}_{0mi} + \frac{\alpha_{mi}}{(1+\kappa_m)}(w_i T - \mathfrak{C}_{0mi} - w_i \mathfrak{L}_{0mi} - w_i D_{0m})$$

The complete indirect utility function when we also replace with expression for d_m is

$$v_{mi} = \frac{\kappa_m^{\kappa_m}\alpha_{mi}^{\alpha_{mi}}(1-\alpha_{mi})^{1-\alpha_{mi}}}{(1+\kappa_m)^{(1+\kappa_m)}w_i^{1-\alpha_{mi}+\kappa_m}}(w_i(T - d_{m0}) - \mathfrak{C}_{0mi} - w_i \mathfrak{L}_{0mi})^{\kappa_m+1},$$

The standard Cobb-Douglas function is a particular case of the Stone-Geary utility function with $\alpha_{mi} = \alpha_m$, $C_{0mi} = \mathfrak{C}_{0m}$ and $L_{0mi} = \mathfrak{L}_{0m}$

$$v_{mi} = \frac{\alpha_{mi}^{\alpha_{mi}}(1-\alpha_{mi})^{1-\alpha_{mi}}}{w_i^{1-\alpha_{mi}}}(d_m - D_{0m})^{\kappa_m}(w_i(T-d_m) - \mathfrak{C}_{0mi} - w_i\mathfrak{L}_{0mi}) \tag{1}$$

$$v_{fj} = \frac{\alpha_{fj}^{\alpha_{fj}}(1-\alpha_{fj})^{1-\alpha_{fj}}}{w_j^{1-\alpha_{fj}}}(d_f - D_{0f})^{\kappa_f}(w_j(T-d_f) - \mathfrak{C}_{0fj} - w_j\mathfrak{L}_{0fj})$$

2.1.2 Utilities for married individuals

Now, let's consider the utilities of matched individuals. Agents are supposed to be egoistic : their utility only depends on their own private consumption and leisure quantity and not on the consumption and leisure quantity of their partner. However, when married, individuals benefit from complementarities in the joint production of a domestic public good. The production of this public good depends on three different inputs : time spent on domestic production, complementarities of characteristics of the partners and some time-varying unobservable heterogeneity z. Let Q_{ijz} denote the produced good made by a man of type i matched with a woman of type j. This household public good has the following form.

$$Q_{ijz}(d_m, d_f) = (\Phi(i,j) + z)F(d_m, d_f)$$

$F(d_m, d_f)$ is a function of domestic work spent by both members. $\Phi(i,j)$ is a function of the partners'type. Higher the complementarities between two types, higher the production. z represents unobservable characteristics of the match at a certain period. z is drawn when the partners meet. When we will consider the dynamic framework, z will be allowed to take different values at different periods. Each couple is then characterized at each period by three indexes : i, the type of the man, j, the type of the woman and z. The utility of a man of type i married with a woman of type j under the circumstance z is then defined as:

$$U_{mijz} = (\Phi(i,j) + z)F(d_m, d_f)(C - \mathfrak{C}_{0mi})^{\alpha_{mi}}(L - \mathfrak{L}_{0mi})^{1-\alpha_{mi}}$$

I assume that matched people incur an additional cost due the match. Let C_c be this cost. It can come from raising children or from higher expectations of the standard of living. The budget constraint of the couple is then

$$(BC)\ C_i + C_j \leq w_i(T - L_i - d_i) + w_j(T - L_j - d_j) - C_c$$

Following the literature on collective models, household are supposed to make Pareto-efficient decisions. This defines a infinity of solutions. I specify which optimum is chosen assuming that the members of the household use a Nash bargaining to share their resources. Then the program of the household can be decentralized as a two-step processus. In the first step, the two members of the household decide which level of public good they want to produce or equivalently which quantity of time they want to invest in public good production. In this same first step, the two members also bargain on the sharing rule that is how they will share total income. Each of them will get a transfer t of money resulting for this sharing. Let t_m^* and t_f^* be the solution of this bargaining. I assume there is no non-labor income, then $t_m^* + t_f^* = -C_c$. In the second step, each individual maximizes his own utility subject to his new budget constraint. As the individual utility is separable between the public good and the private good, we are only interested in maximizing the private sub-utility of the individual to derive his optimal consumption of leisure and private good. Let's consider a couple with a man of type i and a woman of type j under heterogeneity z. They choose the optimal quantity of domestic work spent by the man d_{mijz}, the optimal quantity of domestic work spent by the woman d_{fijz} and the optimal transfer t_{mijz}. Then the program of married man of type i in a couple of type (i, j, z) is :

$$\max_{C,L} u_i(C, L) = (C - \mathfrak{C}_{0mi})^{\alpha_{mi}} (L - \mathfrak{L}_{0mi})^{1-\alpha_{mi}}$$
$$s.c\ C \leq w_i(T - L - d_{mijz}) + t^*_{mijz}$$

To obtain easy reading expressions, I define the following two functions

$$\mathcal{C}_{0mi} = \mathfrak{C}_{0mi} + w_i \mathfrak{L}_{0mi}$$
$$P_{mi} = \frac{w_i^{1-\alpha_{mi}}}{\alpha_{mi}^{\alpha_{mi}} (1 - \alpha_{mi})^{1-\alpha_{mi}}}$$

then $\mathcal{C}_{0mi}(w_i)$ represents a minimal expenditure to spend on leisure and consumption and $P_{mi}(w_i)$ represents an aggregate price index. These functions are gender specific and depend on the type and on the wage of the individual. Then the indirect utility for a man in a couple of type (i, j, z) is

$$v_{mijz} = (\Phi(i, j) + z) F(d_m, d_f) \frac{w_i(T - d_{mijz}) + t^*_{mijz} - \mathcal{C}_{0m}(w_i)}{P_m(w_i)}.$$

Similarly we can simplify the indirect utility expression for single (1) which can be rewritten

$$v_{mi} = \frac{(d_m - D_{0m})^{\kappa_m} (w_i(T - d_m - \mathcal{C}_{0m}(w_i))}{P_m(w_i)}.$$

I just defined utilities for singles and married people, I will now define how these people meet, form couples and break up.

2.2 The marriage market

This subsection presents the meeting process between two singles. I assume that only singles search for a partner. There is no "on the marriage search". Let N be the number of matches. U_f and U_m are the respective number of single women and single men. Let $u_m(i)$ and $u_f(j)$ be the respective density of single men of type i and single women of type j. Let λ denote the instantaneous probability of a meeting between a random single woman and a random single man. Then $\lambda_m = \lambda U_f$ and $\lambda_f = \lambda U_m$ are the respective instantaneous probabilities of an agent among the population m and f of meeting a new person of the populations f and m. Let r be the interest rate.

2.2.1 Present value

The Bellman equation of the present value of a single man of type i, noted W_{mi} is then:

$$rW_{mi} = v_{mi} + \lambda_m \left(\int_z \int_j \max(W_{mijz} - W_{mi}, 0) \mathbb{1}(W_{fijz} > W_{fj}) u_f(j) dG(z) \right) \tag{2}$$

with W_{mijz} the present value of a man of type i married with a woman of type j when the value z is drawn during the formation of the couple. The present value is then the sum of the instantaneous utility of being single plus his expected surplus from a match averaged over all possible values of z. Symmetrically, W_{fj} denotes the present value of a single woman of type j and W_{fijz}, the present value of a woman of type j married with a man of type i with a match quality z. When married, shocks to the couple's match quality z happen regularly. At each shock, the value of being in couple can be modified, and the partners bargain again over the new surplus, the couple can separate if they do not agree on a new sharing rule. This shock is modeled by a new draw of z which is independent from the previous value of z. I suppose this shock happens with probability λ_z. Then the present value of a man of type i married with a woman of type j is

$$rW_{mijz} = v_{mijz} + \lambda_z \int_{z'} \max(W_{mi} - W_{mijz}, W_{mijz'} - W_{mijz}) dG(z'),$$

which gives

$$W_{mijz} = \frac{v_{mijz} + \lambda_z W_{mi} + \lambda_z \int_{z'} \max(W_{mijz'} - W_{mi}, 0) dG(z')}{r + \lambda_z} \tag{3}$$

This present value is the sum of the instantaneous utility v_{mijz} and the expected surplus due to a possible change of z which will lead either to a different surplus if z is high enough, either to a separation otherwise.

2.2.2 Surplus, Nash bargaining and transfers

When a match is formed, the two members start to bargain on the level of production of the public good and on the repartition of the household resources. I model the decision process with a Nash bargaining where the threat point is to stay single. The respective threat points are then the respective outside options of the man and the woman and so their present value as singles. I denote β and $(1 - \beta)$ the respective bargaining power of the man and the woman[10]. Then the Nash bargaining β of the household is the maximization of the following program :

$$\max_{d_m, d_f, t_m, t_f} \quad (W_{mijz} - W_{mi})^\beta (W_{fijz} - W_{fj})^{1-\beta}$$
$$s.c \; t_m + t_f = -C_c$$
$$Q_{ijz} = F(d_m, d_f)(\Phi_{ij} + z)$$

$W_{mijz} - W_{mi}$ is then the surplus of the man and $W_{fijz} - W_{fj}$ is the surplus of the woman. This model has the important property of transferable utility models : both surplus are simultaneously

[10] β is not necessarily constant. I will actually use $\beta(i, j)$ which will depend on the wage ratio w_i/w_j

positive or negative. Either the match generates enough surplus to make both people want to marry or it doesn't and they both refuse to marry. The maximization of this program with respect to t_m leads to the following equality :

$$\frac{P_{mi}(W_{mijz} - W_{mi})}{\beta} = \frac{P_{fj}(W_{fijz} - W_{fj})}{1-\beta}$$

Denoting $S(i,j,z) = \frac{P_{mi}(W_{mijz} - W_{mi})}{\beta}$, we have :

$$\begin{aligned} P_{mi}S_{mijz} &= P_{mi}(W_{mijz} - W_{mi}) = \beta S(i,j,z) \\ P_{fj}S_{fijz} &= P_{fj}(W_{fijz} - W_{fj}) = (1-\beta)S(i,j,z) \end{aligned} \tag{4}$$

and the two individuals i and j decide to marry if and only if they both obtain a positive surplus from the match, that is if $S(i,j,z) > 0$[11]

The maximization of the program with respect to inputs gives the following conditions. The optimal quantities of inputs d_m, d_f solve :

$$\begin{aligned} \frac{\partial F(d_m,d_f)/\partial d_m}{F} &= \frac{w_i}{w_i(T-d_m) + w_j(T-d_f) - \mathcal{C}_{0mi} - \mathcal{C}_{0fj} - C_c} \\ \frac{\partial F(d_m,d_f)/\partial d_m}{\partial F(d_m,d_f)/\partial d_f} &= \frac{w_i}{w_j} \end{aligned} \tag{5}$$

The optimal quantities d_m^*, d_f^* do not depend on z and will be functions of T, w_i,w_j,$\mathcal{C}_{0mi}$,$\mathcal{C}_{0fj}$ and C_c. The production of the domestic good is efficient. The optimal quantities of domestic work are only function of the wages and depend on the specification of the production function. Let R_{ij} be the total resources for a couple of type (i,j) after spending some time in domestic production and in minimum level of consumption and leisure. That is $R_{ij} = w_i(T-d_{mij}) + w_j(T-d_{fij}) - \mathcal{C}_{0mi} - \mathcal{C}_{0fj} - C_c$. Then I can derive the following proposition (the proof is developed in appendix).

Proposition 2.1. *The total surplus is linear in z and has the following expression*

$$\begin{aligned} S(i,j,z) = &(w_i(T-d_{mij}) + w_j(T-d_{fij}) - \mathcal{C}_{0mi} - \mathcal{C}_{0fj} - C_c)F(d_{mij},d_{fij})(\Phi(i,j)+z) \\ &- P_{mi}rW_{mi} - P_{fj}rW_{fj} \\ &+ \lambda_z \int_{z'} \max(S(i,j,z'),0)dz' \end{aligned}$$

[11] S can also be expressed as a form of total surplus

$$S(i,j,z) = \frac{P_{mi}P_{fj}}{\beta P_{fj} + (1-\beta)P_{mi}} \left(\underbrace{W_{mijz} - W_{mi}}_{Man\ surplus} + \underbrace{W_{fijz} - W_{fj}}_{Woman\ surplus} \right)$$

Then the expressions of transfers are (when $\Phi(i,j)+z\neq 0$)

$$t_{mijz}=\beta R_{ij}-(w_i(T-d_{mij})-\mathcal{C}_{0m}(w_i))+\frac{(1-\beta)rP_mW_{mi}-\beta rP_fW_{fj}}{F(d_{mij},d_{fij})(\Phi(i,j)+z)} \tag{6}$$
$$t_{fijz}=(1-\beta)R_{ij}-(w_j(T-d_{fij})-\mathcal{C}_{0f}(w_j))+\frac{\beta rP_fW_{fj}-(1-\beta)rP_mW_{mi}}{F(d_{mij},d_{fij})(\Phi(i,j)+z)}$$

The total surplus is then decomposed in three terms. The first term is the total resource of the household multiplied by the value of the domestic public good. The second term subtracts the sum of the single present value of each partner. The higher this term, the lower the surplus of the couple. The last term is the continuation value, the expected future surplus when shocks happen to z. The remarkable fact of this expression is the linearity of the surplus in z. We have

$$S(i,j,z)=\frac{F(d_{mij},d_{fij})R_{ij}}{r+\lambda_z}(s(i,j)+z)$$

with

$$s(i,j)=\Phi(i,j)-\frac{P_{mi}rW_{mi}+P_{fj}rW_{fj}}{R_{ij}F(d_{mij},d_{fij})}+\frac{\lambda_z}{r+\lambda_z}\int_{z'}\max(s(i,j)+z',0)dz'$$

This result is crucial for the identification of the whole model and comes from the separability of the term $\Phi_{ij}+z$ in the utility function. Linearity in z allows us to link the total surplus of the couple to the match probability. When two people of each population meet, they decide to match if and only if the surplus is positive. Then the matching probability between a man of type i and a woman of type j when they meet can be computed with

$$\begin{aligned} a(i,j) &= \mathbb{P}\left\{s(i,j)+z>0|i,j\right\} \\ &= 1-\mathbb{P}\left\{z\leq -s(i,j)|i,j\right\} \end{aligned} \tag{7}$$

2.2.3 Equilibrium

The characterization of the equilibrium allows us to close the model. To solve for a market equilibrium, we have to describe how new singles enter the market overtime. Burdett and Coles ([6],1999) review the different cases that have been considered in the literature. Here I suppose there is no entry of new singles, but the partnerships of type (i,j) are destroyed at rate $\lambda_z(1-a(i,j))$ whereupon both return to the single market. At the equilibrium, there is equality between inflows and outflows for each type of marriage. We will note $n(i,j)$ the density of couples of type i for the m member and of type j for the f member. Then we have for all couple of type (i,j), the equality between the number of outflows and the number of inflows:

$$\lambda_z(1-a(i,j))n(i,j)N=\lambda u_m(i)u_f(j)U_mU_fa(i,j), \tag{8}$$

that gives

$$a(i,j)=\frac{n(i,j)N}{\frac{\lambda}{\lambda_z}u_m(i)u_f(j)U_mU_f+n(i,j)N} \tag{9}$$

The match probability $a(i,j)$ can be then computed using data after having estimated the parameters λ and λ_z. Using the equation (7) and making some additional assumptions on the law of z, I am able to derive the marriage surplus. Indeed, assuming that the distribution function of z is $F_z(z)$ then the match probability is linked to the surplus function trough this simple relation:

$$s(i,j) = -F_z^{-1}(1 - a(i,j))$$

3 Data

3.1 The sample and variables

I estimate the model using the British Household Panel Survey (BHPS) where I follow individual's marriage history from 1991 to 2008 using the family and individual samples. I merge the individual file with the marriage history file to obtain marriage history anterior to 1991 for married people. The original BHPS sample was 5,050 households containing 9,092 interviewed adults at wave 1 (1991) with a response rate of 74 % of eligible households[12]. All adults and children in the first wave are designated as original sample members. On-going representativeness of the non-immigrant population has been maintained by using a 'following rule' typical of household panel surveys: at the second and subsequent waves, all original sample members are followed (even if they moved house or if their households split up), and there are interviews, at approximately one-year intervals, with all adult members of all households containing either an original sample member, or an individual born to an original sample member whether or not they were members of the original sample. The sample therefore remains broadly representative of the population of Britain as it changes over time[13]. I keep households composed of heterosexual couples and single member households who are between 22 and 40 year old at the time of interview. I limit my sample to working employees declaring their usual gross pay per month, the number of hours normally work per week (including paid and unpaid overtime hours) and the number of hours they spend a week doing housework. When married, both spouses have to work and declare their wage and hours to be included in the sample. I define the hourly wage as the number of hours normally work per month (without overtime) divided by the usual gross pay per month.

I consider two different variables to define the agent's type. Wages must be part of the type as most of the analysis is made on labor income and resource sharing. However, it is quite restrictive

[12] The sample was a stratified clustered design with 250 Primary Sampling Units in England, Scotland and Wales and was designed to be representative of the GB population (excluding Northern Ireland and North of the Caledonian Canal)

[13] The BHPS data are made up of five samples, the original BHPS from 1991 to present. This is the main BHPS sample (n=5,050 households), the former European Community Household Panel survey low-income sub-sample from 1997 to 2001 (Waves 7 to 11) (n=1,000 households), the Welsh extension from 1999 (Wave 9) (n=1500 households), the Scottish extension from 1999 (Wave 9) (n=1500 households) and the Northern Ireland extension from 2001 (Wave 11) (n=1900 households). I only keep the original sample.

to assume that agents only differ by their productivity on the marriage market. Heterogeneity of individuals varies in an infinite number of dimensions which can be important in couples' formation. One of the most important observable dimension of heterogeneity on the marriage market must be education or one's social group according to sociologists (Mare [24] (2008), Bozon, Héran [4] (1991)). Indeed, the correlation between partners in a couple of education is around 0.6 however, some heterogeneity features of education or social group are already captured by the wage. It would be more interesting to study the impact of other variables on the match. The BHPS provides us with some alternatives. We could think of the Body Mass Index observed in 2004 and 2006 which could be a proxy for physical attractiveness. However, I prefer to use in this paper some information on family values available during the whole period. This information reflects how individuals value the marriage institution. In the survey, individuals have to express their opinion on some statements about marriage, cohabitation and divorce. They can qualify their answer with 5 items : Strongly agree - Agree - Neither agree nor disagree - Disagree - Strongly disagree. The answers are not available each year and questions asked changed in 1998. Table 1 displays which statements are proposed each year.

Table 1: Family Value Questions

Do you agree with the following statements ?

Years 1992, 1994, 1996	Years 1998, 2000, 2002, 2004, 2006, 2008
1. Divorce is better than unhappy marriage (both periods)	
2. Bible Gods word and true	
3. Man should be the head of the household	
4. Cohabiting is always wrong	
	5. Cohabitation is alright
	6. Marital status is irrelevant for children
	7. Homosexual relationships are wrong
	8. Parents ought stay together for children

Using this questionnaire, I build a Family Value Index which is high if the answers are conservative about religion, marriage and family[14]. Agents will be defined by their wage and their Family

[14] I give some points to each answer to question i. Let's $A(i)$ be the number of points given to the answer of question i, then $A(i) = 1$ *if answer is "Strongly disagree"*, $A(i) = 2$ *if answer is "Disagree"*, $A(i) = 3$ *if answer is "Neither agree nor disagree"* , $A(i) = 4$ *if answer is "Agree"*, $A(i) = 5$ *if answer is "Strongly agree"*. Then the index is built the following way

$$I_{fv1991-1996} = \frac{5}{4}[(6 - A(1)) + A(2) + A(3) + (6 - A(4))]$$

$$I_{fv1998-2008} = 6 - A(1) + (6 - A(5)) + (6 - A(6)) + A(7) + A(8)$$

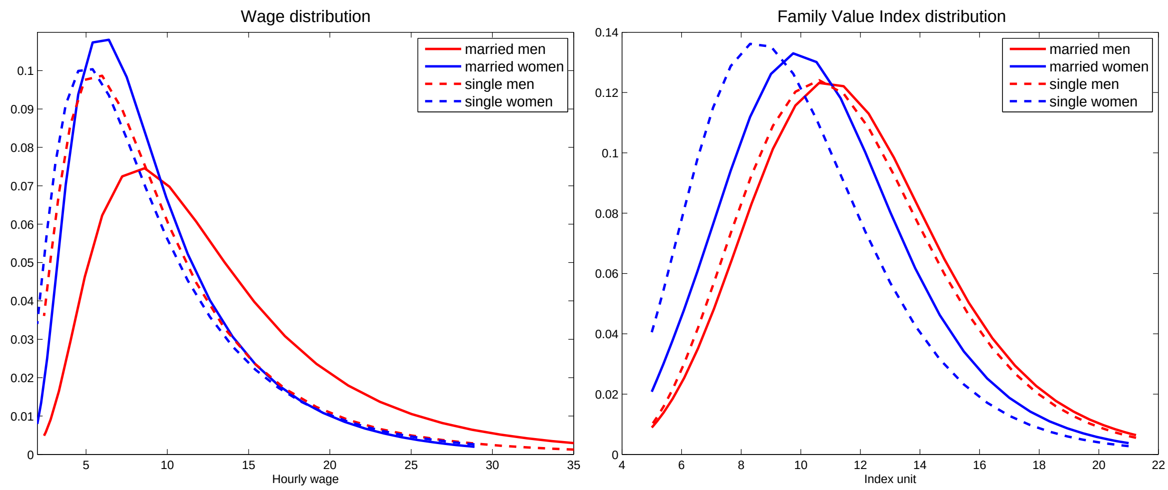
Value Index: $i = (wage_i, f_i)$. I trim the 1 % top and bottom of all these variables. Wages, work hours and domestic work hours are declarative data and must contain important measurement errors. By selecting individuals the same way for each year between 1991 and 2008, I obtain 18 final samples whose sizes vary between 2255 individuals in 1991 and 3456 individuals in 1999.

My analysis has two important limits. First, I need to restrict my sample to working people as I do not model the extensive participation to the labor market jointly with the formation of couples. I am then only considering the marriage market of working people composed of working singles and bi-working couples. However, married women's participation to the labor market has increased from 1991 to 2001 and my samples become more and more representative overtime. Since 1999, more than 75 % of married women between 22 and 40 years old are working and more than 90 % of men (This figures are presented in appendix on figure 25). Second, I don't model the evolution of wages with age. Married individuals who are older in average than singles have then higher wages. This could lead to overestimate the attractiveness of high wage men on the marriage market. To limit the bias, I restrict the sample to the age range between 22 and 40 years old. In my sample, married men are in average only 3 years older than single men. Married women are in average only 1.5 years older (This figures are presented in appendix on figure 24)

3.2 Data description

The left panel of figure 1 represents the wage distribution for different marital status in 1999. Married men have higher wages. The right panel represents the distribution of the Family Value Index. Matched individuals are more conservative than singles and men seem to be more conservative than women.

Figure 1: Distribution of wages and Family Value Index for different marital status in 1999. BHPS



In 1999, wage correlation among couples was around 0.35 and F.V.I correlation is around 0.44. Figure 2 represents the average market and domestic work conditional on wages. Labor supply

and takes value from 5 to 25

of women increases sharply with wages whereas labor supply of men seems much less elastic. Married men work in average 3 hours more a week than single men. On the contrary, single women work more than married women by around 2 hours a week. The result is symmetric for domestic work: married women work more at home than single women by about 5 hours a week. Domestic work of married women is strongly decreasing with wage. Domestic work of men is inelastic with an average wage of 6 hours a week.

Figure 2: Average market and domestic work hours conditional on wages in 1999. BHPS

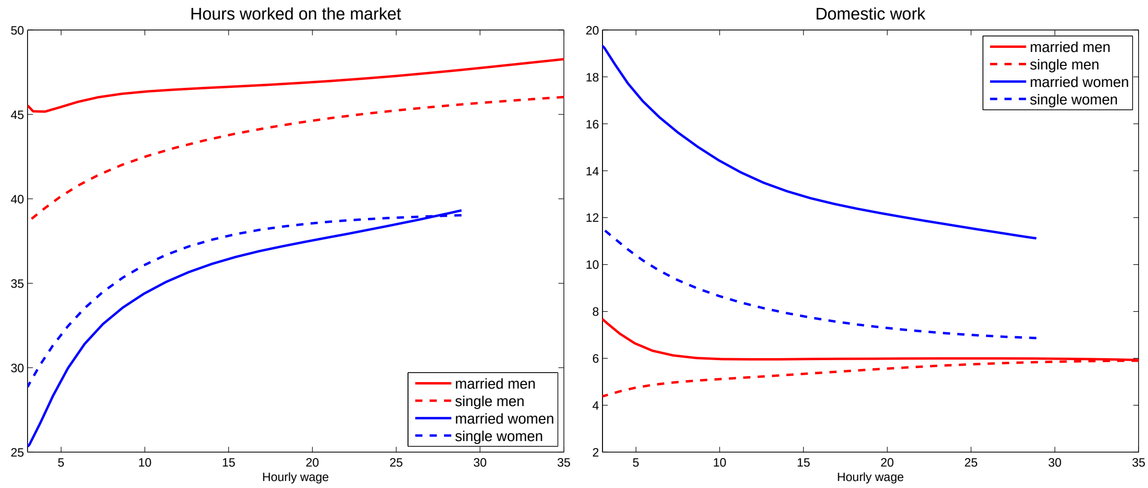
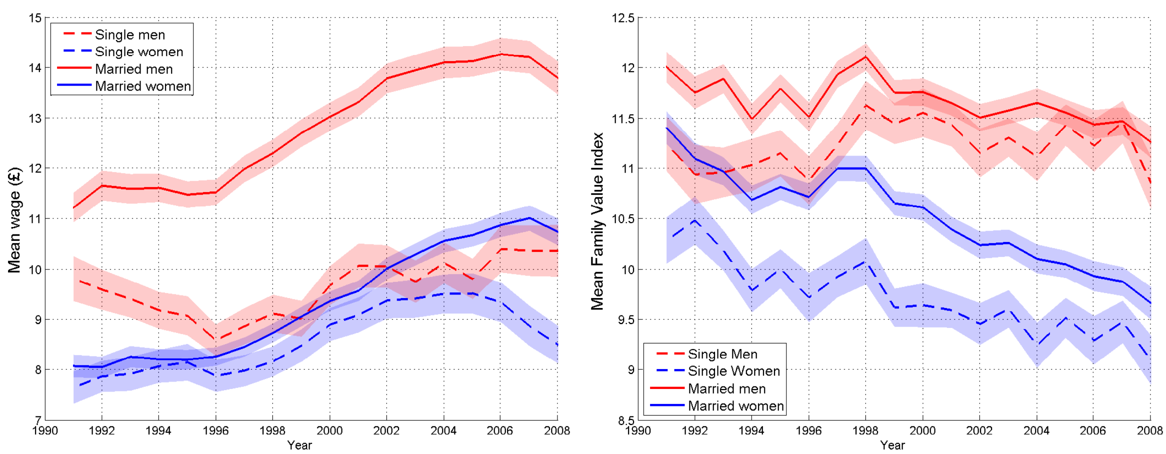


Figure 3 represent wage and FVI trends for different marital status. Married people's wages have increased whereas single men's wages have remained constant. Single women's wage have also increased a little bit but less than married women. The average of the FVI is decreasing overtime, particularly since the year 2000 and particularly for single women and single mothers. The discontinuity in 1998 is certainly due to the change in questions making arbitrarily individuals more conservative. Since 1998, the average has decreased strongly for women but has remained constant for men.

Figure 3: Evolution of the average wage and Family Value Index from 1991 to 2008. BHPS



I represent the trends of market hours and domestic work hours by sex and marital status on figure 4 and 5. Market work has raised by 2 hours a week for married women and has diminished by 1 hour a week for married men. Market work of single women has remained constant whereas it has decreased by 4 hours a week for single men.

The most remarkable fact is a decreasing trend in domestic work for married women and mothers. They used to do housework 20 hours a week in average in 1991, whereas in 2008, they spend 12 hours a week doing housework. Domestic work of single women has diminished by 2 hours a week whereas domestic work for single and married men has only been reduced by 0.5 hours a week. We note that in 2008, married men work only 1 hour more a week than single men whereas married women work about 4 hours more than single women.

Figure 4: Evolution of the average quantity of market work by month from 1991 to 2008. BHPS

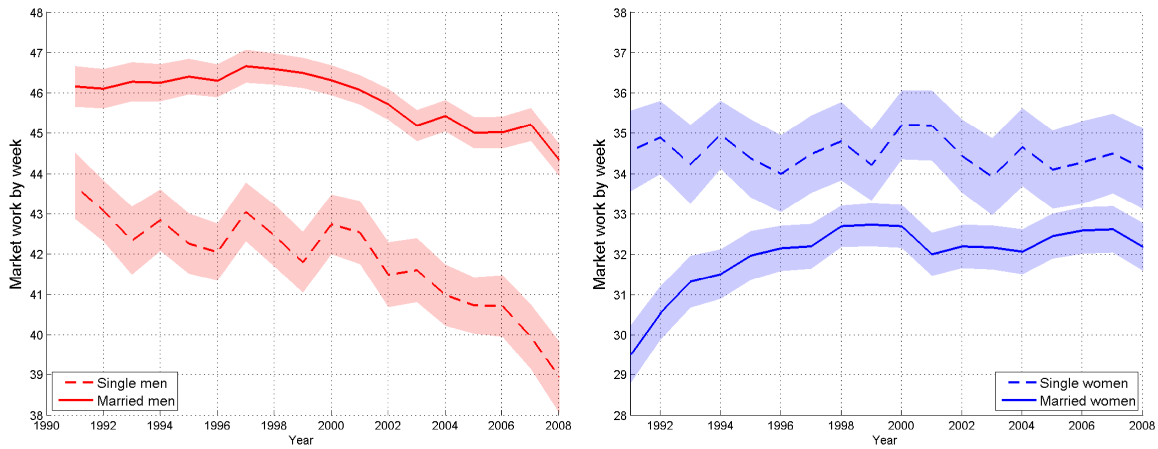
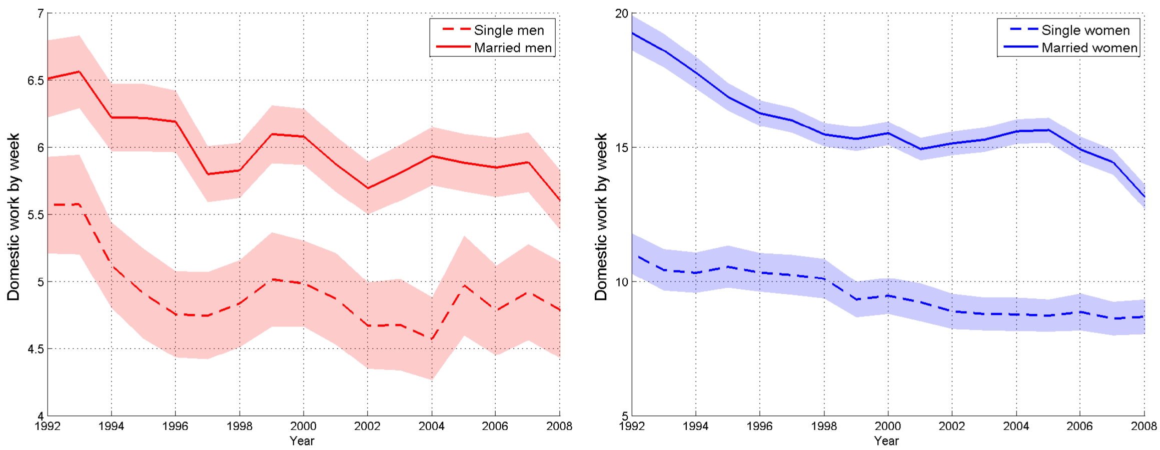


Figure 5: Evolution of the average quantity of domestic work by month from 1991 to 2008. BHPS



4 The estimation strategy

4.1 Estimation of λ and λ_z with a duration model

The object of interest is to estimate the likelihood of a type i agent marrying a type j agent. As in Wong (2003), the method used is maximum likelihood. Agents can be single or married at the time of the interview. Information on the duration of singlehood or marriage is obtained by following single agents before and after the interview. Let T_{0b} (T_{0f}) be the elapsed (residual) duration of singlehood for the single people at the time of the interview. Therefore, the duration of singlehood is $T_0 = T_{0b} + T_{0f}$. If this duration is left censored, $C_{0b} = 1$ ($C_{0b} = 0$ otherwise). If this duration is right censored, $C_{0f} = 1$ ($C_{0f} = 0$ otherwise). Consider a type i man who is single at first interview. Let T_{0b} and T_{0b} be i.i.d. and have an exponential distribution with parameter $\lambda U_f \int_j u_f(j)a(i,j)dj$, that is the probability to find a woman with whom the match will be formed. Then for an agent of type i, the individual contribution of singlehood duration until and including the time of exit into marriage with a woman of type j or censoring is

$$L_{0i} = u_m(i)\left(\lambda U_f \int_j a(i,j)u_f(j)\right)^{1-C_{0b}+1-C_{0f}} e^{-\left(\lambda U_f \int_j u_f(j)a(i,j)\right)(T_{0b}+T_{0f})} u_f(j)^{(1-C_{0f})}$$

where $T_{0f} > 0$ and $T_{0f} > 0$. Events occurring after exit from being singlehood are independent of the events up to exit. Therefore, their probability is independent of the likelihood of being singlehood. The event immediately following type i's singlehood duration is the realization of whom to match with. This event is given by the density of accepted type $u_f(j)$.

I note T_{1b} (T_{1f}) the elapsed (residual) duration of marriage for the married people at the time of the interview. If this duration is left censored, I note $C_{1b} = 1$ ($C_{1b} = 0$ otherwise). If this duration is right censored, I note $C_{1f} = 1$ ($C_{1f} = 0$ otherwise). The contribution to the loglikelihood of a man of type i married with a woman of type j which separates after a certain period is then

$$L_{1ij} = \frac{n(i,j)N}{L_m}\left(\lambda_z(1-a(i,j))\right)^{1-C_{1b}+1-C_{1f}} e^{-\lambda_z(1-a(i,j))(T_{1b}+T_{1f})}.$$

I observe the total duration of partnership of around 10 % of couples and the total length of singlehood for around 13 % of singles. On the non-censored observation, couples stay together for an average of 13 years whereas singles stay alone for 7 years in average. This figures are presented on graph ?? and graph 20 in appendix.

4.2 Specification of the domestic production functions

As previously described in the section on utilities, the specification of domestic production function for single men and women is

$$Q(d_m) = (d_m - D_{0m})^{\kappa_m}$$
$$Q(d_f) = (d_f - D_{0f})^{\kappa_f}.$$

People enjoy the part of domestic work which is superior to the required level D_{0m} for single men and D_{0f} for single women. The maximization of utility under the budget constraint of singles leads to the following formula for domestic work

$$d_{m0}(w_i) = D_{0m} + \frac{\kappa_m}{(1+\kappa_m)w_i}(w_i T - \mathcal{C}_{0m}(w_i) - w_i D_{0m})$$
$$d_{f0}(w_j) = D_{0f} + \frac{\kappa_f}{(1+\kappa_f)w_j}(w_j T - \mathcal{C}_{0f}(w_j) - w_j D_{0f}).$$

The domestic production function for couples depend on domestic works of both partners in the following way

$$Q(d_m, d_f) = (d_m - D_{0m})^{\kappa_m}(d_f - D_{0c})^{\kappa_f}.$$

The minimum level of housework required for married women can be different than for single women : $D_{0c} \neq D_{0f}$. This flexibility has been chosen to match the observed domestic work of married women who work much more at home than single women whereas domestic work of married men and single men are very similar. The quantities of domestic works are defined during the Nash Bargaining as explained in the previous section and solve the equation system (5) which gives the two following identifying equations.

$$d_{mij} = D_{0m} + \frac{\kappa_m}{1+\kappa_m+\kappa_f}\frac{1}{w_i}((w_i + w_j)T - \mathcal{C}_{0m}(w_i) - \mathcal{C}_{0f}(w_j) - w_i D_{0m} - w_j D_{0c} - C_c)$$
$$d_{fij} = D_{0c} + \frac{\kappa_f}{1+\kappa_m+\kappa_f}\frac{1}{w_j}((w_i + w_j)T - \mathcal{C}_{0m}(w_i) - \mathcal{C}_{0f}(w_j) - w_i D_{0m} - w_j D_{0c} - C_c)$$

I jointly estimate the parameters κ_m, κ_f, D_{0f}, D_{0m} and D_{0c} with a system of linear regressions. I implicitly assume that I observe d_{mi0}, d_{fj0}, d_{mij} and d_{fij} with some error terms uncorrelated with wages and with preferences.

4.3 Preference functions : inference from hours

To compute the single present value, the public good and the average of transfers, I need the identification of $\mathcal{C}_{0f}$ and $\mathcal{C}_{0m}$. If P_f and P_m are known functions and if κ_m, κ_f, D_{0f}, D_{0m} and D_{0c} are known, $\mathcal{C}_{0f}$ and $\mathcal{C}_{0m}$ can be recovered from working hours of single individuals. Using Roy's identity, you can write the following equations for working hours of single and married men[15]

[15] You also remark that $h_{mi} = T - L_{mi} - d_{mi}$ and that $\mathcal{C}'_{0m}(w_i) = L_{0m}$. You don't derive in Roy identity the preference functions L_{0m} and C_{0m}. They are functions of characteristics of the individual and reflect heterogeneity in preferences but are not changing instantaneously. They are assumed constant for each individual during his optimization

$$h_{mijz} = T - d_{mij} - \mathcal{C}'_{0m}(w_i) - \frac{P'_m(w_i)}{P_m(w_i)}(w_i(T - d_{mij}) + t_{mijz} - \mathcal{C}_{0m}(w_i)) \tag{10}$$

$$h_{mi0} = T - d_{mi0} - \mathcal{C}'_{0m}(w_i) - \frac{P'_m(w_i)}{P_m(w_i)}(w_i(T - d_{mi0}) - \mathcal{C}_{0m}(w_i)).$$

Using the equation for singles[16] you can write the following linear differential equation :

$$\mathcal{C}'_{0m}(w_i) - \frac{P'_m(w_i)}{P_m(w_i)}\mathcal{C}_{0m}(w_i) = T - h_{mi0} - d_{mi0} - \frac{P'_m(w_i)}{P_m(w_i)}w_i(T - d_{mi0})$$

whose solution is

$$\mathcal{C}_{0m}(w_i) = P_m(w_i)\int_0^{w_i} \frac{T - h_{mi0} - d_{mi0} - \frac{P'_m(w_i)}{P_m(w_i)}w_i(T - d_{mi0})}{P_m(w)}dw.$$

Then if the aggregate price indexes P_m and P_f are known, we can recover the functions $\mathcal{C}_{0m}(w_i)$ and $\mathcal{C}_{0f}(w_j)$. These aggregate price indexes can be recovered using transfers and the difference of market hours between married people and single people. Using (10), we can write :

$$h_{mijz} - h_{mi0} = d_{mi0} - d_{mij} - \frac{P'_m(w_i)}{P_m(w_i)}(t_{mijz} + w_i(d_{mi0} - d_{mij})). \tag{11}$$

Then integrating the preceding equation, we get

$$\int_{z|z>-Sxy} h_{mijz} + d_{mij} - h_{mi0} - d_{mi0} = -\frac{P'_m(w_i)}{P_m(w_i)}\left(\int_{z|z>-Sxy} t_{mijz} + w_i(d_{mi0} - d_{mij})\right).$$

We will consequently regress the ratio $\Delta\overline{H_{mij}} = \overline{h_{mijz}}^{z|z>-Sxy} + d_{mij} - h_{mi0} - d_{mi0}$ on $\overline{t_{mijz}}^{z|z>-Sxy} + w_i(d_{mi0} - d_{mij})$ to obtain

$$\frac{P'_m(w_i)}{P_m(w_i)} = -\frac{\int_j \left(\Delta\overline{H_{mij}}\right)\left(\overline{t_{mijz}} + w_i(d_{mi0} - d_{mij})\right)n(i,j)dj}{\int_j(\overline{t_{mijz}} + w_i(d_{mi0} - d_{mij}))^2 n(i,j)dj}. \tag{12}$$

Then P_m can be recovered using transfers and the observation of domestic and market work. However, transfers are also function of $\mathcal{C}_{0m}$ and $\mathcal{C}_{0f}$. Besides, the preference parameters, κ_m, κ_f, D_{0m}, D_{0f} and D_{0c} are also estimated using $\mathcal{C}_{0m}$ and $\mathcal{C}_{0f}$. These functions are then solved numerically. Using initial values for these functions we estimate κ_m and κ_f, compute transfers, then P_m and P_f using (12) and we estimate new functions for $\mathcal{C}_{0m}$ and $\mathcal{C}_{0f}$ until convergence. This method seems to work well as it always converges and the solution found doesn't depend on initial values.

[16] Similarly for women

$$h_{fijz} = T - d_{fij} - \mathcal{C}'_{0f}(w_j) - \frac{P'_f(w_j)}{P_f(w_j)}(w_j(T - d_{fij}) + t_{fijz} - \mathcal{C}_{0f}(w_j))$$

$$h_{f0j} = T - d_{fj0} - \mathcal{C}'_{0f}(w_j) - \frac{P'_f(w_j)}{P_f(w_j)}(w_j(T - d_{f0j}) - \mathcal{C}_{0f}(w_j))$$

4.4 Bargaining power

The parameter β is estimated by minimizing the errors in market hours prediction with the whole model. However, I obtain better predictions of estimated hours if I allow β to vary with wages of the members of the couple. I define the bargaining power β as a linear function of the within household wage ratio such that $\beta_{ij} = \beta_0 + \beta_1 \log(\frac{w_i}{w_j})$.

5 Estimation results

There are 11 parameters and 5 functions to estimate in the model. These parameters are displayed in table 2.

Table 2: Parameters of the model

Discount rate	r
Quality shocks	$F_z(z)$, λ_z
Meeting parameters	λ
Bargaining parameters	β_0, β_1
Domestic production	κ_f, κ_m, D_{0m}, D_{0f} , D_{0fc}
Preference functions	$P_m(w_i)$, $P_f(w_i)$, $\mathcal{C}_m(w_i)$, $\mathcal{C}_f(w_j)$
Cost	C_c

All parameters can't be estimated. The discount rate r is set at 3 % per year. The distribution of the match quality shock $F_z(.)$ is modeled with a centered Gaussian distribution of variance σ_z^2 set to 0.1[17]. The parameters λ and λ_z are estimated independently using the exponential duration model previously described. The domestic production parameters and the preference functions are estimated together as described in the previous section. I fix the cost C_c at 800 £ a month which is the minimum required to obtain realizable predictions of market hours. As 65 % of married couples in my sample have children, this must represent an average additional cost supported by parents to raise children[18]. The bargaining parameters which fit the best the data are $\beta_0 = 0.5$ and $\beta_1 = 0.15$. I keep this estimate in 1999. However for the longitudinal analysis, I fix $\beta_0 = 0.7$ and $\beta_1 = 0$ to make the interpretation of transfers evolutions simpler. The closing gender wage gap would have an effect on β plus an effect on single present values and it would be difficult to disentangle the two effects. Table 3 presents the parameter estimates in 1999 where the sample is the largest. The parameters of quality shocks and meeting give an average duration of singlehood of 10 years for single men and 8 years for single women and a average duration of couples of 19 years.

[17] I derive an estimation method of σ_z in appendix

[18] However, I should also take into account that many single women also incur an additional cost for raising children. 30 % of single women in my sample have children, which concerns less than 2 % of men.

Table 3: Parameters estimation

Quality shocks[a]	Meeting parameters[a]	Domestic production parameters				
λ_z	λ	κ_f	κ_m	D_{0m}	D_{0f}	D_{0c}
0.0030	0.00028	0.042	0.018	3.6	6.6	8.4
(0.00014)	*(0.0017)*	*(0.009)*	*(0.003)*	*(0.32)*	*(0.62)*	*(1.5)*

(a)*Standard errors are obtained by bootstraps (100 replications)*

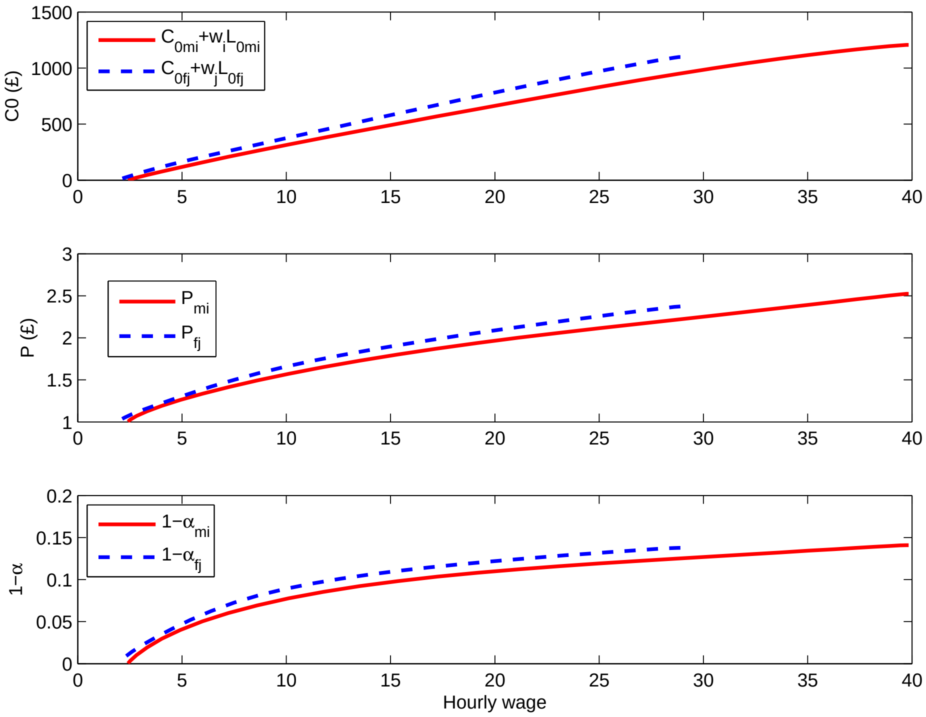
The preference functions are represented on figure 6. The top panel represents the minimal amount of consumption and shows that it increases almost linearly with wages for both men and women. The slope is a little bit higher for men. The middle panel represents the price index and shows that it is also increasing with wage. The bottom panel of figure 6 represents the corresponding preference functions in the direct utilities : α_{mi} and α_{fj}. Remind that

$$\mathcal{C}_{0mi} = \mathfrak{C}_{0mi} + w_i \mathfrak{L}_{0mi}$$

$$P_{mi} = \frac{w_i^{1-\alpha_{mi}}}{\alpha_{mi}^{\alpha_{mi}}(1-\alpha_{mi})^{1-\alpha_{mi}}}$$

Preference for leisure $(1-\alpha)$ increases with wage and is higher for women than for men. Low wage women enjoy more leisure than low wage single men.

Figure 6: Preference for consumption and leisure



Evolution of preferences over time Estimating these functions and parameters each year, we obtain some variations. Figure 7 shows the evolution of the estimated parameters. The blade area represents the 90 % confidence interval. D_{0f} and D_{0c} have decreased from 1991 to 1999 then stabilized. The minimum required in domestic production is lower for single women (6 hours a week) than for married women (10 hours a week). D_{0m} has remained stable at 4 hours a week. The preference parameters for domestic production κ_m and κ_f did not follow any trend, they fluctuate around 0.035 for women and around 0.015 for men.

Figure 7: Evolution of domestic production function parameters

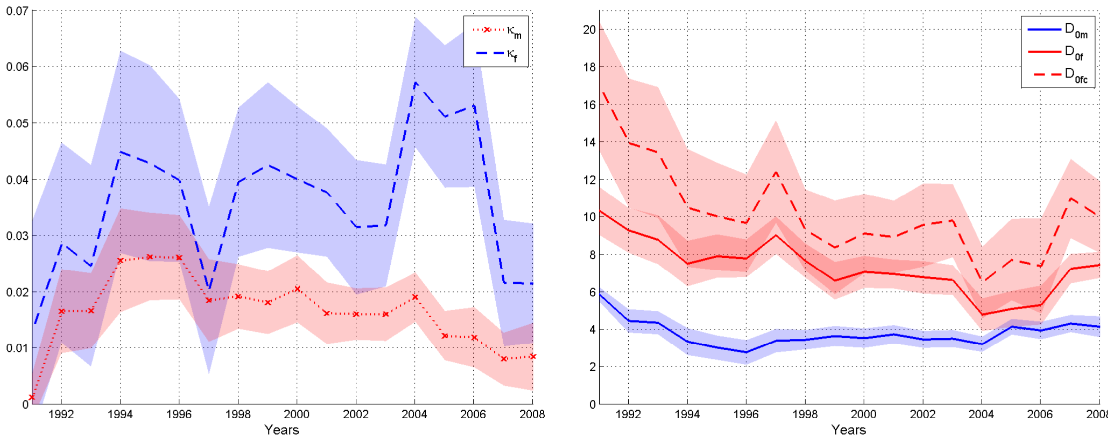
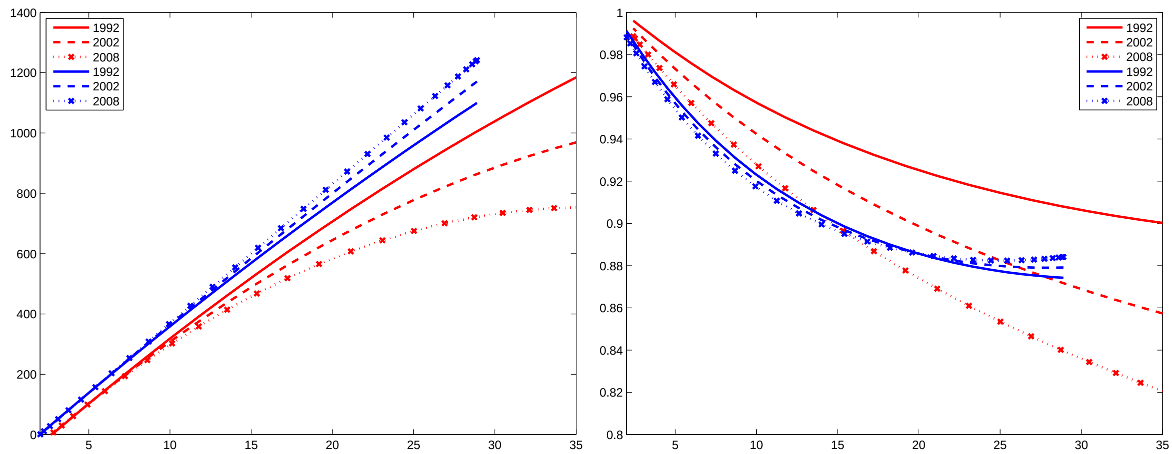


Figure 8 represents the evolution of preferences for consumption and leisure over years. It seems that men preferences have more changed than women preferences. Men have decreased their minimum level of consumption and leisure and have decreased their preferences for consumption relative to leisure. This results seem confirm the intuition of Aguiar et al. ([1], 2007). On the contrary, women have slightly increased their minimum level of consumption and leisure and have very slightly increased their preference for leisure relative to consumption.

Figure 8: Evolution of preferences for consumption and leisure

(a) $C_0 + wL_0$ (b) α

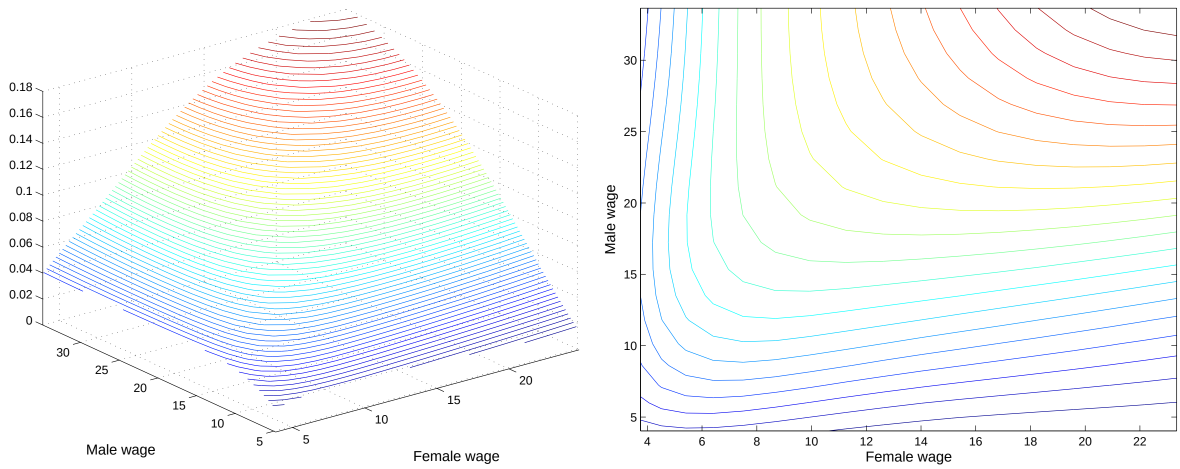


5.1 Cross section analysis in 1999

5.1.1 Matching patterns

This section presents the estimation of the match probability $a(wage_i, f_i, wage_j, f_j)$ which is the probability that a man of type $(wage_i, f_i)$ and a woman of type $(wage_j, f_j)$ match if they happen to meet. I represent this 4 variable function on a 3D graph. The left panel of figure 9 shows the expected match probability conditional on wages[19]. The left panel is a 3D plot whereas the right panel represents the level curves. The left panel shows that the matching probability is strongly increasing in both wages. The probability that a man with a wage rate of 30£ matches with a woman of wage rate 25£ when he meets her is 0.2 whereas the probability that he marries a woman of wage rate 5£ when he meets her is 0.06. This figure also shows a little dissymmetry more visible on the right panel. Women with low wages have higher chances to marry than men with low wage. The probability that a rich man marries a low wage woman is higher than the probability that a rich woman marries a poor man, even if they have the same probability of meeting. Figure 10 represents the match probability conditional on the Family Value Index. The expected probability of matching is lower in average meaning that the F.V.I explains less of the matching probability. The highest expected match probability conditional on F.V.I reaches 0.14 and is obtained for individuals with high family value index. In comparison, the maximum of match probability conditional on wages reached 0.2 for the highest wages of men and women. The right panel shows that the matching probability is higher when the two F.V.I are high and close.

Figure 9: Expected Marriage probability conditional on wages

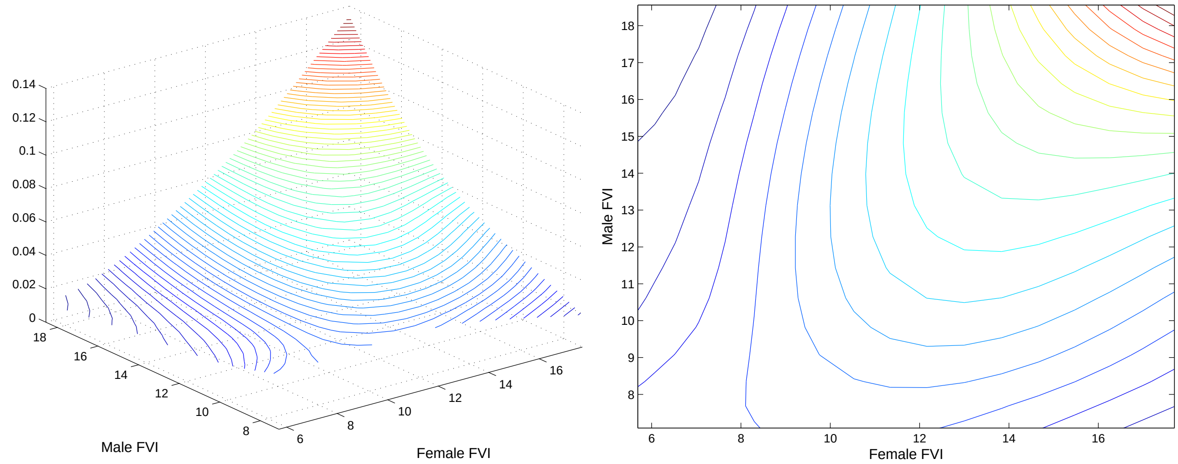


[19] The actual formula for the conditional expectation represented on the graph is

$$a(w_i, w_j) = \mathbb{E}(a(w_i, f_i, w_j, f_j)|w_i, w_j) = \int_{f_i}\int_{f_j} a(w_i, f_i, w_j, f_j) n_{|(w_i,w_j)}(w_i, f_i, w_j, f_j)$$

with $n_{|(w_i,w_j)}(w_i, f_i, w_j, f_j)$ the density of couple of type (w_i, f_i, w_j, f_j) conditional to (w_i, w_j)

Figure 10: Expected Marriage probability conditional on Family Value Index



I represent the affinity factor Φ_{ij} conditional on wages on figure 11. Its shape is slightly different from the total surplus. Low wage women have affinity with low wage men. It is not the higher the wage the better anymore. Same wage partners have high complementarities in public good production. When we look at the affinity factor conditional on the Family Value Index on figure 12 the best match is also reached for individuals with conservative family values. It is low for female with low FVI and increases sharply with women FVI. Women with conservative family values are valuable for production of the public good.

Figure 11: Affinity factor conditional on wages

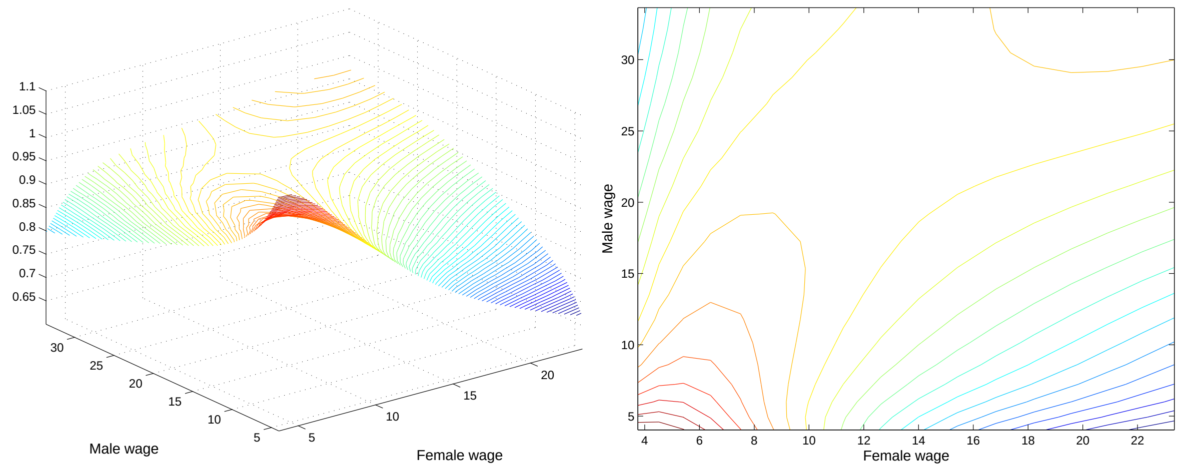


Figure 12: Affinity factor conditional on Family Value Index

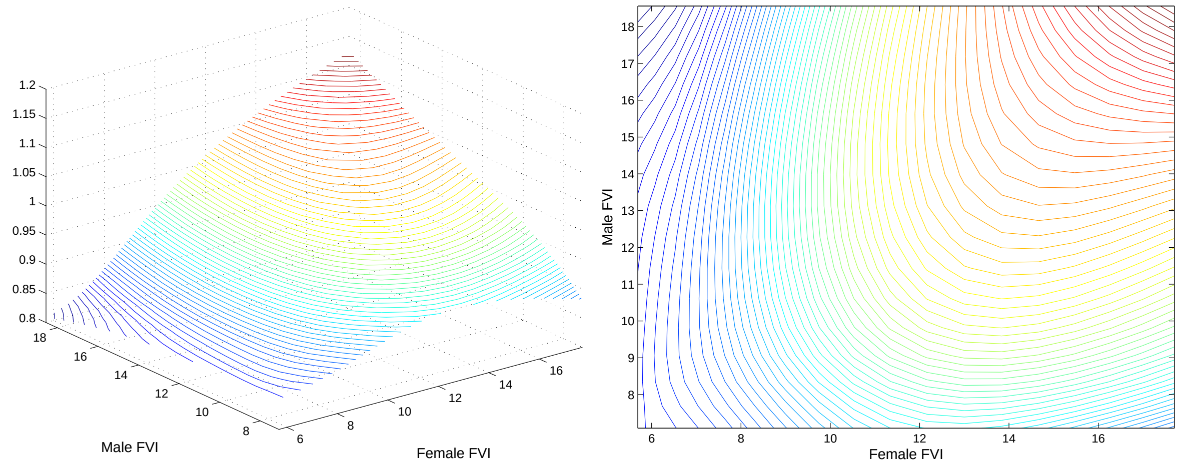
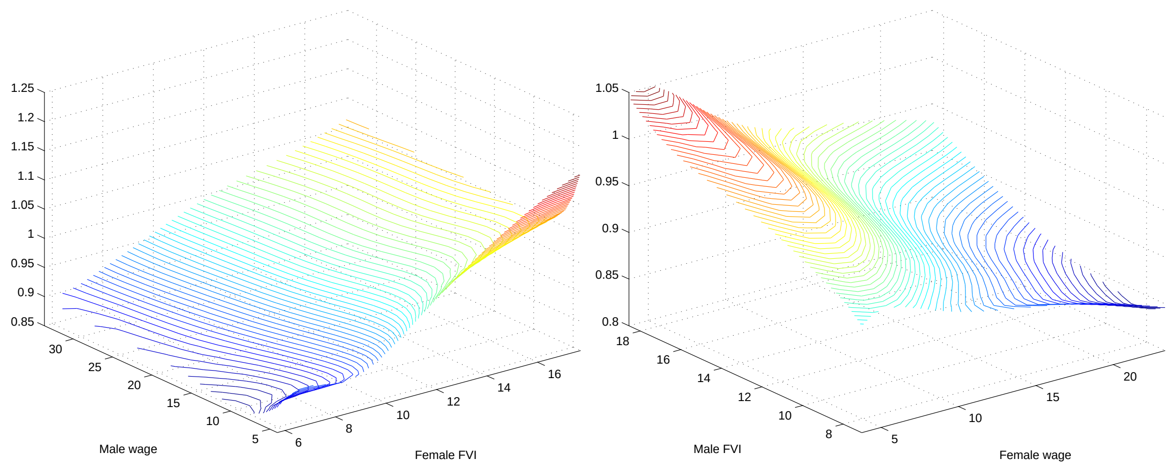


Figure 13 represents cross characteristics preferences. The left panel show that complementarities in public production is high for high FVI women and low wage men. Women's FVI seem to have a stronger positive impact on the public production factor than men's wage. Interestingly, the right panel shows that public production is much higher for low wage women with high FVI male and can be very low for high wage women with low wage men. Public production seems to be linked with strong beliefs in the importance of traditional family structure and may be in opposition with women work.

Figure 13: Affinity factor conditional on certain characteristics



The previous graphs show the shape of the affinity factor with respect to 2 out of the 4 characteristics : w_i, w_j, f_i, f_j. It would be interesting to better understand the contribution of each variable to this factor and their interaction. To this end, I perform a nonparametric regression of using Generalized Additive Models (GAMs). I present in table 4 the generalized R^2 obtained for different specifications of the interactions of variables (I put a note on GAM in appendix). The characteristics of the woman contribute more to the affinity factor. Her FVI explains 24 % of the variance. The family value of the woman explains more than her wage. The interaction of the partners wage plus the interactions of the partners FVI account for almost all the variance : 92.4

%. The interaction of the male characteristics plus the interaction of the women characteristics account for 44.2 % of the variance. There exist strong complementarities between partners. The public good is not only a sum of contributions of each partner.

Table 4: Comparisons of different models

Specifications $\mathbb{E}(\Phi \vert w_m, f_m, w_f, f_f)$	Generalized R^2
$g(w_m)$	3.3%
$g(f_m)$	9.4%
$g(w_f)$	6.7%
$g(f_f)$	23.8%
$g(w_m, f_m)$	13.0 %
$g(w_f, f_f)$	31.2 %
$g(w_m, w_f)$	41.0 %
$g(f_m, f_f)$	51.4 %
$g(w_m, f_f)$	27.6 %
$g(w_f, f_m)$	17.4 %
$g(w_m, w_f, f_m, f_f)$	96.2 %

5.1.2 Transfers and Inequalities

In this section, I consider different measures of resource sharing within households. The usual sharing rule derived in collective models with household production is the conditional sharing rule. This rule represents how the resources are shared after each member had spent some time in domestic production. However, this rule neglects public production. It doesn't represent how individual contribute to domestic production. A woman can get a large part of the rest of the total income, but she may have also contributed much more to domestic production than her husband and she finally doesn't benefit so much from the couple surplus.

Another measure of how resources are shared can be the generalized sharing rule which decentralizes via personal prices the spending in public consumption. The generalized sharing rule is then equal to the conditional sharing rule plus one's spending in domestic production. This the generalized sharing rule which has been named t_m in the present paper. However this rule neglects price of public consumption. Indeed, men and women could have different marginal propensity to spend time and money in public production, then it could be less costly for a woman to spend more time in housework than for a man. To be clearer, the generalized transfers are t_m and t_f such that $t_m + t_f = -C_c$. The conditional transfers are $t_{mc} = t_m - w_m d_m$ and $t_{fc} = t_f - w_m d_m$ such that $t_{mc} + t_{fc} = -w_m d_m - w_f d_f - C_c$.

We can also consider the measure developed in Chiappori, Meghir (2013), the Money Metric

Welfare Index (henceforth MMWI) which corresponds to the monetary amount that an agent would need to reach alone the same utility level that she reaches when she is in couple. Chiappori and Meghir (2013) argue that the Money Metric Welfare Index fully characterizes the utility level reached by the agent. If we give M to a single individual of type i, he would reach the following utility level

$$v_{mi} = \frac{(d_m - D_{0m})^{\kappa_m}(w_i(T - d_{mi0}) + M - \mathcal{C}_{0m}(w_i))}{P_m(w_i)}.$$

When he is married with a woman of type j under circumstance z he reaches

$$v_{mijz} = (\Phi_{ij} + z)F(d_m, d_f)\frac{w_i(T - d_{mij}) + t_{mijz} - \mathcal{C}_{0m}(w_i)}{P_m(w_i)}.$$

To equalize these both utilities, the man should receive

$$\begin{aligned} M_{mijz} &= -(w_i(T - d_{mi0}) - C_{0m}(w_i)) \\ &+ \frac{1}{(d_m - D_{0m})^{\kappa_m}}(w_i(T - d_{mij}) + t_{mijz} - \mathcal{C}_{0m}(w_i))F(d_m, d_f)(\Phi_{ij} + z). \end{aligned}$$

The couple brings M_{ijz} to the man and M_{fijz} to the woman. This transfer increases with domestic production of the couple and with the affinity factor. It decreases with the domestic production as single and with the single total resource. It also increases with the conditional sharing rule. To sum up, I have available three instruments to measure within household inequalities, the generalized sharing rule, the conditional sharing rule and the MMWI. Although I don't observe the random variable z, I can compute the average of transfers for all possible value of z which allow people to match. The expression of average within household generalized transfers are

$$\begin{aligned} \overline{t_{mijz}} &= \mathbb{E}(t_{mijz}|i, j, z > -s(i, j)) \\ \overline{t_{fijz}} &= \mathbb{E}(t_{fijz}|i, j, z > -s(i, j)). \end{aligned}$$

Using Discrete Cosine Transform, I compute the three measures of transfers for each men and women of each couple. In 1999, the median of the MMWI's share of the woman ($\frac{M_f}{M_f+M_m}$) is 0.32 that is 50% of married women get less than 32% of the surplus generated by the couple. The median of the woman conditional transfer is - 658 £ which is the share of the children cost[20] supported by the woman plus the total opportunity cost of her domestic work. The median of the woman generalized transfer is equal to - 200 £ which is only the share of the children cost supported by the woman.

Table 5: Median value of transfers to the women in 1999

MMWI woman'share ($\frac{M_f}{M_m+M_f}$)	0.20
Generalized Sharing Rule	-228 £
Conditional Sharing Rule	-701 £

[20] equal to 800 £

However the median is not very informative. Let's study more precisely these transfers on particular types of household. I consider 9 different types of household which depend on the position of each partner in the wage distribution. I consider 3 categories, those whose wages are below the first quartile ($w \leq q_{25}$), those whose wages are between the first quartile and the third quartile ($w \in [q_{25} - q_{75}]$) and those whose wages are over the third quartile ($w > q_{75}$). The average of these transfers among each type of household are presented in table 6

Table 6: Average values of transfers to the women according to the household type

Woman Wage Quantile	$\leq q_{25}$			$[q_{25} - q_{75}]$			$> q_{75}$		
Man Wage Quantile	$\leq q_{25}$	$[q_{25} - q_{75}]$	$> q_{75}$	$\leq q_{25}$	$[q_{25} - q_{75}]$	$> q_{75}$	$\leq q_{25}$	$[q_{25} - q_{75}]$	$> q_{75}$
Woman share of MMWI $(\frac{M_f}{M_m+M_f})$	0.23	0.23	0.21	0.24	0.21	0.21	0.19	0.20	0.19
Conditional Sh.Rule	-605	-522	-435	-882	-791	-702	-1441	-1281	-1234
Generalized Sh.Rule	-244	-116	27	-270	-275	-114	-676	-383	-390

To obtain a complete characterization of these transfers, I regress them on individuals characteristics. Indeed, even if we have the exact formula for all these transfers, the impact of different variables is not obvious in the expression $\frac{(1-\beta)rP_m W_{mi} - \beta r P_f W_{fj}}{F(d_{mij}, d_{fij})(\Phi(i,j)+z)}$ for instance. Regression results are presented in table 7. The first column displays the result for the average share welfare for women that is $\frac{M_f}{M_f+M_m}$, the second column displays the results for the generalized transfer. The transfer increases in woman's wage and decreases in her husband wage. It also increases in FVI of both members. The arbitrage between optimal value of the FVI and the wage is here well described. Let us consider an average couple where the man has an FVI of 11.7 and an hourly wage of 12.5 £ and the woman has an hourly wage of 9 £ and a FVI index of 10.7. An increase of 1 £ in man's wage leads to a decrease in 63 £ in the woman's conditional transfer and a decrease in 10 % in her welfare share. An increase of 1 £ in woman's wage leads to a increase in 54 £ in her conditional transfer and a increase in 11 % in her welfare share. An increase of 1 point in man's FVI leads to a increase in 6 £ in the woman's conditional transfer and a decrease in 1 % in her welfare share whereas an increase of 1 point in woman's FVI wage leads to an increase in 29 £ in her transfer and an increase in 3 % in her welfare share.

Table 7: Determinants of the MMWI and the conditional sharing rule in 1999

Variables	**Woman share of MMWI (%)**	**Sharing rule t_f (£/mth)**
Constant	−0.23	−926.03
	(12.22)	(114.02)
Wage male	−16.80	−45.93
	(0.43)	4.04
Wage male square	0.28	−0.70
	(0.01)	(0.11)
Wage female	18.10	65.66
	(0.66)	(6.16)
Wage female square	−0.38	−0.63
	(0.03)	(0.2)
FVI male	2.44	54.38
	(1.60)	(14.91)
FVI male square	−0.07	−2.06
	(0.06)	(0.6)
FVI female	5.83	39.14
	(1.57)	(14.64)
FVI female square	−0.15	−0.49
	(0.07)	(0.65)
R^2	80.1 %	75.5 %

5.1.3 Prediction of hours

Using the model, I compute the predicted working hours of married people. The graph 14 shows a very good prediction of market hours for both men and women. I also compute what would be the working hours of individuals in two extremal case. The first case is when there is no possibility of transfers. Then married individuals still benefit from complementarity in domestic production but each member keeps his own labor income and pay the half of the children cost. The dash line with little stars shows what would be the number of working hours if the individual didn't get any transfer from his spouse (positive or negative). The second case is when married individuals share equally all their resources. More precisely the expression of men labor supply in these three cases are

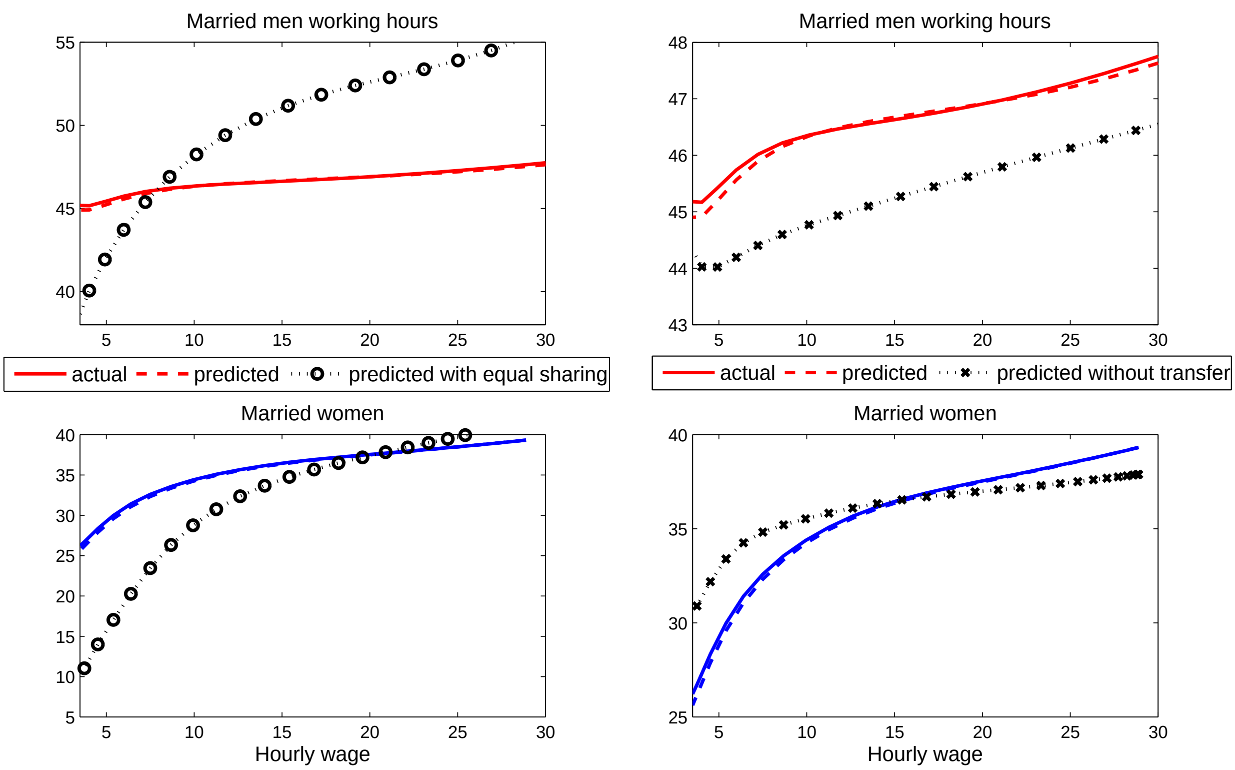
Hours with transfers $\quad h_{mijz} = T - d_{mij} - \mathcal{C}'_{0m}(w_i) - \frac{P'_m(w_i)}{P_m(w_i)}(w_i(T - d_{mij}) + t_{mijz} - \mathcal{C}_{0m}(w_i))$

Hours without transfers $\quad h_{mijz} = T - d_{mij} - \mathcal{C}'_{0m}(w_i) - \frac{P'_m(w_i)}{P_m(w_i)}(w_i(T - d_{mij}) - \mathcal{C}_{0m}(w_i) - \frac{C_c}{2})$

Hours with equal sharing $\quad h_{mijz} = T - d_{mij} - \mathcal{C}'_{0m}(w_i) - \frac{P'_m(w_i)}{P_m(w_i)}(w_i(T - d_{mij}) + w_j(T - d_{fij}) - \mathcal{C}_{0m}(w_i) - \mathcal{C}$

In the last two equations, there is no "bargaining effects". Labor supplies depends on the standards income effect and substitution effects. When man's wage rises, his labor supply tend to increase through substitution effect with the decrease in $\frac{P'_m(w_i)}{P_m(w_i)}$ and the decrease in domestic work d_{mij} (and the increase in wife's domestic work). It tends to decrease through the income effect with the rise in $w_iT - \mathcal{C}_{0m}(w_i)$. Whereas the first equation also includes a "bargaining effect" which acts like an income effect through an increase in the transfer t_m due to an higher wage. This bargaining effects then tends to reduce man labor supplies when his wage rises. Resource sharing mostly benefit to low wage women who can work much less as if they didn't get any transfer. On the contrary, married men should work less than they do to compensate the transfer they give to their wife. If individuals shared equally their resources, as men have generally higher wages, they would work much more as they would give more than half of their resources to their wife. On the contrary, women would work much less. The actual working hours lie between the two extremal cases.

Figure 14: Prediction of working hours in 2009



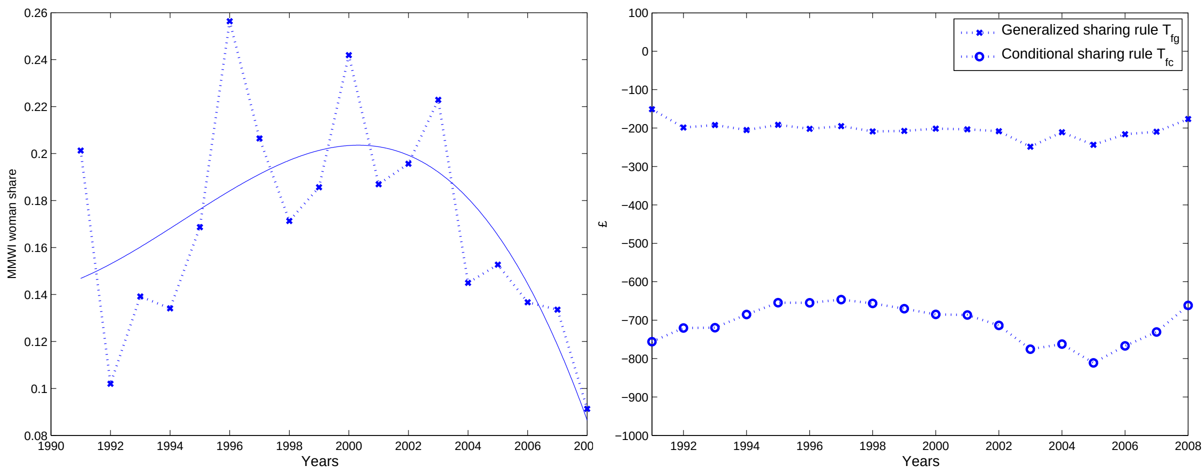
5.2 Evolution from 1991 to 2008

5.2.1 Transfers and inequalities

How did these transfers evolve overtime ? I represent on figure 15 the evolution of the median women's share of the total surplus that is $\frac{M_f}{M_f+M_m}$ and the evolution of their conditional and generalized sharing rule. We observe the increase in women's share of the surplus overtime. The median of the generalized sharing rule has also increased a little from 1991 to 1995 whereas the median of the generalized sharing rule has remained stable. This would mean that most of the increase in women welfare would come from the decrease in their domestic work which is higher than the decrease of domestic work of single women.

Figure 15: Evolution of transfers

(a) MMWI (b) Monetary transfers



To obtain a complete picture of inequalities, I compute the variance of the logarithm of the individual MMWI and the variance of the logarithm of the individual monetary resource that is $w_i(T - d_{mij} + t_m - \mathcal{C}_{0mi})$ for the man and $w_j(T - d_{fij} + t_f - \mathcal{C}_{0fj})$. As in Lise-Seitz, 2012 [23], I decompose this variance in two terms : the inter-household variance and the intra-household variance such as

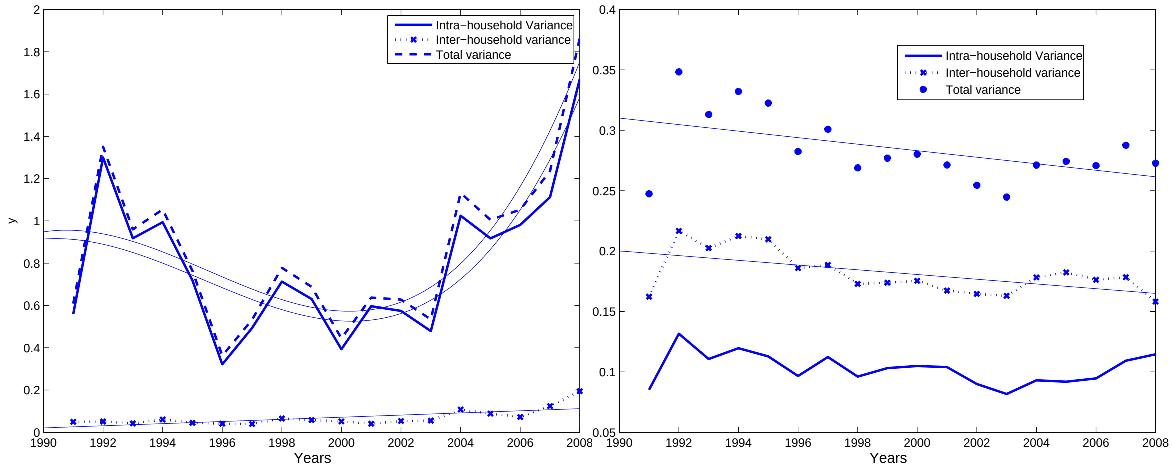
$$\mathbb{V}(M_{mg}) = \mathbb{E}(\mathbb{V}(M_{mg}|g \in (i,j)) + \mathbb{V}(\mathbb{E}(M_{mg}|g \in (i,j)))$$

Left panel of figure 16 shows that the total variance of the woman's share of the surplus decreases overtime and that this decrease is mostly due to the decrease of the intra-household variance. On the contrary, we observe on the right panel a decrease in the total variance of the logarithm of resource, this decrease is mostly due to the decrease of the inter-household variance.

Figure 16: Variance decomposition

(a) Log of individual MMWI

(b) Log of individuals resources



5.2.2 Prediction of hours

In a similar way that in section 5.1.3, I compute the predicted hours for each individual of each sample and the predicted hours with (i) no within household transfers and (ii) equal sharing. I present the evolution of the average of working hours by marital status on figure 17. Without transfer, men would work 1 hour less in average that is about 2.5 % less. To the contrary, married women would work more by 2 hours that is about 4.5 % more. This gap seems constant overtime for both men and women. If there was equal sharing, men would have worked 2 hours more from 1991 to 1998, 3 hours more until 2006, then 4 hours more. On the opposite, with equal sharing, women would have worked 4 hours less from 1991 to 2002 then 4 hours from 2002 to 2007.

Figure 17: Evolution of working hours

(a) Men

(b) Women

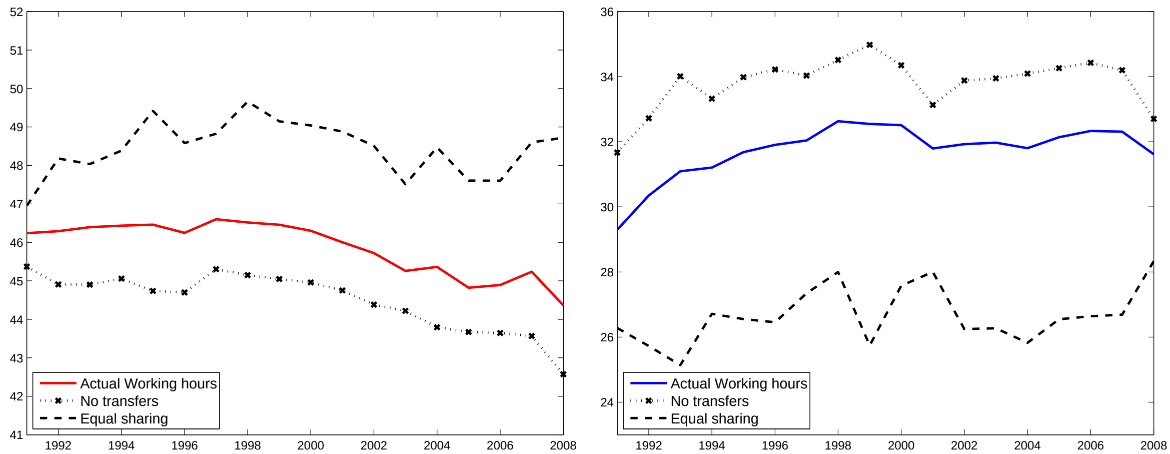
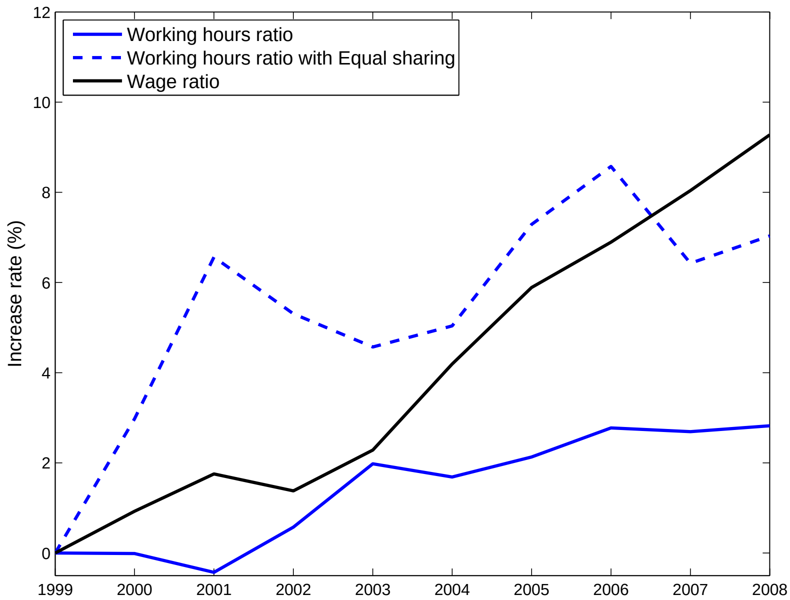


Figure 18 show the within household wage ratio growth and the within household work ratio growth from 1999 to 2008. The wage ratio has increased by 9 % over these years (women wages have relatively increased more than their male partners) whereas the work ratio has only increased by 3 %. If people share resources equally, the work hour ratio would have increased by 8 % (dashed line on Figure 18). This confirm the result found by Jones et al. ([21], 2003) and Knowles ([22],(2013), married men should have decreased much more their labor supply than they actually did.

Figure 18: Increase rate of working hours ratio and wage ratio



6 Simulations

6.1 Characterization of the equilibrium

In this section I check that we can go backward, that is, calculate the equilibrium distribution of characteristics and match probabilities from the previous nonparametric estimates of the structural parameters, namely, the complementarities in characteristics $\Phi(i,j)$ and the preference parameters. Let $n(i)$ denotes the density of married men of type i, then $U_m u_m(i) = L_m l_m(i) - Nn(i)$ and $n(i) = \int_j n(i,j)dj$. Besides, remind that

$$n(i,j) = \frac{\lambda u_m(i) U_m u_f(j) U_f a(i,j)}{N\lambda_z(1-a(i,j))},$$

then we obtain the following equation for the distribution of type i among single men

$$U_m u_m(i) = \frac{l_m(i) L_m}{1 + \frac{\lambda}{\lambda_z} U_f \int_j \frac{a(i,j)u_f(j)}{1-a(i,j)}}$$

Similarly, we obtain the expression for density of single women. Now, we will compute the equilibrium expression of the present value of single men and single women. We obtain the following Bellman equation for a single individual using equation (2) and (4)

$$\begin{aligned}
s_{mi} &= P_m(rW_{mi} - v_{mi}) \\
&= \lambda \int \int_{z\,j} \max(P_m S_{mij}(t_{mijz}), 0) u_f U_f(j) dG(z) \\
&= \lambda \int \int_{z\,j} \left(\max \left(\frac{\beta}{r+\lambda_z} F(d_{mij}, d_{fij}) R_{ij}(s(i,j)+z), 0 \right) u_f(j) U_f dG(z) \right) \\
&= \frac{\beta\lambda}{r+\lambda_z} \int_j F(d_{mij}, d_{fij}) R_{ij} \left(\int_z \max(s(i,j)+z, 0) dG(z) \right) u_f(j) U_f.
\end{aligned}$$

We first compute the inner integral on z :

$$\begin{aligned}
\int_z \max(s(i,j)+z, 0) dG(z) &= s(i,j)a(i,j) + \int_{-s(i,j)}^{+\infty} z dG(z) \\
&= s(i,j)a(i,j) + \sigma \int_{-\frac{s(i,j)}{\sigma}}^{+\infty} v d\Phi(z) \\
&= s(i,j)a(i,j) + \sigma\phi\left(\frac{s(i,j)}{\sigma}\right) \\
&= \mu(a(i,j)),
\end{aligned}$$

then we obtain the following formula for the present value of a single man of type i and a woman of type j

$$\begin{aligned}
s_{mi} &= \frac{\beta\lambda}{r+\lambda_z} \int_j F(d_{mij}, d_{fij}) R_{ij} \mu(a(i,j)) u_f(j) U_f \\
s_{fj} &= \frac{(1-\beta)\lambda}{r+\lambda_z} \int_i F(d_{mij}, d_{fij}) R_{ij} \mu(a(i,j)) u_m(i) U_m
\end{aligned} \tag{13}$$

An equilibrium is a fixed point of (u_m, u_f, W_m, W_f) of the following system of equations where the first two equations determine equilibrium wage distributions for singles and the last two equations determine equilibrium present values of single men and single women.

$$\begin{aligned}
u_m(i) &= \frac{l_m(i) L_m}{U_m + \frac{\lambda}{\lambda_z} U_f U_m \int_j \frac{a(i,j) u_f(j)}{1-a(i,j)}} \\
u_f(j) &= \frac{l_f(j) L_f}{U_f + \frac{\lambda}{\lambda_z} U_m U_f \int_i \frac{a(i,j) u_m(i)}{1-a(i,j)}} \\
P_{mi}(rW_{mi} - v_{mi}) &= \frac{\beta\lambda}{r+\lambda_z} \int_{fj} F(d_{mij}, d_{fij}) R_{ij} \mu(a(i,j)) u_f(j) U_f \\
P_{fj}(rW_{fj} - v_{fj}) &= \frac{(1-\beta)\lambda}{r+\lambda_z} \int_{mi} F(d_{mij}, d_{fij}) R_{ij} \mu(a(i,j)) u_m(i) U_m,
\end{aligned}$$

where $a(i,j)$ solves the following fixed point equation

$$a(i,j) = 1 - F_z\left(-\Phi(i,j) + \frac{rP_m W_{mi} + rP_f W_{fj}}{F(d_{mij}, d_{fij}) R_{ij}} - \frac{\lambda_z}{r+\lambda_z} \mu(a(i,j))) \right).$$

Despite the lack of a global contraction mapping property, The standard fixed-point iteration algorithm, $x_{n+1} = Tx_n$ works well in practice, even starting far from the equilibrium (for instance with $s_m = 0$ and $u_m(i) = l_m(i)$). For each dataset, the algorithm converges to the equilibrium observed in the data. I obtain the fixed point (u_m, u_f, W_m, W_f) which corresponds to the density and present value we observe in the data.

6.2 Simulation of other equilibria

What would change if all women had higher wages whereas men's wages wouldn't change ? Using my sample in year 2001, I slightly change the wage distribution of men and women and look at its impact on different outcomes. I consider the following different scenarios :

- **Scenario 1:** Women's distribution of wage is uniform on $[10£ - 20£]$ and the distribution of men doesn't change.
- **Scenario 2:** Men's distribution of wage is uniform on $[10£ - 20£]$ and the distribution of women doesn't change.
- **Scenario 3:** All single women with a wage inferior to 10 £ receive a transfer of 500 £ each month.

Table 8: Simulation exercises

	2009 data	Simulated equilibrium	Scenario 2	Scenario 3	Scenario 4
Matching pattern					
U_m	424	418	433	350	695
U_f	420	418	451	324	695
Social surplus					
$\mathbb{E}(s(x,y)$ $+\mathbb{E}_{\|z>-s(x,y)}(s(x,y)+z))$		$1.713 * 10^6$	$1.99 * 10^6$	$1.924 * 10^6$	$1.782 * 10^6$
Labor supply					
Married men	46.5	45.4	44.0	47.1	46.1
Married women	32.7	31.7	35.8	31.0	30.3
Single men	41.8	41.7	41.6	43.3	41.4
Single women	34.2	34.1	37.5	34.0	27.2

Scenario 1 When women's wages are higher, there are more single people. Women would not like to match with men with lower wages than them. They prefer to stay single. Married women work more by 4 hours due to a substitution effect reduced by a bargaining effect and married men work less by 1 hour due to income effect reduced by a bargaining effect. Single men work the same and single women work 3.5 hours more because of substitution effects. The social surplus is higher[21]. Quelles sont les femmes mariées ici ? Les plus riches ou les moins riches ? à vérifier

Scenario 2 When men's wages are higher, there are less single people. All women want to match with higher wage men. Married women work 0.7 hours less because of two opposite

[21] I compute in appendix how I derive that formula for the total social surplus

effects, a negative income effect and a positive bargaining effect due to a decrease in their transfer. Married men work more by 1.7 hours due to a positive substitution effect reduced by a bargaining effect. Single women work the same and single men work more by 1.6 hours whereas they get lower wages than married men (through selection effects). The social surplus is higher but less than in Scenario 1.

Scenario 3 When single women get high subsidies, they prefer to stay single than loosing it. There are a lot of single individuals. Married men work 1 hour more because they loose some bargaining power and married women work 1 hour less because they increase their bargaining power. Single men work the same and single women work less by 7 hours due to a big income effect. The social surplus is higher (because money come from nowhere here) but much less than in the last two scenarios.

These simulation exercises would be very interesting to simulate the impact of taxation and family policy programs on matching patterns and labor supplies. This would require the introduction of taxation and children. I propose in the two following subsections a way to introduce these two important features.

7 Extensions

7.1 Extension to taxation

Many countries use joint taxation: taxes are based on the household income level and not on the individual income. Even in countries which use individual taxation as a basis, there can be a bit of joint taxation to give some benefits to low income families. The estimation of collective models with taxation is a little bit trickier. Donni ([16],2003) and Donni and Moreau ([25],2002) showed that the decentralization process still applies but needs additional concepts as shadow wages and shadow non labor income. The household budget constraint with taxation is

$$C_i + C_j + C_c \leq g[w_i(T - d_i - L_i m) + w_j(T - d_j - L_j)]$$

with g representing the total labor income revenue net of taxation. Donni defines shadow wages ω_m and ω_f as

$$\omega_{mij} = w_i g'[w_i(T - d_{mij} - L_i) + w_j(T - d_{mij} - L_j)]$$
$$\omega_{fij} = w_j g'[w_i(T - d_{mij} - L_i) + w_j(T - d_{fij} - L_j)],$$

And the shadow non labor income as

$$\eta = g[w_i(T - d_{mij} - L_i) + w_j(T - d_{fij} - L_j)] - \omega_{mij}(T - d_{mij} - L_i) - \omega_{fij}(T - d_{fij} - L_j).$$

The household decentralization process is the following. First the members bargain over the quantity of domestic production they want to produce and about the sharing rule such that

$t_{mij} + t_{fij} = \eta$. Then each of them maximizes his own utility under his budget constraint

$$\max_{c_i, l_i} u_{mi}(C_i, L_i)$$
$$s.c\ C_i \leq \omega_{mij}(T - L_i - d_{ij}) + t_{mijz}.$$

If we consider income support for low income family, we could have a non convex budget set and it would be difficult to solve analytically the model (Salanié, 2003, [26]). However, if we consider a negative marginal tax rate (as for instance, we can consider the WFTC in the UK) for low income household when they are working, we may still have a convex budget set. In this case, the model can be derived similarly. Wages are replaced by their shadow wages. Equations of resulting surplus and transfers are derived in appendix.

7.2 Extension to children

Children are not taken into account in this setting. However, I propose a way to introduce children in that kind of model at the cost of two additional strong hypothesis. First, I assume that when two people decide to match, they immediately and necessarily have children. Second when a couple separates, it is always the woman who keeps the children[22]. The marriage market is then composed of single men without children, single women with or without children and couples with children. The model is still identified.

In this variation, I assume that women's preferences for leisure and consumption are the same for women with children than for women without children. However, I assume that having children incur a cost for both single mothers and couples with children. Single men without children and single women without children do not pay this cost. Finally I assume that married and single mothers value similarly domestic production with the same preference parameter κ_{fc} and need the same minimum quantity of housework D_{0c}. The present value of single mothers is different from the present value of single women without children. The outside option for married women is now to be a single mother, which changes a little bit the bargaining terms. I present the program of a single woman in appendix as well the new Nash bargaining and the resulting surplus.

The equilibrium on the market is also different. Single men can match with single women without children and with single women with children. Then there exist two different match probability. $a(x, y)$ is the match probability of a single woman of type j without children with a single man of type i when she meets him whereas $a(x, y)^c$ is the match probability of a single mother of type j with a single man of type i when she meets him. The equilibrium condition becomes

$$\lambda_z(1 - a_{ij}^c)Nn_{ij} = U_m u_m(i)\lambda(U_f u_f(j)a_{ij} + U_f^c u_f^c(j)a_{ij}^c)$$

Then, we can still recover s_{ij}^c from data by adding hypothesis on the distribution on z and derive the model. I present the solution in appendix. The estimation is however quite cumbersome.

[22] In my data, around 65 % of couples have children, 25 % of single women have children whereas less than 2 % of single men have children. Cf figure 22 in Appendix

8 Conclusion

This paper proposes a model which identifies the impact of matching preferences and marital sorting on intra-household allocation and labor supply.
Considering matching pattern, this model identifies the total surplus formed by a match. I disentangle what comes from preferences and complementarities of characteristics and what comes from resource sharing, and productivity. I show that if total surplus increases in wages of both members of the household, complementarities in characteristics can be higher for same wage couples.
Relative wages and family value indexes have a large impact on the allocation of resources. High wage women and conservative women get higher share of the surplus of the couple.
Finally, I show that bargaining effects are significant on labor supply of married individuals. At an aggregate level, bargaining effects reduce labor supply of married women by 2 hours a week and increase married men labor supply by one hour a week.
It is important to take heterogeneity into account. The analysis of the evolution over 18 years on the BHPS shows that welfare of married women and within household inequalities of resources have remained stable over these years. I show that men preferences for leisure have increased.
Finally, simulations show that initial distribution of characteristics have strong impact on matching patterns and resulting labor supplies which confirm the need to model the marriage market together with the sharing rule. These results call for further research in modeling exhaustive participation and fertility decisions.

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APPENDIX 1 : proof of proposition 2.1

Using equation (3) the surplus of a marriage for a i-type man with a j-type woman is

$$P_{mi}S_{mijz} = \frac{F(d_{mij}, d_{fij})(\Phi_{ij} + z)\left(w_i(T - d_{mij}) + t_{mijz} - \mathcal{C}_{0mi}\right) - rP_{mi}W_{mi} + \lambda_z \int_{z'} \max(P_{mi}S_{mijz}, 0)dG(z')}{r + \lambda_z},$$

then we obtain the following formulas for transfers using equation (4)

$$\begin{aligned} t_{mijz}F(d_{mij}, d_{fij})(\Phi_{ij} + z) &= (r + \lambda_z)\beta S(i,j,z) + rP_{mi}W_{mi} - (w_i(T - d_{mij}) - \mathcal{C}_{0mi})F(d_{mij}, d_{fij})(\Phi_{ij} + z) \\ &\quad - \lambda_z \int_{z'} \max(P_{mi}S_{mijz}, 0)dG(z') \qquad (14) \\ t_{fijz}F(d_{mij}, d_{fij})(\Phi_{ij} + z) &= (r + \lambda_z)(1 - \beta)S(i,j,z) + rP_{fj}W_{fj} - (w_j(T - d_{fij}) - \mathcal{C}_{0fj})F(d_{mij}, d_{fij})(\Phi_{ij} + z) \\ &\quad - \lambda_z \int_{z'} \max(P_{fj}S_{fijz}, 0)dG(z'). \end{aligned}$$

As we have $t_{mijz} + t_{fijz} = -C_c$, we can compute the total surplus by summing the last two equations

$$\begin{aligned} &F(d_{mij}, d_{fij})(\Phi_{ij} + z)(w_i(T - d_{mij}) + w_j(T - d_{fij}) - \mathcal{C}_{0mi} - \mathcal{C}_{0fj} - C_c) \\ &= (r + \lambda_z)S(i,j,z) + rP_{mi}W_{mi} + rP_{fj}W_{fj} - \lambda_z \int_{z'} \max(S_{ijz}, 0)dG(z'). \qquad (15) \end{aligned}$$

Using $R_{ij} = w_i(T - d_{mij}) + w_j(T - d_{fij}) - \mathcal{C}_{0mi} - \mathcal{C}_{0fj} - C_c$, in equation (15), we obtain

$$(\Phi_{ij} + z)F(d_{mij}, d_{fij})R_{ij} = (r + \lambda_z)S(i,j,z) + rP_{mi}W_{mi} + rP_{fj}W_{fj} - \lambda_z \int_{z'} \max(S_{ijz}, 0)dG(z').$$

Assuming $R_{ij} > 0$ and $F(d_{mij}, d_{fij}) > 0$, we get

$$S(i,j,z) = \frac{F(d_{mij}, d_{fij})R_{ij}}{r + \lambda_z}\left(\Phi(i,j) + z - \frac{rP_{mi}W_{mi} + rP_{fj}W_{fj} - \lambda_z \int_{z'} \max(S(i,j,z'), 0)dG(z')}{F(d_{mij}, d_{fij})R_{ij}}\right).$$

Then using the formula (14), we obtain the expressions for transfers.

APPENDIX 2 : Additional data description

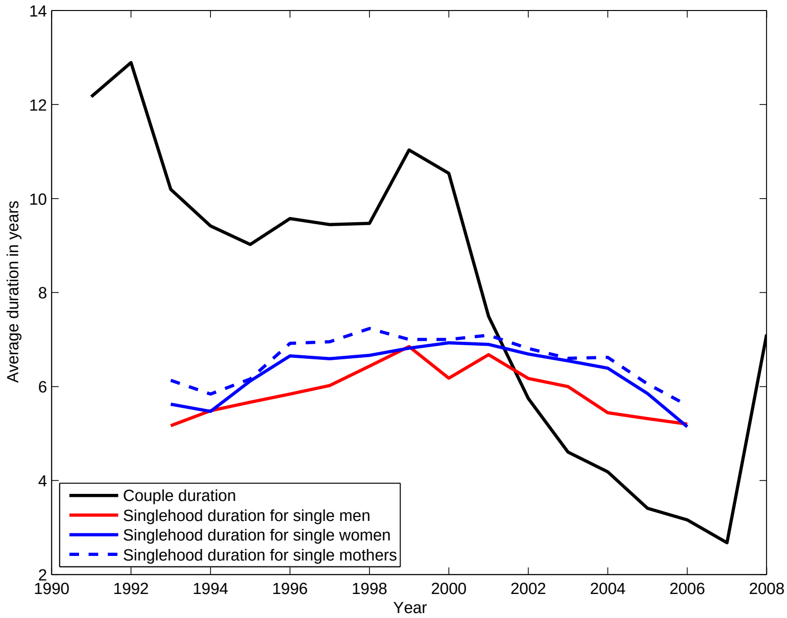


Figure 19: Evolution of the average observed duration of couples and singlehood from 1991 to 2008. Author's computations from the BHPS population of employed people aged 22-40.
Lecture : in 1992, the average partnership duration of couples for which we observe the total duration (date of formation and date of separation) is 13 years.

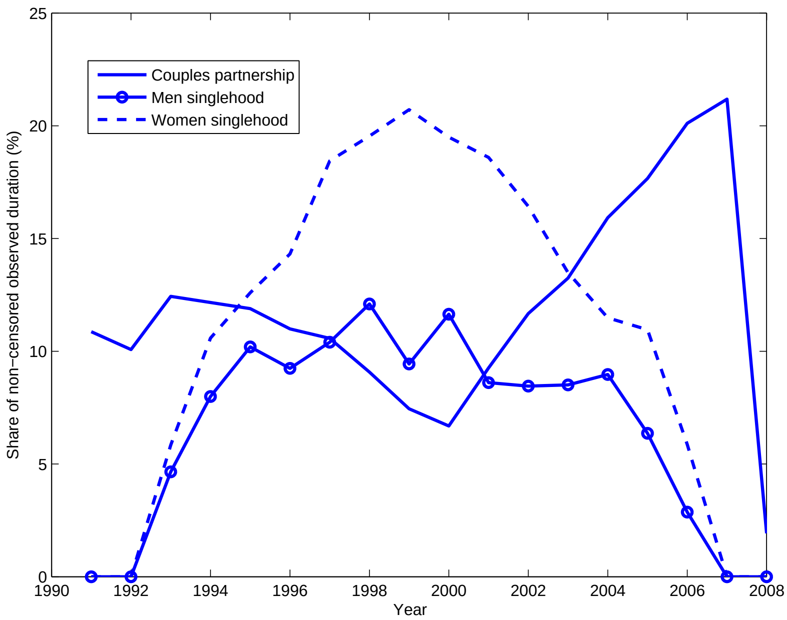


Figure 20: Evolution of the share of total duration observed from 1991 to 2008. Author's computations from the BHPS population of employed people aged 22-40
Lecture : in 1992, we observe the total partnership duration of 10 % of couples.

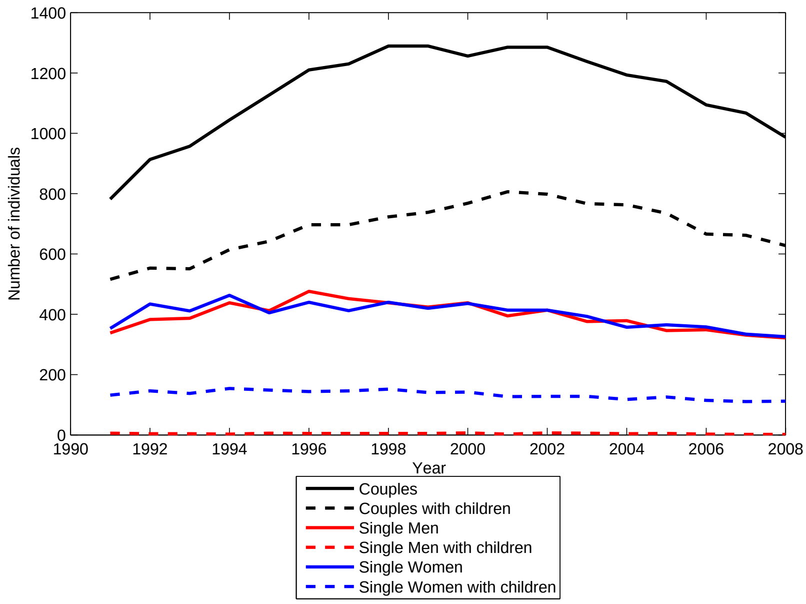


Figure 21: Evolution of sample size from 1991 to 2008. Author's computations from the BHPS population of employed people aged 22-40.

Lecture : in 1992, the sample is composed of 913 couples (of which 553 have children), 383 single men (of which 4 have children) and 434 single women (of which 146 have children)

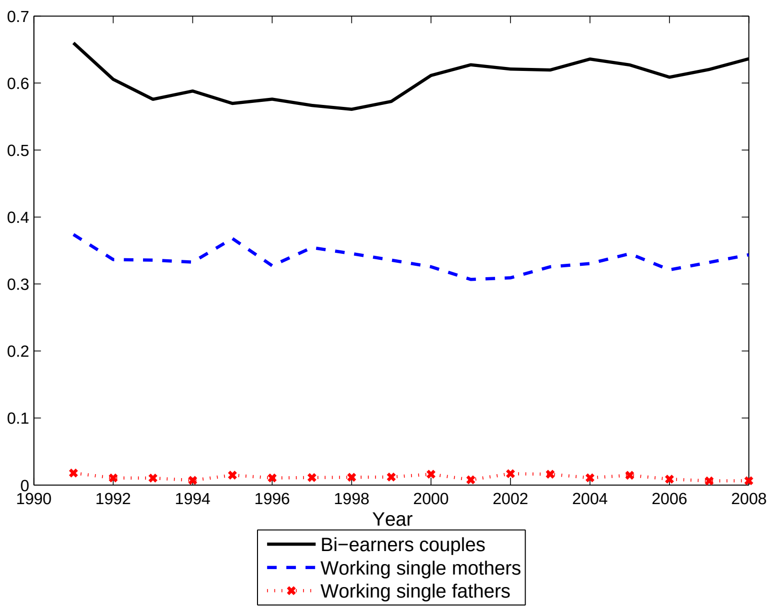
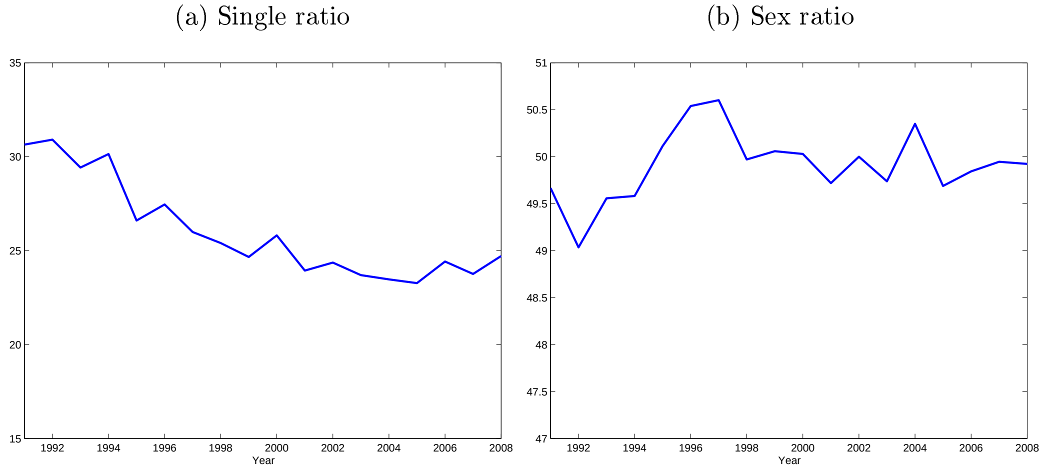


Figure 22: Evolution of sample size from 1991 to 2008. Author's computations from the BHPS population of employed people aged 22-40.

Figure 23: Evolution of the sample composition from 1991 to 2008. BHPS



Author's computations from the BHPS population of employed people aged 22-40

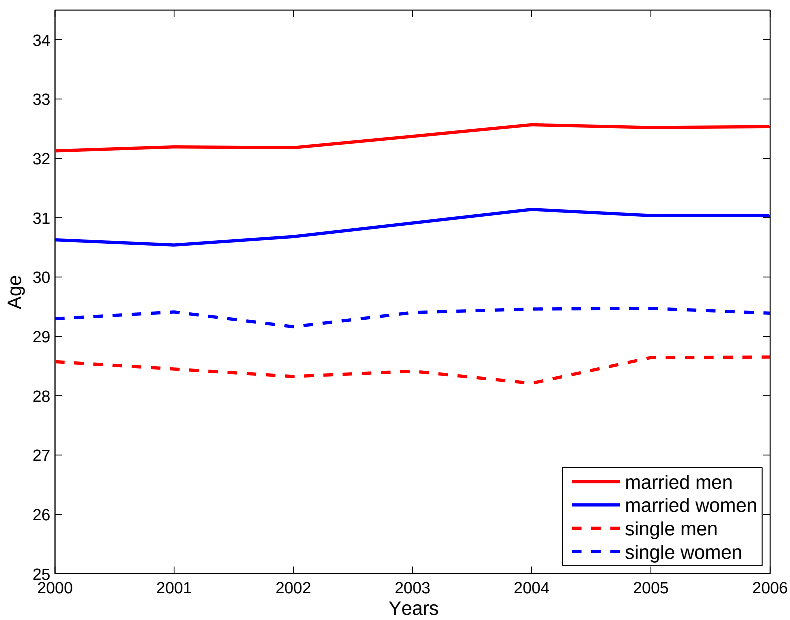


Figure 24: Evolution of mean age from 1991 to 2008. Author's computations from the BHPS population of employed people aged 22-40

Lecture : in 1992, single men are 28.5 year old in average whereas married men are 31.8, single women are 29 year old and married women are 30 year old in average.

(a) Employment rate for all (b) Employment rate according to marital status

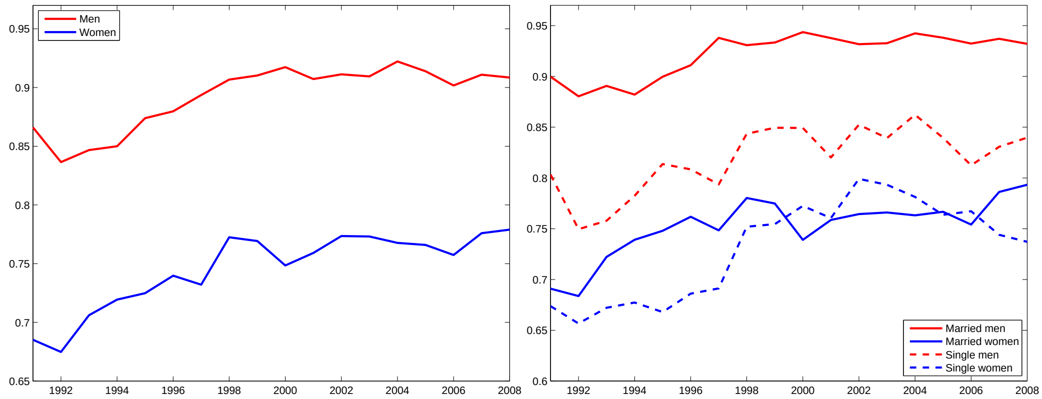


Figure 25: Evolution of the employment rate from 1991 to 2008. Author's computations from the BHPS population of people aged 22-40

Lecture : in 2000, 75 % of women and 90 % of men have a job. 85 % of single men and 93 % of married men have a job.

(a) Domestic work ratio (b) Wage ratio

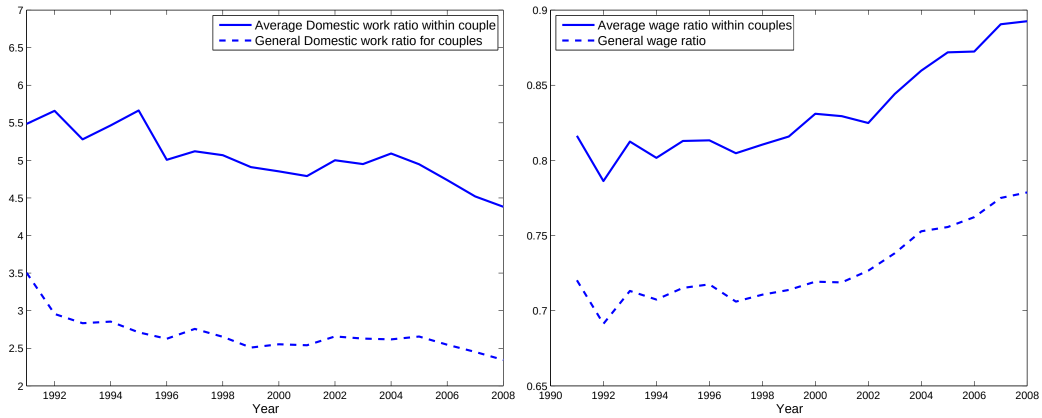


Figure 26: Evolution of within household ratio from 1991 to 2008. Author's computations from the BHPS population of employed people aged 22-40 living in couples. The within domestic work ration of couples is the average of the ratio of women domestic work over their husband domestic work. The general domestic work ratio is the ratio of the average domestic work of married women of the average domestic work of married men. This is the ratio generally used in macroeconomic studies.

APPENDIX 3 : Additional results

Figure 27: Evolution of labor supplies with 1999 preferences

(a) Men (b) Women

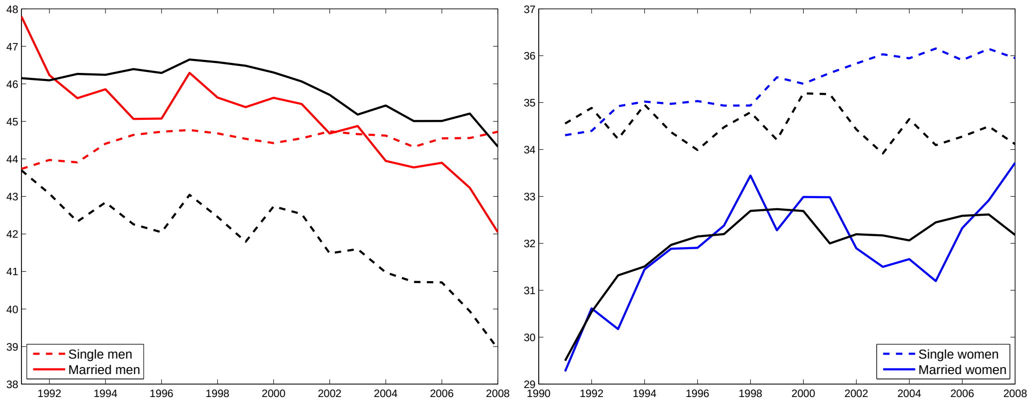
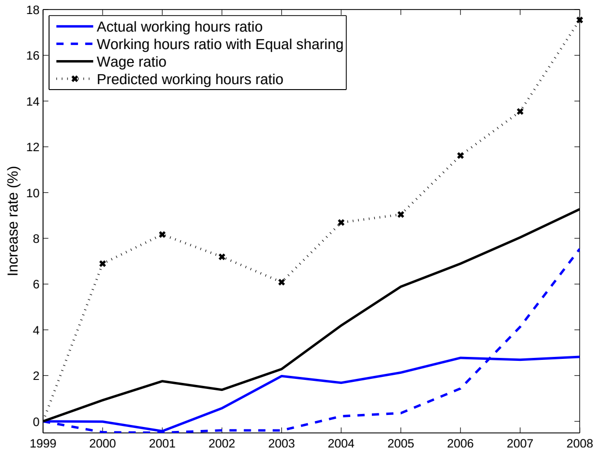


Figure 28: Increase rate of working hours ratio and wage ratio with 1999 preferences



APPENDIX 4 : Computational details

The computational method is the one used by Jacquemet and Robin ([20], 2012). It is adapted to fit a 4D-dimensional model. All functions are discretized on a compact domain using Tchebychev grids. For example, let $[\underline{x}, \overline{x}]$ denote the support of male wages, I construct a grid of $n+1$ points as

$$x_j = \frac{\underline{x} + \overline{x}}{2} + \frac{\overline{x} - \underline{x}}{2} \cos(\frac{j\pi}{n}),\ j = 1 \dots n$$

To estimate wage densities $n(x, y)$, $u_m(x)$ and $u_f(y)$ on those grids, I use kernel density estimators with twice the usual bandwidth to smooth the density functions in the tails. This is important as, for instance, I divide n by $u_m u_f$ to calculate a. Additional smoothing is thus required. To be computed on Matlab, we must have $10^{-16} < a(i, j) < 1$.

8.1 The Clenshaw-Curtis quadrature

Many equations involve integrals. Given Tchebychev grids, it is natural to use Clenshaw-Curtis quadrature to approximate these integrals. The Clenshaw-Curtis method allows to calculate quadrature weights w'_k such that

$$\int_{-1}^{1} f(x)dx = \sum_{k=0}^{N} w'_k f(cos(\theta_k)) + R_n$$

with R_n, an approximation error. The quadrature weights are

$$w_0 = \frac{1}{N}\left(1 + \sum_{j=1}^{\frac{N}{2}} \frac{2}{1-(2j)^2}\right)$$

$$w_{\frac{N}{2}} = \frac{1}{N}\left(1 + \sum_{j=1}^{\frac{N}{2}} \frac{2(-1)^j}{1-(2j)^2}\right)$$

$$w_k = \frac{2}{N}\left(1 + \frac{(-1)^k}{1-N^2} + \sum_{j=1}^{\frac{N}{2}-1} \frac{2}{1-(2j)^2} cos\left(\frac{2jk\pi}{N}\right)\right) \forall k = 1, ..., \frac{N}{2} - 1$$

Algorithm of Jorg Waldvogel

J.Waldvogel ([28],2006) derives a simple algorithm to obtain the weights of the Clenshaw-Curtis quadrature using matrices, Féjer'quadrature and Discrete Fourier Transfor. He shows that the weights $w = (w_0, w_1, ..., w_{N-1})$ of the Clenshaw-Curtis quadrature rule are given by the inverse discrete Fourier transform of the vector $v+g$, where g and v are defined below, and with $w_0 = w_N$.

$$v_k = \frac{2}{1-(2k)^2} \;,\; k = 0, 1, ..., \left[\frac{N}{2}\right] - 1,$$

$$v_{\left[\frac{N}{2}\right]} = \frac{N-3}{2\left[\frac{N}{2}\right] - 1} - 1$$

$$v_{n-k} = v_k \;,\; k = 0, 1, ..., \left[\frac{N-1}{2}\right]$$

$$g_k = -w_0 \;,\; k = 0, 1, ..., \left[\frac{N}{2}\right] - 1,$$

$$g_{\left[\frac{N}{2}\right]} = w_0\left[(2 - mod(N, 2))N - 1\right]$$

$$g_{n-k} = g_k \;,\; k = 0, 1, ..., \left[\frac{N-1}{2}\right]$$

The matlab code is then given by

```
   function [x,wcc] = ccquad(n)
% Clenshaw-Curtis quadratures by DFTs n>1
```

```
% Nodes:  x_k = cos(kπ/n), k = 0,...,n
% wcc = weights
% Compute ∫_{-1}^{1} f(x)dx = f * wcc for f = [f(x_0)...f(x_n)]

K = [0 : n]'; x = cos(Kπ/n);
N = [1 : 2 : n − 1]'; l = length(N); m = n − l;
v0 = [2./N./(N − 2); 1/N(end); zeros(m, 1)];
v2 = −v0(1 : end − 1) − v0(end : −1 : 2);

%Clenshaw-Curtis nodes: k = 0,1,...,n; weights: wcc, wcc_n = wcc_0
g0 = −ones(n, 1); g0(1 + l) = g0(1 + l) + n; g0(1 + m) = g0(1 + m) + n;
g = g0/(n^2 − 1 + mod(n, 2)); wcc = real(ifft(v2 + g));
wcc = [wcc; wcc(1)];
```

8.2 Kernel density estimation

To estimate the wage density of single men, I consider the sample of observed wages of single men : $(x_1, x_2, \ldots, x_{U_m})$. I assume it is an i.i.d sample drawn from some distribution with an unknown density u. Its kernel density estimator is :

$$\hat{u}_h(x) = \frac{1}{U_m h} \sum_{i=1}^{U_m} K\left(\frac{x - x_i}{h}\right)$$

where $K()$ is the kernel. It is a symmetric but not necessarily positive function that integrates to one. I use the normal kernel $K(x) = \phi(x)$, where ϕ is the standard normal density function. This function is estimated over the points x which are the nodes of the clenshaw-curtis quadrature previously exposed.

The bandwith $h > 0$ is a smoothing parameter. Intuitively one wants to choose h as small as the data allows, however there is always a tradeoff between the bias of the estimator and its variance. The optimal bandwidth is of order of $U_m^{-1/5}$, it minimizes the asymptotic mean integrated squared error which is then of order of $U_m^{-4/5}$.

I double this bandwidth to smooth my data.

Non-parametric estimation of joint density To estimate the joint density of wages for couples, $n(x, y)$, I consider the sample of observed wages of N couples. The estimation of the joint density of x and y, $n(x, y)$ on the N points (x_i, y_i) could be made with the Parzen Rosenblatt estimator on $\mathbb{R}^2$

$$\hat{n}_h(x, y) = \frac{1}{Nh^2} \sum_{i=1}^{N} \mathbf{K}\left(\frac{1}{h}\begin{pmatrix} x_i - x \\ y_i - y \end{pmatrix}\right)$$

With $\mathbf{K}$ should be a Parzen-Rosenblatt kernel of $\mathbb{R}^2$ which means a function defined on $\mathbb{R}^2$ integrable and integrates to 1. $\mathbf{K}$ is bounded and we have

$$\lim_{||x|| \to +\infty} ||x||^2 \mathbf{K}(x) = 0$$

K could also be decomposed as : $\mathbf{K}(x, y) = \mathbf{K}_1(x)\mathbf{K}_1(y)$, with $\mathbf{K}_1$: $\mathbb{R} \to \mathbb{R}$. Then $\mathbf{K}_1$ could be the probability distribution function of a standard normal distribution.
Then we have

$$\hat{f}_h(x, y) = \frac{1}{Nh^2} \sum_{i=1}^{N} \mathbf{K}_1 \left(\frac{x_i - x}{h} \right) \mathbf{K}_1 \left(\frac{y_i - x}{h} \right)$$

This function is then evaluated for N values of x and y (the nodes of the Clenshaw-Curtis quadrature).

Non-parametric estimation of : $\mathbb{E}(H|x, y)$ In this paper, I need to estimate the expected value of working hours of women conditional on their wage y and on their husband's wage x, We observe the value taken by three variables x, y and H on a population of size N. Then, the estimation of $\mathbb{E}(H|x, y)$ could be made with the Nadaraya-Watson estimator

$$\widehat{\mathbb{E}}(H|x, y)_h = \frac{\sum H_i \mathbf{K} \left(\frac{1}{h} \begin{pmatrix} x_i - x \\ y_i - y \end{pmatrix} \right)}{\sum \mathbf{K} \left(\frac{1}{h} \begin{pmatrix} x_i - x \\ y_i - y \end{pmatrix} \right)}.$$

$\mathbf{K}$ is a bidimensional Parzen Rosenblatt kernel which can be decomposed as : $\mathbf{K}(x, y) = \mathbf{K}_1(x)\mathbf{K}_1(y)$. And $\mathbf{K}_1$ could be the probability distribution function of a standard normal distribution ($x \to \frac{1}{\sqrt{2\pi}} e^{-x^2}$).

$$\widehat{\mathbb{E}}(H|x, y)_h = \frac{\sum_{i=1}^{N} H_i \mathbf{K}_1 \left(\frac{x_i - x}{h} \right) \mathbf{K}_1 \left(\frac{y_i - x}{h} \right)}{\sum_{i=1}^{N} \mathbf{K}_1 \left(\frac{x_i - x}{h} \right) \mathbf{K}_1 \left(\frac{y_i - x}{h} \right)}$$

In this paper, I use the generalization of these estimations to 4 variable functions as I consider the joint density of four characteristics (w_i, f_i, w_j, f_j) in couples.

8.3 Interpolation

The fact that CC quadrature relies on Tchebychev polynomials of the first kind also allows us to interpolate functions very easily between points $y_0 = f(x_0), \ldots, y_n = f(x_n)$ using Discrete Cosine Transform (DCT).

$$f(x) = \sum_{k=0}^{n} Y_k T_k(x) \tag{16}$$

where Y_k are the OLS estimates of the regression of $y = (y_0, \ldots, y_n)$ on Tchebychev polynomials

$$T_k(x) = \cos\left(k \arccos\left(\frac{x - \frac{\underline{x}+\overline{x}}{2}}{\frac{\overline{x}-\underline{x}}{2}}\right)\right)$$

but are more effectively calculated using FFT. A MATLAB code for DCT is, with $y = (y_0, \ldots, y_n)$:

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
f = @(x) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))*(0:n))*Y(1:n+1);
```

A bidimensional version is

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
Y = Y(:,[1:n+1 n:-1:2]);
Y = real(fft(Y'/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)]';
f=@(x,y) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))*(0:n))*Y(1:n+1,1:n+1)...
        *cos((0:n)'*acos((2*y'-(ymin+ymax))/(ymax-ymin)));
```

I also use a 4D dimensional version to evaluate transfers which depend on 4 variables : w_m, w_f, f_m, f_f. The fact that the grid $(x_0, \ldots, x_n)$ is not uniform and is denser towards the edges of the support interval allows to minimize the interpolation error and thus avoids the standard problem of strong oscillations at the edges of the interpolation interval (Runge's phenomenon). Another advantage of DCT is that, having calculated $Y_0, \ldots, Y_n$, then polynomial projections of $y = (y_0, \ldots, y_n)$ of any order $p \leq n$ are obtained by stopping the summation in (16) at $k = p$. Finally, it is easy to approximate the derivative f' or the primitive $\int f$ simply by differentiating or integrating Chebyshev polynomials using

$$cos(k \arccos x)' = \frac{k \sin(k \arccos x)}{\sin(\arccos x)}$$

and

$$\begin{aligned} \int cos(k \arccos x) &= x \textbf{ if } k = 0 \\ &= x^2/2 \textbf{ if } k = 1 \\ &= \frac{\cos((k+1)x)}{2(k+1)} - \frac{\cos((k-1)x)}{2(k-1)} \textbf{ if } k \geq 1 \end{aligned}$$

In calculating an approximation of the derivative, it is useful to smoothen the function by summing over only a few polynomials. Derivatives are otherwise badly calculated near the boundary.

APPENDIX 5 : Extension to children

8.4 Present value

The program for single mothers is

$$\max_{d,C,L} (d - D_{0c})^{\kappa_{fc}} (C - \mathfrak{C}_{0fj})^{\alpha_{fj}} (L - \mathfrak{L}_{0fj})^{1-\alpha_{fj}}$$
$$s.c\ C \leq w_i(T - L - d) - C_c,$$

with $D_{0c} \neq D_{0f}$ and $\kappa_{fc} \neq \kappa_f$.
Then the indirect utility for a j-type single women with children is

$$v^c_{fj} = \frac{(d_f - D_{0c})^{\kappa_{fc}} (w_j(T - d_{fij}) - \mathcal{C}_{0f}(w_j) - C_c)}{P_f(w_j)}.$$

The single present value for a single women without children reads

$$rW_{fj} = v_{fj} + \lambda U_m \int_z \int_i \max(W_{fijz} - W_{fj}, 0) \mathbb{1}(W_{mijz} > W_{mi}) u_m(i) dG(z).$$

The single present value for a single women with children reads

$$rW^c_{fj} = v^c_{fj} + \lambda U_m \int_z \int_i \max(W_{fijz} - W^c_{fj}, 0) \mathbb{1}(W_{mijz} > W_{mi}) u_m(i) dG(z).$$

A single man can now meet either a woman without children or a single mother. His present value is

$$rW_{mi} = v_{mi} + \lambda U_f \int_z \int_j \max(W_{mijz} - W_{mi}, 0) \mathbb{1}(W_{fijz} > W_{fj}) u_f(j) dG(z)$$
$$+ \lambda U_{fc} \int_z \int_j \max(W_{mijz} - W_{mi}, 0) \mathbb{1}(W_{fijz} > W^c_{fj}) u_{fc}(j) dG(z).$$

When the couple breaks up, the woman becomes a single mother and the man becomes a single man without children. The present values of members of a couple are:

$$rW_{fijz} = v_{fijz} + \lambda_z \int_z \max(W_{fijz'} - W^c_{fj}, 0) \mathbb{1}(W_{mijz} > W_{mi}) u_m(i) dG(z)$$
$$rW_{mijz} = v_{mijz} + \lambda_z \int_z \max(W_{mijz'} - W_{mj}, 0) \mathbb{1}(W_{fijz} > W^c_{fj}) u_f(j) dG(z).$$

8.5 Surplus, Nash bargaining and transfers

When the two members of the couple bargain, the outside option for the man is still his single present value whereas the outside option of the woman is now a single mother present value. The Nash bargaining is now modeled by the following program

$$\max (W_{mijz'} - W_{mi})^{\beta} (W_{fijz'} - W^c_{fj})^{1-\beta}$$
$$s.c\ t_m + t_f = -C_c,$$

whose solution gives

$$P_{mi}S_{mijz} = P_{mi}(W_{mijz} - W_{mi}) = \beta S(i,j,z) \tag{17}$$
$$P_{fj}S_{fijz} = P_{fj}(W_{fijz} - W^c_{fj}) = (1-\beta)S(i,j,z).$$

where $S(i,j,z)$ is still linear in z and equals $\frac{R_{ij}F(d_{mij},d_{fij})}{r+\lambda_z}(s^c_{ij}+z)$.
When a single woman without children meets a man, she knows that she will get children if she marries him and that her outside option will be the one of a single mother. In this case, the surplus for a woman without children is different for a woman with children. And a match of a woman without children of type j with a man of type i under circumstance z can be valuable for the man and not for the woman, whereas it would have been valuable for a single mother of type j. The match probability between a single mother of type j who meets a single man of type i is

$$a^c_{ij} = \mathbb{P}(W_{fijz} - W^c_{fj} > 0) = \mathbb{P}(z > -s^c_{ij}),$$

whereas the probability that a woman type j without children match with a man of type i when she meets one can be written :

$$a_{ij} = \mathbb{P}(W_{fijz} - W_{fj} > 0 \;\&\; W_{mijz} - W_{mi} > 0).$$

After some algebra[23] we can similarly write

$$a_{ij} = \mathbb{P}\left(z > -s^c_{ij} + \max\left(0, (W_{fj} - W^c_{fj})\frac{P_{fj}(r+\lambda_z)}{R_{ij}F(d_{mij},d_{fij})(1-\beta)}\right)\right).$$

This last expression shows us that a match will be more valuable for a single woman without children if her single value with a child is large. It reminds us the result of the job market search : the reservation wage of non-participants lowers when the unemployment benefits for the unemployed rise.
$(W_{fj} - W^c_{fj})$ is complex to derive and can be computed recursively as follows.

[23]

$$\begin{aligned}
a_{ij} &= \mathbb{P}(W_{fijz} - W_{fj} > 0 \;\&\; W_{mijz} - W_{mi} > 0) \\
&= \mathbb{P}(W_{fijz} - W^c_{fj} + W^c_{fj} - W_{fj} > 0 \;\&\; W_{fijz} - W^c_{fj} > 0) \\
&= \mathbb{P}\left(\frac{(1-\beta)R_{ij}F(d_{mij},d_{fij})}{(r+\lambda_z)P_{fj}}(s^c_{ij}+z) + W^c_{fj} - W_{fj} > 0 \;\&\; z > -s^c_{ij}\right) \\
&= \mathbb{P}\left(z > (W_{fj} - W^c_{fj})\frac{P_{fj}(r+\lambda_z)}{R_{ij}F(d_{mij},d_{fij})(1-\beta)} - s^c_{ij} \;\&\; z > -s^c_{ij}\right)
\end{aligned}$$

$$
\begin{aligned}
rW_{fj} - rW^c_{fj} &= v_{fj} - v^c_{fj} + \lambda U_m \int_z \int_i \max(W_{fijz} - W_{fj}, 0)\mathbb{1}(W_{mijz} > W_{mi}) u_m(i) dG(z) \\
&\quad - \lambda U_m \int_z \int_i \max(W_{fijz} - W^c_{fj}, 0)\mathbb{1}(W_{mijz} > W_{mi}) u_m(i) dG(z) \\
&= v_{fj} - v^c_{fj} + \lambda U_m \int_z \int_i (\max(W_{fijz} - W_{fj}, 0) - \max(W_{fijz} - W^c_{fj}, 0))\mathbb{1}(W_{fijz} > W^c_{fj}) u_m(i) dG(z) \\
&= v_{fj} - v^c_{fj} + \lambda U_m \int_z \int_i (\max(W_{fijz} - W_{fj}, 0) - W_{fijz} + W^c_{fj})\mathbb{1}(W_{fijz} > W^c_{fj}) u_m(i) dG(z) \\
&= v_{fj} - v^c_{fj} + \lambda U_m (W^c_{fj} - W_{fj}) \int_i a_{ij} u_m(i) - \lambda U_m \int_z \int_i (W_{fijz} - W^c_{fj})\mathbb{1}(W_{fj} > W_{fijz} > W^c_{fj}) u_m(i) dG(z),
\end{aligned}
$$

then

$$
\begin{aligned}
\left(r + \lambda U_m \int_i a_{ij} u_m(i)\right) P_{fj}(W_{fj} - W^c_{fj}) &= P_{fj}(v_{fj} - v^c_{fj}) - \lambda U_m \int_z \int_i (P_{fj} W_{fijz} - P_{fj} W^c_{fj})\mathbb{1}(0 < W_{fijz} - W^c_{fj} < W_{fj} - W^c_{fj}) u_m(\\
&= P_{fj}(v_{fj} - v^c_{fj}) - \lambda U_m \frac{1-\beta}{(r+\lambda_z)} \int_i R_{ij} F(d_{mij}, d_{fij}) \int_{z > -s^c_{ij}}^{z > -s^c_{ij} + \frac{P_{fj}(W_{fj} - W^c_{fj})(r+\lambda_z)}{(1-\beta) R_{ij} F(d_{mij}, d_{fij})}} (s^c_{ij} + z)
\end{aligned}
$$

with

$$
\int_{z > -s^c_{ij}}^{z > -s^c_{ij} + \frac{P_{fj}(W_{fj} - W^c_{fj})(r+\lambda_z)}{(1-\beta) R_{ij} F(d_{mij}, d_{fij})}} (s^c_{ij} + z) dG(z) =
$$

$$
s^c_{ij}(a^c_{ij} - a_{ij}) + \sigma \left(\phi\left(\frac{s^c_{ij}}{\sigma}\right) - \phi\left(\frac{s^c_{ij}}{\sigma} + \frac{P_{fj}(W_{fj} - W^c_{fj})(r+\lambda_z)}{\sigma(1-\beta) R_{ij} F(d_{mij}, d_{fij})}\right)\right).
$$

We obtain a complex recursive formula for $W_{fj} - W^c_{fj}$. I use a fixed point algorithm to estimate it. First I suppose that $W_{fj} - W^c_{fj} = 0$ and I compute a_{ij} as follows

$$
\begin{aligned}
a_{ij} &= \mathbb{P}(W_{fijz} - W_{fj} > 0 \;\&\; W_{mijz} - W_{mi} > 0) \\
&= \mathbb{P}(W_{mijz} - W_{mi} > 0) \\
&= a^c_{ij},
\end{aligned}
$$

with

$$
\lambda_z (1 - a^c_{ij}) N n_{ij} = U_m u_m(i) \lambda (U_f u_f(j) a_{ij} + U^c_f u^c_f(j) a^c_{ij}).
$$

Then I find a new value for $W_{fj} - W^c_{fj}$ using

$$
W_{fj} - W^c_{fj} = \frac{v_{fj} - v^c_{fj}}{r + \lambda U_m \int_i a_{ij} u_m(i)}.
$$

Finally I use my new estimate as an initial value and resume the process until convergence.

APPENDIX 6 : Extension to taxation

Let R denotes labor income and τ_1, τ_2, τ_3 denote the subsidies rates. Let us consider the function g which represents the total labor income net of transfer and taxes such that

$$g(R) = R + \tau_1 R \mathbb{1}_{R \leq A_1} + \tau_2 R \mathbb{1}_{A_1 < R \leq A_2} + \tau_3 R \mathbb{1}_{R > A_2},$$

with $\tau_1 > \tau_2 > \tau_3 > 0$ and $g'(R) = \frac{g(R)}{R}$. Then the shadow wages and income are

$$\begin{aligned}
\omega_{mij} &= w_m(1 + \tau_1 \mathbb{1}_{R \leq A_1} + \tau_2 \mathbb{1}_{A_1 < R \leq A_2} + \tau_3 \mathbb{1}_{R > A_2}) \\
\omega_{fij} &= w_f(1 + \tau_1 \mathbb{1}_{R \leq A_1} + \tau_2 \mathbb{1}_{A_1 < R \leq A_2} + \tau_3 \mathbb{1}_{R > A_2} \\
\eta &= 0.
\end{aligned}$$

The indirect utility to be single remains

$$v_{mi} = \frac{(d_m - D_{0m})^{\kappa_m}(w_i(T - d_m) - \mathcal{C}_{0mi}(w_i))}{P_{mi}(w_i)},$$

and the indirect utility when married becomes

$$v_{mij} = (\Phi(i,j) + z)F(d_m, d_f)\frac{\omega_{mij}(T - d_{mij}) + t'_{mij} - C_{0mi}(\omega_{mij})}{P_{mi}(\omega_{mij})}.$$

Then the surplus equation becomes

$$s(i,j) = \Phi(i,j) - \frac{rP_m W_m + rP_f W_f}{F(i,j)Rij(\omega_i, \omega_j)} + \frac{\lambda_z}{r + \lambda_z}\int_{z'} \max(s(i,j) + z', 0)dz',$$

where the single present value reads

$$P_m(w_i)(rW_{mi} - v_{mi}) = \frac{\beta\lambda}{r + \lambda_z}\int_{fj} F(i,j)Rij(\omega_i, \omega_j)\mu(a(i,j))du_f(j),$$

and the new transfer is

$$t'_{mij} - \omega_{mij}d_{mij} = \beta Rij(\omega_i, \omega_j) + \frac{(1-\beta)rP_m W_m - \beta r P_f W_f}{F(i,j)(\Phi(i,j) + z)}.$$

If we assume that α_m, $\mathfrak{C}_{0m}$ and $\mathfrak{L}_{0mi}$ are constant across men and α_f, $\mathfrak{C}_{0f}$ and $\mathfrak{L}_{0f}$ are constant across women, then

$$\begin{aligned}
\mathcal{C}_{0mi} &= \mathfrak{C}_{0m} + w_i \mathfrak{L}_{0m} \\
P_{mi} &= \frac{w_i^{1-\alpha_m}}{\alpha_m^{\alpha_m}(1-\alpha_m)^{1-\alpha_m}}.
\end{aligned}$$

In that case we obtain $\frac{P'_{mi}}{P_{mi}} = \frac{1-\alpha_m}{w_i}$ and the labor supply equations for married men and single men rewrite

$$\begin{aligned}
h_{mijz} &= T - d_{mij} - \mathfrak{L}_{0m} - \frac{1-\alpha_m}{\omega_i}(\omega_i(T - d_{mij}) + t'_{mijz} - \mathfrak{L}_{0m}\omega_i - \mathfrak{C}_{0m}) \\
h_{mi} &= T - d_{mi0} - \mathfrak{L}_{0m} - \frac{1-\alpha_m}{w_i}(w_i(T - d_{m0}) - \mathfrak{L}_{0m}w_i - \mathfrak{C}_{0m}).
\end{aligned}$$

h_{mijz} can be rewritten

$$
\begin{aligned}
h_{mijz} &= T - d_{mij} - \mathfrak{L}_{0m} - \frac{1-\alpha_m}{\omega_i}(\omega_i(T-d_{mij}) + t'_{mijz} - \mathfrak{L}_{0m}\omega_i - \mathfrak{C}_{0m}) \\
&= T - d_{mij} - \mathfrak{L}_{0m} - \frac{1-\alpha_m}{g'w_i}(g'w_i(T-d_{mij}) + t'_{mijz} - \mathfrak{L}_{0m}g'w_i - \mathfrak{C}_{0m}) \\
&= T - d_{mij} - \mathfrak{L}_{0m} - \frac{1-\alpha_m}{w_i}(w_i(T-d_{mij}) + \frac{t'_{mijz} - \mathfrak{C}_{0m}}{g'} - \mathfrak{L}_{0m}w_i)
\end{aligned}
$$

and h_{mi} doesn't change. Then $\mathfrak{C}_{0m}, \mathfrak{L}_{0m}$, α_m, $\mathfrak{C}_{0f}, \mathfrak{L}_{0f}$ and α_f are still identifiable.

APPENDIX 7 : Estimation of σ_z

Assuming that affinity is constant over time, we can identify σ using different cohorts. Indeed, as Φ_{ij} is not totally proportional to σ because of the terms $\frac{P_f v_{fj} + P_m v_{mi}}{F(d_{mij}, d_{fij})R_{ij}}$, we can compare cohort t with cohort t' and subtracting the two affinity matrixes to obtain σ_z. Remind that

$$
\Phi(i,j) = s(i,j) + \frac{rP_{mi}W_{mi} + rP_{fj}W_{fj}}{F(d_{mij}, d_{fij})R_{ij}} - \frac{\lambda_z}{r+\lambda_z}\left(a(i,j)s(i,j) + \sigma_z\phi\left(\frac{s_{ij}}{\sigma_z}\right)\right)
$$

with

$$
s(i,j) = -\sigma_z\Phi^{-1}(1-a(i,j))
$$

The single present value for a i-type man is

$$
\begin{aligned}
rP_{mi}W_{mi} &= P_m v_{mi} + s_{mi} \\
&= P_m v_{mi} + \frac{\beta\lambda}{r+\lambda_z}\int_j F(d_{mij}, d_{fij})R_{ij}\left(s(i,j)a(i,j) + \sigma_z\phi\left(\frac{s(i,j)}{\sigma_z}\right)\right)u_f(j)U_f \\
&= P_m v_{mi} + \frac{\beta\lambda}{r+\lambda_z}\int_j F(d_{mij}, d_{fij})R_{ij}\left(-\sigma_z\Phi^{-1}(1-a(i,j))a(i,j) + \sigma_z\phi\left(\Phi^{-1}(1-a(i,j))\right)\right)u_f(j)U_f \\
&= P_m v_{mi} + \frac{\sigma_z\beta\lambda}{r+\lambda_z}\int_j F(d_{mij}, d_{fij})R_{ij}\left(-\Phi^{-1}(1-a(i,j))a(i,j) + \phi\left(\Phi^{-1}(1-a(i,j))\right)\right)u_f(j)U_f
\end{aligned}
$$

and similarly the single present value for a j-type woman is

$$
rP_{fj}W_{fj} = P_f v_{fj} + \frac{\sigma_z(1-\beta)\lambda}{r+\lambda_z}\int_j F(d_{mij}, d_{fij})R_{ij}\left(-\Phi^{-1}(1-a(i,j))a(i,j) + \phi\left(\Phi^{-1}(1-a(i,j))\right)\right)u_m(i)U_m
$$

Then

$$\Phi(i,j) = \sigma_z \left(-\Phi^{-1}(1-a(i,j)) \left(1 - \frac{\lambda_z}{r+\lambda_z} a(i,j)\right) - \frac{\lambda_z}{r+\lambda_z}\phi\left(\frac{s_{ij}}{\sigma_z}\right) + \frac{\frac{\beta\lambda}{r+\lambda_z}\int_j F(d_{mij}, d_{fij})R_{ij}\left(-\Phi^{-1}(1-a(i,j))a(i,j) + \phi\left(\Phi^{-1}(1-a(i,j))\right)\right)u_f(j)U_f}{F(d_{mij}, d_{fij})R_{ij}} + \frac{\frac{(1-\beta)\lambda}{r+\lambda_z}\int_j F(d_{mij}, d_{fij})R_{ij}\left(-\Phi^{-1}(1-a(i,j))a(i,j) + \phi\left(\Phi^{-1}(1-a(i,j))\right)\right)u_m(i)U_m}{F(d_{mij}, d_{fij})R_{ij}} \right) + \frac{P_f v_{fj} + P_m v_{mi}}{F(d_{mij}, d_{fij})R_{ij}}$$

APPENDIX 8 : Generalized Additive Models

This explanation has been developed in Chiappori, Ghandi, Salanié and Salanié ([12], 2012). Generalized additive models (GAM) were introduced by Hastie and Tibshirani (1986). They model a variable y_i by assuming that its distribution around its mean belongs to the exponential family and by modeling the mean as a sum of smooth functions of subvectors of the covariates (X_i). To estimate my GAM models, I use the methods described by Wood (2006); I use his implementation in the mgcv package of R, which incorporates the improved algorithm of Wood (2008). More precisely, one writes

$$\mathbb{E}(y_i|X) = \sum_{j=1}^{J} f_j(X_i^j)$$

where each X_i^j is a user-defined subvector of X_i, and the f_j are to be estimated; and the user also chooses the distribution of the error term $(y_i - \mathbb{E}y_i)$ within the exponential family. Modeling starts by choosing a rich family of basis functions (typically splines) (b_{jk}) for $k = 1...K_j$ with a maximal order K_j chosen large enough. Then

$$f_j(X_i^j) = \sum_{k=1}^{K_j} \beta_{jk} b_{jk}(X_i^j)$$

Finally, the generalized R^2 cited in the text are defined as the ratio $1 - \frac{\mathbb{E}\mathbb{V}(y|X)}{\mathbb{V}(y)}$