

Informality and government enforcement in Latin America*

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Abstract

This paper analyzes how the informality responds to the quality of the labor enforcement and the bundle of benefits that the formal workers receive in different countries of Latin America. Countries with different levels of informality were compared, highlighting the features that could induce these different levels. In a general equilibrium framework, the government chooses a level of government enforcement and a bundle of benefits maximizing the workers' utility subject to a budget constraint, a representative firm chooses the share of workers in formality and informality that they want to hire, and the workers offer a share of time in formality and informality. I estimate the main parameters of the model, the production function, the quality of government enforcement and the quality of benefits, for five countries: Argentina, Brazil, Colombia, Peru and Uruguay. Differences in the quality functions of the government enforcement and benefits are found, as well as in the fines established to enforce the agents.

JEL codes: E26, H53, O17, O54.

Keywords: Informality, labor regulation, government enforcement, Latin America.

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1 Introduction

1.1 Motivation

Latin America is one of the regions where informality has been identified in depth. About 50% of salaried workers¹ are employed informally, if we define informal workers as those who are not covered by labor regulation, such as taxation, the right to the health system and the right to a pension income in retirement (Portes et al. (1989) and Schneider (2012)). Informal work defines an underdevelopment phase of these economies and can be better understood as multicausal and complex, and in which the government can play a crucial role. Although informality is present in each country of the region and is also present among different levels of education, as is shown in Table 1, the figures are extremely heterogeneous between different countries.

The role of the government in informality can be conceived in three main dimensions: first, the administration receives taxes and contributions from the formal workers; second, the regulator monitors the firms looking for informal jobs, fines them and eliminates these jobs; and third, the administration brings benefits to the formal workers through the health system, pensions and unemployment payments. These three assignments are carried out in different degrees and lead to different levels of enforcement to firm and workers. **The aim of this paper is to shed light on how the labor market responds to the quality of the government enforcement and the quality of the bundle of benefits brought by the public institutions.** To reach this objective I compare the informality performance in different countries, given the quality of the enforcement of the public sector. **I develop a model to capture these relationships and I estimate it for five countries with different levels of informality in order to capture the heterogeneity.** These five countries are Argentina, Brazil, Colombia, Peru and Uruguay.

In this paper I will only measure the informality of those who declare that their employer does not pay the necessary contributions in order to give them the right to a pension in old age. The decision to use this definition is due to the fact that this question is present in all the household surveys and its consequences have been widely analyzed by the literature (Holzmann and Takayama (2009), Joubert (2012) and Ceni Gonzalez (2013)). Levels of informality are clearly heterogenous among countries not only at all the educative levels² but also

¹Salaried workers are those who has a salary paid by a employer.

²High educated workers are defined as those who at least finished high school, and the low

where other benefits are considered in the definition of informality, such as health benefits or the 13th salary as is shown in Table 2.

Country	Mean	Low education	High education
Argentina	36.1%	39.7%	15.2%
Brazil	21.6%	26.8%	14.3%
Colombia	42.7%	46.9%	14.8%
Peru	51.6%	66.8%	25.2%
Uruguay	19.4%	25.0%	6.5%

Table 1: Informality rates among salaried workers in 2009. Source: CEDLAS, World Bank.

Analyzing the nature of the informality, there are three strands in the literature. The first one has extensively claimed that there are two separate segmented markets, which have different rules, and have been related with low and high productivity sectors. From the firm point of view, formal and informal workers are treated as two different inputs. This concept has been discussed by the empirical literature using data from Mexico, Colombia, Argentina and Uruguay³, where the evidence suggests that it is the workers who decide whether to be formal or informal employees. The second strand focuses on the workers' individual decision to be in each sector given their characteristics as a unique labor market. Finally, the third one proposes a moderate dualism, which is considered in most of the recent theoretical papers. Workers and firms can decide to operate optimally either formally or informally. Galiani and Weinschelbaum (2007) present a model of an economy with a continuum of firms and workers in which formal and informal jobs and their wages are endogenous. The main feature which determines formality is the managerial ability which drives the difference in human capital between these two sectors.

Amaral and Quintin (2006) present a dynamic model where managers can either self-finance part of their capital with savings or borrow funds from an intermediary. In this way, the most talented managers self-select into the formal sector in which formal managers operate with more physical capital than informal managers and informal employers self-finance more intensely than the formal ones. The model reproduces the main macro elements of labor markets in developing nations.

The main controversial point in these models is that the worker's decision does not play any role in the equilibrium, which is in contrast to the literature

educated the one that did not.

³Magnac (1991), Maloney (2004), Pratap and Quintin (2006) and Bucheli and Ceni (2010)

which largely accepts that it does, especially in the middle and high educated workers. The central argument against this dualism is the mobility between these sectors, which is clearly observed in these economies, if workers move among sectors it means that their intrinsic characteristics are not so different, then it can be considered as the same input in the production function. Mobility is a permanent feature of this phenomenon (Fields, 2011). Indeed, there were a significant number of annual changes among sectors in Argentina between 2003 and 2011 (Table 3)

My model introduces a novel general equilibrium framework, where the three main characters in the economy: households, firms and government optimize the level of informality, government enforcement and the benefits that formal workers receive. Additionally, **I allow a loose definition of dualism through the definition of the production function of the representative firm which includes formal and informal workers, where the level of substitution is specifically estimated.** In contrast with the main papers in the literature, which focus on entrepreneurial ability and how it determines in which sector they develop their activity; firms hire both formal and informal workers contemporaneously, and enter in the production function with different levels of substitutability by education. If both inputs are perfect substitutes, the optimal is the corner solution and inputs can be treated as only one. But, if the level of substitutability is lower, both inputs coexist in the production function. Theoretically this is one of the main contributions of this paper.

Country	Pensions	Health	13th month	Holidays
Argentina	36.1%	35.9%	34.6%	34.4%
Brazil	21.6%	-	-	-
Colombia	42.7%	42.4%	53.7%	28.2%
Peru	51.6%	51.3%	-	-
Uruguay	19.4%	21.5%	25.3%	-

Table 2: Informality levels using different benefits in 2009. Source: CEDLAS.

1.2 Incentives: employees, firms and government

There are cost and benefits for those either in formality or informality, and these are borne by employees and employers. Firstly, in formality the employees are obliged to pay contributions and taxes, but they have the right to receive benefits in the present and in the future (such as the right to be covered by the health system, enjoy holidays, receive some extra payments and a pension

Probability of yearly change of sector			
	Unemployment	Formal	Informal
Unemployed (-1)	0.337	0.254	0.409
Formal (-1)	0.024	0.909	0.067
Informal (-1)	0.086	0.240	0.674

Table 3: Probability of change of sector in Argentina, based in the multinomial model 2003-2010. (only men)

for the elderly). However, some workers do not value some of these benefits, because the services which are provided are of poor quality, or the government commitment is too weak *i.e.* they believe that in the future a form of survival pension will be available for everyone. Additionally, informality is attractive for some workers because it is a more flexible sector, which allows easier entrance for unemployed workers or for those who want to acquire experience without signing a formal contract.

The World-Bank (2010) also focused on governmental policies designed to improve the life quality of the poorest population, such as health insurance or Conditional Cash Transfer programs. In the case of Colombia, the government carries a program to provide health services to the population below a formal income threshold, so the workers prefer to be informal because there are no additional (or there are lower) benefits to working formally.

Secondly, firms in informality do not pay any contributions and taxes because they are not monitored by the government or the fines are too low. Moreover, there is no social punishment because this is not viewed as a crime. However, in formality they benefit from government protection against probable abuse from criminal activities, such as blackmail (Loayza et al., 2009).

Finally, the role of government emerges as crucial in both academic and political discussion about informality. The weakness and corruption of governments play a negative role in the analysis. Conversely, there is a trade-off about the burden of taxes and regulations: on one hand stricter laws can dissuade firms and workers from formality, but, on other hand stricter regulation and monitoring should prevent informality (Loayza et al., 2009). Moreover, informality plays a role in these economies as a source of flexibility in the cyclical phase of recessions by absorbing the unemployment which could be generated by the impossibility of paying the cost of the formality. The counter cyclical behavior leads governments to allow this practice in order to moderate the

impact of the economic downturn (Loayza and Rigolini, 2011).

Ihrig and Moe (2004) analyse the size of the shadow economies with a two sector model when the taxes and the enforcement of the taxes change. The main objective of their paper is to explore the size and the dynamic of the shadow economy cross-country, and when the enforcement of the taxation changes exogenously. The definition of shadow economies is more reductive than informality because it only focus on the taxation side of the problem. Moreover, the government role is exogenous in contrast with my paper when the government choices are endogenous.

1.3 Research question

What are the **underlying differences** (e.g. in enforcement technology, the ability to provide public benefits, the production technology, the skill composition of the labor force) that explain jointly the **choice of governments on the level of government enforcement and benefits, and the reaction of firms and workers in terms of operating in formality/informality?** My aim is to compare the informality performance in different countries given the quality of the bundle of benefits, the costs, fines and quality of the government engorcement.

The main objective is to explain the heterogeneity, regarding the fact that this phenomenon is present in all countries in Latin America, and **explore the features that could explain these differences, focusing on costs, benefits and enforcement.** I estimate the main parameters which characterize each country (Argentina, Brazil, Colombia, Perú and Uruguay) in order to assess the differences in the informality levels.

This paper is structured as follows. In Section 2 I introduce the data and the main variables of the paper, Section 3 provides the model, with one period and two types of workers with different educative levels, Section 4 presents the main results of the estimation and the experiments, and finally, in Section 5 I present the main conclusions.

2 Data and main variables

I use data from five countries with different levels of informality: Argentina, Brazil Colombia, Peru and Uruguay. In order to estimate the production function I use data form the National Accounts and the household surveys

for each one. These surveys have a socioeconomic purpose and they are crucial in identifying workers in different productive sector in the economy. The identification of the formal workers⁴ is directly assessed by asking if the employer pays the contribution in order to obtain the right for a pension in retirement. The high educated workers are identified as those who declare that they completed high school (completed high school and higher), and the low educated all the other ones (uncompleted high school and lower).

I estimate the informality by education and sector quarterly, and the GDP for each sector quarterly. In this way the data base to estimate the production function has 1,162 observations (414 for Argentina, 88 for Brazil, 108 for Colombia, 240 for Peru and 312 for Uruguay).

For Argentina, I use the Permanent Household Survey (EPH in Spanish) carried out by the National Institute of Statistics and Census (INDEC in Spanish) for the period 1995-2010. The sample is restricted to the urban regions, covering 28 large urban centers where 70% of the urban population of Argentina live⁵.

For Brazil, I use the Continuous Household Survey (PNAD)⁶, conducted by IBGE⁷ in September of each year between 1996 and 2007. The survey is carried out only in September so I only have one observation per year.

In the case of Colombia, I use the Continuous Household Survey (ECH) between 2002 and 2005 and the Large Integrated Household Survey (GEIH) between 2007 and 2010, conducted by the National Bureau of Statistics (DANE). The question about the social contribution is only present in the second quarter in the ECH and in the first half of the year in the GEIH, so the number of observation are limited.

I use the National Household Survey (ENAHO) of Peru carried out by the National Institute of Statistics and Informatics (INEI) in the period 2001-2010. The sample includes all urban and rural areas in all the country.

In the case of Uruguay, I use the Continuous Household Survey (ECH) conducted by the National Statistics Institute (INE), between 1997 and 2010 throughout the whole year. The ECH is a survey carried out in urban areas between 1997 and 2005, where more than 90% of the Uruguayan population is,

⁴Note that, informal workers are those who are not covered by labor regulation, such as taxation, the right to the health system and the right to a pension income in retirement

⁵Urban population account for the 90% for the total population of Argentina, so the survey gives a good representation of the country.

⁶Pesquisa Nacional por Amostra de Domicilios

⁷Instituto Brasileiro de Geografia e Estatística

so the survey gives a good representation of the country. From 2006 the survey includes rural areas as well.

Finally, to estimate the government enforcement and the quality of the benefits that the formal workers receive, I use some indicators collected by the InterAmerican Development Bank. In particular for the quality of the government enforcement I use two indicators: compliance with the law⁸ and confidence in the judiciary system⁹. For the benefits that the government provides I use the citizens' perception of the taxes being well spent¹⁰.

2.1 Some facts from the data

In this framework, the literature basically assigns three roles to the government: collecting taxes, providing benefits and monitoring and regulating the economy. Empirical analysis show an ambiguous relation between unofficial economy and the level of taxation, and a positive relation with corruption using large cross-country data base of entrepreneurs. Johnson et al. (1998), using simple OLS regression, find a positive relation between regulation bureaucracy, tax burden and corruption with higher unofficial activities. Friedman et al. (2000) go further with a larger number of countries and find that the taxes have a negative effect on the unofficial economy, and it is the corruption and the bureaucracy which have a positive effect.

First, in Figure 1 we can observe the relationship between informality and the social contributions (taxes) of both employee and employer. There is a slightly negative relation between them, i.e. countries with a high level of contribution have less informality. This relation is observed both with the total, employer and employee contribution, and it could be interpreted as a part of the institutional framework.

Second, I explore the relation between informality and the quality of the benefits. In Figure 2 we can assess that there is a clear negative relation between the perception that taxes are well spent by the government and informality.

Finally, I consider the relation between informality with the government's enforcement, measured by citizen compliance with the law (Figure 3). There is

⁸This indicator represents the percentage of those surveyed who respond that they believe that citizens comply with the law very much or a fair amount. Source: Latinobarometer.

⁹It measures the percentage of firms that agree with the statement: I am confident that the judicial system will enforce my contractual and property rights in business disputes. Source: World Bank.

¹⁰This indicator represents the percentage of answers to the question: Do you believe that the government spends your tax dollars well?

Figure 1: Informality by social contribution in Latin America

(a) Employer and employee contribution

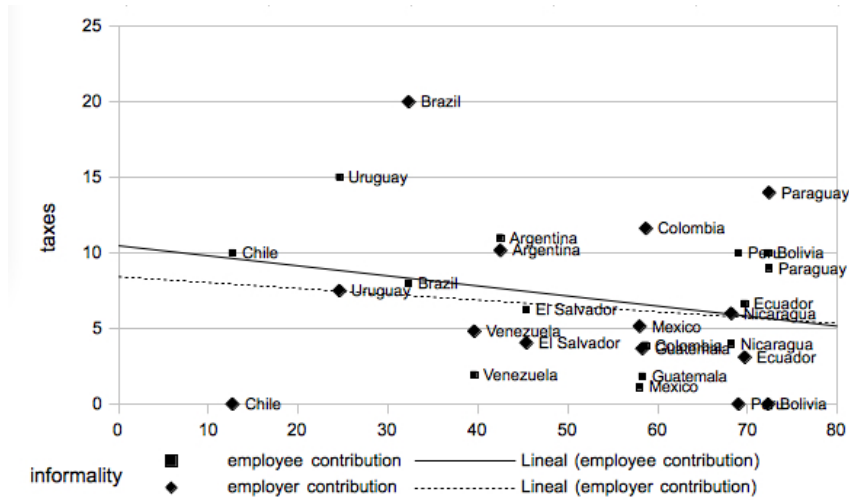
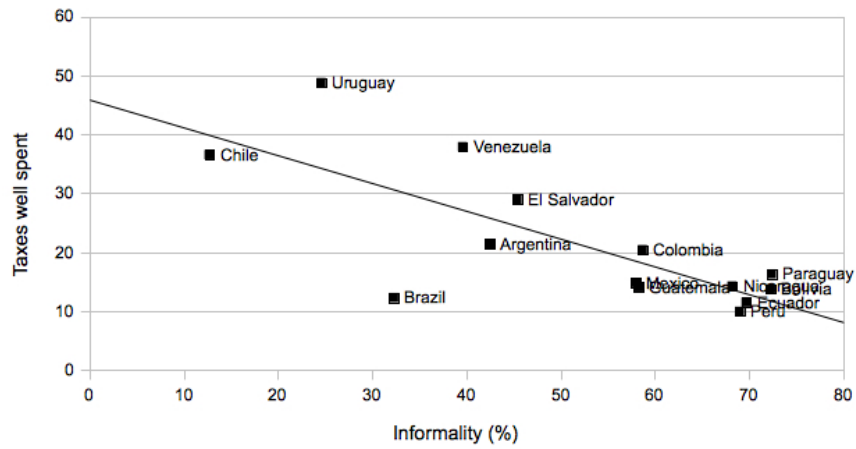


Figure 2: Informality by benefits in Latin America

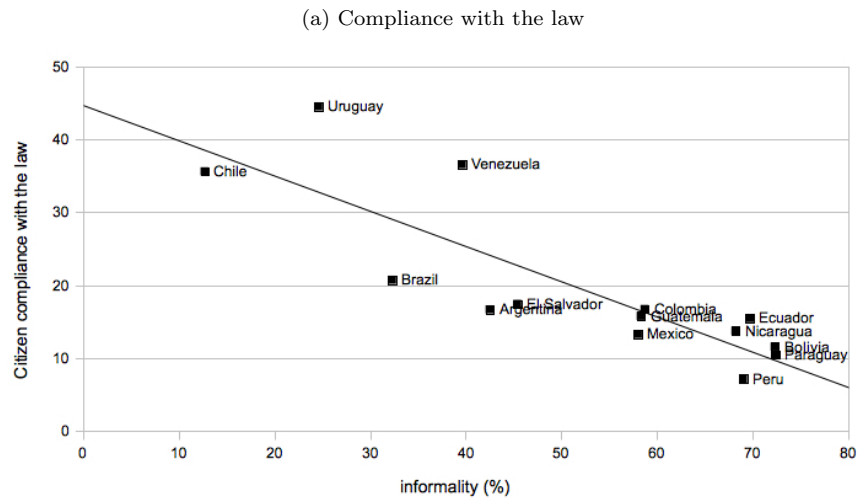
(a) Taxes well spent



a clear negative relation between them, which is in line with the literature of how the relation between informality and institutions is.

These three figures show that two of the three relations between government's role and informality appear clearly in the cross-country data. The benefits that the workers receive and the level of government enforcement in the labor market seem to play an important role in the level of informality, and in the process of formalization.

Figure 3: Informality by benefits in Latin America



3 Model

I develop a simple model to analyze the relation between the levels of informality by education (θ_L^i, θ_H^i) and the quality of the government enforcement ($q(e)$) and the quality of benefits ($K(\kappa)$). The households and firms decide on the level of informality (formality), and simultaneously there is a government choosing the level of enforcement in the labor market, and the benefits that the formal workers would enjoy.

3.1 Optimization problem: households and firms

Representative household problem: In a representative household, there are a continuous of x workers with a low level of education ($s = L$), and $1 - x$ with a high level ($s = H$). This household maximizes its consumption deciding the share of informal (θ_s^i) and formal (θ_s^f) work for each level of education s . There is no utility of leisure, the worker decides how to split their total number of hours (x and $1 - x$) between formality and informality. The consumption is determined by the revenues from formal and informal work, a lump sum tax (T) and the profit from firms (Π) which is fixed at zero. The formal revenues are the wage of formal hours $\omega_s^f \theta_s^f$ and they also receive ($K(\kappa)$) representing the quality of the benefits that the formal workers enjoy. The informal revenues are the informal wage $\omega_s^i \theta_s^i$ for the hours worked in informality, and there is also a share ($\phi_2 q(e)$) of this total wage which is lost. This loss depends on the quality of the enforcement ($q(e)$) and a parameter ϕ_2 , and represents a market imperfection in the informal labor market.

$$\max_{C, \{\theta_s^i, \theta_s^f\}} U(C) \quad (1)$$

Subject to

$$C = \sum_s \left[\omega_s^i \theta_s^i (1 - \phi_2 q(e)) + \omega_s^f \theta_s^f (1 + K(\kappa)) \right] - T + \Pi \quad (2)$$

The maximum of low skill hours is x :

$$\theta_L^i + \theta_L^f \leq x \quad (3)$$

and the maximum of high skill hours is $1 - x$:

$$\theta_H^i + \theta_H^f \leq 1 - x \quad (4)$$

Lagrangian function:

$$L = U(C) + \lambda_1^{HH} \left[C - \sum_s \left[\omega_s^i \theta_s^i (1 - \phi_2 q(e)) - \omega_s^f \theta_s^f (1 + K(\kappa)) \right] + T - \Pi \right] + \lambda_2^{HH} \left(\theta_L^i + \theta_L^f - x \right) + \lambda_3^{HH} \left(\theta_H^i + \theta_H^f - (1 - x) \right) \quad (5)$$

The interior solution to the household problem ($\theta_s^j \neq 0$) implies workers offering formal and informal hours in the labor market, then wages in informality after the market imperfection loss is equal to the formal wage plus the benefits:

$$\theta_L^i \frac{\partial L}{\partial \theta_L^i} = 0 : \quad \theta_L^i \left(-\lambda_1^{HH} \left(\omega_L^i (1 - \phi_2 q(e)) \right) + \lambda_2^{HH} \right) = 0 \quad (6)$$

$$\theta_L^f \frac{\partial L}{\partial \theta_L^f} = 0 : \quad \theta_L^f \left(-\lambda_1^{HH} \left(\omega_L^f (1 + K(\kappa)) \right) + \lambda_2^{HH} \right) = 0 \quad (7)$$

$$\theta_H^i \frac{\partial L}{\partial \theta_H^i} = 0 : \quad \theta_H^i \left(-\lambda_1^{HH} \left(\omega_H^i (1 - \phi_2 q(e)) \right) + \lambda_3^{HH} \right) = 0 \quad (8)$$

$$\theta_H^f \frac{\partial L}{\partial \theta_H^f} = 0 : \quad \theta_H^f \left(-\lambda_1^{HH} \left(\omega_H^f (1 + K(\kappa)) \right) + \lambda_3^{HH} \right) = 0 \quad (9)$$

Representative firm problem: A representative firm, decides to hire a share of informal l_s^i and formal workers l_s^f for each educative level s . The firm pays the formal workers $\omega_s^f l_s^f$ plus taxes τ , I am considering that the net wage (after taxes) and the taxes are only paid by the firm. Informal workers receive $\omega_s^i l_s^i$ but the firm faces a proportional fine $\phi_1 q(e)$ if that job is monitored.

$$\max_{l_s^i, l_s^f} \quad \Pi = \quad y(l_s^i, l_s^f) - \sum_s \left((1 + \phi_1 q(e)) \omega_s^i l_s^i + \omega_s^f l_s^f (1 + \tau) \right) \quad (10)$$

F.O.C.

$$l_s^i : \quad \frac{\partial y(l_s^i, l_s^f)}{\partial l_s^i} - \left(\omega_s^i (1 + \phi_1 q(e)) \right) = 0 \quad (11)$$

$$l_s^f : \quad \frac{\partial y(l_s^i, l_s^f)}{\partial l_s^f} - \left(\omega_s^f (1 + \tau) \right) = 0 \quad (12)$$

The market clearing condition equalizes the share of hours in formality and informality for each level of education, that the firm demands and the worker

supplies:

$$l_s^i = \theta_s^i \quad ; \quad l_s^f = \theta_s^f \quad (13)$$

The definition of the production function is one of the contributions of this paper. In the literature most of the papers either introduce the formal and informal as substitutes as in Ihrig and Moe (2004), or treat it as complements modeled in a Cobb-Douglas framework. The functional form which I choose is the CES function as in Dolado et al. (2001), Giuliodori and Stucchi (2010), Cappellari et al. (2011) who model the coexistence of temporary and permanent workers to reflect the fact that there are two types of workers who are not different in essence but in the contract way. In the informality literature, Ulyssea (2010) also presents a model with a CES production function with formal and informal intermediate goods. The CES function allows me to introduce the loose form of market duality, if formal and informal are perfect substitutes the solution tends to be a corner solution. But if there is an imperfect substitution, formality and informality coexist in the production function.

Remark 1 *The CES production function of the representative firm including contemporaneously formal and informal workers captures the market duality through the level of substitutability δ_j . If inputs have high substitutability, it shows that both inputs are more similar than in the case when the parameter goes to the complementarity. The production function is:*

- $y = \gamma l_H^{\rho_1} l_L^{\rho_2}$
- $l_H = \left[\psi_1 (l_H^f)^{-\delta_1} + (1 - \psi_1) (l_H^i)^{-\delta_1} \right]^{\frac{-(\psi_1)}{\delta_1}} \quad l_L = \left[\psi_2 (l_L^f)^{-\delta_2} + (1 - \psi_2) (l_L^i)^{-\delta_2} \right]^{\frac{-(\psi_2)}{\delta_2}}$

The level of substitutability is determined by δ_j , if it is close to -1, both inputs are perfect substitutes. Conversely if both inputs are complements, $\delta_j \rightarrow \infty$.

From the F.O.C. of the firm problem

$$\begin{aligned}
& \gamma v_2 \rho_2 \theta_H^{\rho_1} \theta_L^{\rho_2 - 1} \left(\psi_2 (\theta_L^f)^{-\delta_2} + (1 - \psi_2) (\theta_L^i)^{-\delta_2} \right)^{\frac{-(v_2 + \delta_2)}{\delta_2}} (1 - \psi_2) (\theta_L^i)^{-\delta_2 - 1} \\
& \qquad \qquad \qquad = \omega_L^i (1 + \phi_1 q(e)) \\
& \gamma v_2 \rho_2 \theta_H^{\rho_1} \theta_L^{\rho_2 - 1} \left(\psi_2 (\theta_L^f)^{-\delta_2} + (1 - \psi_2) (\theta_L^i)^{-\delta_2} \right)^{\frac{-(v_2 + \delta_2)}{\delta_2}} \psi_2 (\theta_L^f)^{-\delta_2 - 1} \\
& \qquad \qquad \qquad = \omega_L^f (1 + \tau) \\
& \gamma v_1 \rho_1 \theta_H^{\rho_1 - 1} \theta_L^{\rho_2} \left(\psi_1 (\theta_H^f)^{-\delta_1} + (1 - \psi_1) (\theta_H^i)^{-\delta_1} \right)^{\frac{-(v_1 + \delta_1)}{\delta_1}} (1 - \psi_1) (\theta_H^i)^{-\delta_1 - 1} \\
& \qquad \qquad \qquad = \omega_H^i (1 + \phi_1 q(e)) \\
& \gamma v_1 \rho_1 \theta_H^{\rho_1 - 1} \theta_L^{\rho_2} \left(\psi_1 (\theta_H^f)^{-\delta_1} + (1 - \psi_1) (\theta_H^i)^{-\delta_1} \right)^{\frac{-(v_1 + \delta_1)}{\delta_1}} \psi_1 (\theta_H^f)^{-\delta_1 - 1} \\
& \qquad \qquad \qquad = \omega_H^f (1 + \tau)
\end{aligned}$$

3.2 Equilibrium: households and firms

I will focus only on the interior solution ($\theta_s^{j*} \neq 0$), firms where both formal and informal workers coexist. The relative informal wages depends positively on the quality of benefits and the quality of the government enforcement:

$$\begin{aligned}
\frac{\omega_L^{i*}}{\omega_L^{f*}} &= \frac{(1 + K(\kappa))}{(1 - \phi_2 q(e))} \\
\frac{\omega_H^{i*}}{\omega_H^{f*}} &= \frac{(1 + K(\kappa))}{(1 - \phi_2 q(e))}
\end{aligned} \tag{14}$$

The relative size of the informal sector depends on the relative wage, the relation between fines from being in the informality ($\phi_1 q(e)$), and the contribution paid by the formal employer (τ), while the differences among educative levels is given by the formal shares (ψ_1 and ψ_2) and the level of substitutability (δ_1 and δ_2) of the production function.

$$\begin{aligned}
\frac{\theta_L^{i*}}{\theta_L^{f*}} &= \left[\frac{\psi_2}{1 - \psi_2} \frac{(1 + K(\kappa))}{(1 - \phi_2 q(e))} \frac{(1 + \phi_1 q(e))}{1 + \tau} \right]^{\frac{-1}{\delta_2 + 1}} \\
\frac{\theta_H^{i*}}{\theta_H^{f*}} &= \left[\frac{\psi_1}{1 - \psi_1} \frac{(1 + K(\kappa))}{(1 - \phi_2 q(e))} \frac{(1 + \phi_1 q(e))}{1 + \tau} \right]^{\frac{-1}{\delta_1 + 1}}
\end{aligned} \tag{15}$$

The share of informality in both educative level in equilibrium depend

negatively on the level of enforcement e (Equations 16) and the benefits κ (Equations 17).

$$\frac{\partial \frac{\theta_L^{i^*}}{x - \theta_L^{i^*}}}{\partial e} = -A_1 \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} (\phi_1 + \phi_2) \frac{\partial q}{\partial e} < 0; \quad \frac{\partial \frac{\theta_H^{i^*}}{x - \theta_H^{i^*}}}{\partial e} = -A_2 \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} (\phi_1 + \phi_2) \frac{\partial q}{\partial e} < 0 \quad (16)$$

$$\frac{\partial \frac{\theta_L^{i^*}}{x - \theta_L^{i^*}}}{\partial \kappa} = -A_1 (1 + \phi_1 q(e)) \frac{\partial K(\kappa)}{\partial \kappa} < 0; \quad \frac{\partial \frac{\theta_H^{i^*}}{x - \theta_H^{i^*}}}{\partial \kappa} = -A_2 (1 + \phi_1 q(e)) \frac{\partial K(\kappa)}{\partial \kappa} < 0 \quad (17)$$

$$A_j = \frac{1}{\delta_j + 1} \left[\frac{\psi_j}{1 - \psi_j} \frac{1 + \mathbb{K}(\kappa)}{1 - \phi_2 q(e)} \frac{1 + \phi_1 q(e)}{1 + \tau} \right]^{-\frac{\delta_j + 2}{\delta_j + 1}} \frac{\psi_j}{1 - \psi_j} \frac{1}{(1 - \phi_2 q(e))(1 + \tau)} > 0 \quad j\{1, 2\}$$

3.3 Optimization problem: Government

The government maximizes the consumer's utility by choosing benefits κ and the level of enforcement e as a carrot and a stick. For those workers in formality there is a carrot which is benefits $K(\kappa)$, and there is a stick for those in informality given by the quality of the government enforcement ($q(e)$) and the level of fines ϕ_1 .

The government equalizes the resources (from the social contribution in the formal jobs τ , the fines in the informal jobs $\phi_1 q(e)$ and a lump sum tax (T) with the spending *e.g.* the cost of the government enforcement and providing benefits which is given by the function $B(e, \kappa)$ (Equation 20)

Remark 2 *This maximization allows the government to choose the level of enforcement and benefits such that informality can exist in the labor market in its optimal choices. This possibility goes in the direction that some level of informality can be allowed by the government through the relaxation of the quality of the government enforcement and benefits. This point could be controversial if informality is perceived as a pure negative feature of the economy, although in this paper I use an agonistical approach about informality.*

$$\max_{e, \kappa} U(C) \quad (18)$$

Subject to

$$C = \sum_s \left[\omega_s^i \theta_s^i (1 - \phi_2 q(e)) + \omega_s^f \theta_s^f (1 + K(\kappa)) \right] - T + \Pi \quad (19)$$

$$\sum_s \left[\tau \omega_s^f \theta_s^f + \phi_1 q(e) \theta_s^i \right] + T = B(e, \kappa) \quad s = \{L, H\} \quad (20)$$

F.O.C.

$$e : \quad \frac{\partial U}{\partial C} \left(\sum_s \omega_s^i \theta_s^i (\phi_1 - \phi_2) \frac{\partial q(e)}{\partial e} - \frac{\partial B(e, \kappa)}{\partial e} \right) = 0 \quad (21)$$

$$\kappa : \quad \frac{\partial U}{\partial C} \left(\sum_s \omega_s^f \theta_s^f \frac{\partial K(\kappa)}{\partial \kappa} - \frac{\partial B(e, \kappa)}{\partial \kappa} \right) = 0 \quad (22)$$

3.4 Equilibrium: Government

The functional forms which are chosen to estimate this model are such that the quality functions (enforcement e , and benefits κ) are increasing and concave where a and d are the quality parameters. The cost function of the government enforcement and benefits are quadratic

$$\begin{aligned} q(e) &= a\sqrt{e} & K(\kappa) &= d\sqrt{\kappa} \\ B(e, \kappa) &= b_1 e^2 + b_2 \kappa^2 + b_3 \end{aligned} \quad (23)$$

The equilibrium enforcement and benefits are:

$$e^* = \left(\frac{a(\phi_1 - \phi_2)(\omega_L^{i*} \theta_L^{i*} + \omega_H^{i*} \theta_H^{i*})}{4b_1} \right)^{\frac{2}{3}}$$

The level of enforcement e depends positively on the mass of salaries in the informality ($\omega_L^i \theta_L^i + \omega_H^i \theta_H^i$), the ratio of the quality and cost function parameters $\frac{a}{b_1}$, and the term $\phi_1 - \phi_2$, which is the difference between the fines which the firms pay for any informal job monitored and the worker loss in the informality. If the fines were equal to the workers' losses, the level of government enforcement would be zero because the effect of the informality is solved within the market, and the action of the government is not necessary.

The level of benefits κ that the government chooses depend positively on the mass of salaries in the formality ($\omega_L^f \theta_L^f + \omega_H^f \theta_H^f$), and the ratio of the quality and cost function parameters $\frac{d}{b_2}$.

$$\kappa^* = \left(\frac{d(\omega_L^{f*} \theta_L^{f*} + \omega_H^{f*} \theta_H^{f*})}{4b_2} \right)^{\frac{2}{3}}$$

The level of taxes is given by the equalized budget constraint of the government:

$$T^* = \sum_s B(e^*, \kappa^*) - \tau \omega_s^{f*} \theta_s^{f*} - \phi_1 \omega_s^{i*} \theta_s^{i*} q(e^*)$$

Definition 1 *Given the set of parameters, there is a unique equilibrium which determines the level of informality in each level of education (θ_s^{i*}) working, the level of government enforcement over informal jobs (e^*), the benefits that the government brings to the formal ones (κ^*), and the lump sum taxes collected from the households (T^*).*

3.5 Comparative statics

In this section, I analyze what the effect is of the exogenous parameters in the main model equilibrium outcomes (θ_s^i , e and κ). I focus not only on the comparative statics in respect to single parameters¹¹, but also considering the effect of a couple of them¹².

In the Equations in 24, I analyze the comparative statics of the equilibrium relative size of the informality (θ_s^{i*}) in respect to the fines (ϕ_1) and informal market imperfection or informal wage loss (ϕ_2). These shares depend negatively on the fines that the firm has to pay if the informal job is monitored (ϕ_1) and it is uncertain on the share of informal wage that the worker loses if their job is monitored (ϕ_2). Figure C.1 shows the simulated performance of a grid of ϕ_1 and ϕ_2 , the informality is more sensitive with a change of ϕ_1 than ϕ_2 .

Additionally, I consider the comparative statics of the informality shares respect to the quality functions parameters, these shares decrease when the parameters of the quality of monitoring and benefits function (a and d) increase. The effect of a on the informality is higher than the effect of d , as is shown in Figure C.3. In Figures C.4 and C.5, there are the effect of the quality and cost parameters of the enforcement and the benefits, negative in the quality and

¹¹The analytical development is presented in the Appendix C.

¹²The graphical analysis is presented in the Appendix C, where there is a numerical exercise fixing the parameters as Argentina, and then computing the solution on a grid of two parameters. The parameters for Argentina are shown in Table 6.

positive in costs.

$$\begin{aligned} \frac{\theta_s^{i*}}{\partial \phi_1} < 0 & \quad \frac{\theta_s^{i*}}{\partial \phi_2} ?? \\ \frac{\theta_s^{i*}}{\partial a} < 0 & \quad \frac{\theta_s^{i*}}{\partial d} < 0 \end{aligned} \quad (24)$$

The comparative statics in the Equations in 25 show that the government enforcement e^* is decreasing in the fines parameters (ϕ_1), decreasing in the quality of benefits (d) and unknown sign respect to the quality of the government enforcement (a). The equilibrium behavior when these parameters change is also shown in Figures C.3 and C.4, in which the equilibrium is solved by fixing the other parameters. The effect of a on e^* is positive with this set of parameters, which was uncertain in the analytical analysis, and the effect of b_1 is negative (but the intensity depend also on the level of a as is shown in Figure C.4).

$$\frac{\partial e^*}{\partial \phi_1} < 0 \quad \frac{\partial e^*}{\partial a} ?? \quad \frac{\partial e^*}{\partial d} < 0 \quad (25)$$

$$\frac{\partial \kappa^*}{\partial \phi_1} > 0 \quad \frac{\partial \kappa^*}{\partial a} > 0 \quad \frac{\partial \kappa^*}{\partial d} > 0 \quad (26)$$

The benefits κ increases in fines (ϕ_1), the quality parameter of the enforcement (a) and the benefits (d), as is shown in Figure C.4. In Figures C.3 and C.5, there are the effect on κ of d and b_2 , which are positive and negative respectively.

In the set of Equations in 27 and 28 I present the effect of the production function parameters on the model outcomes. In the case of the level of substitutability, the signs depend on the terms B_1 or B_2 . If these terms are higher than one, a higher level of substitutability ($\delta_j \rightarrow -1$) leads to lower level of informality, but if B_1 or B_2 are between 0 and 1, higher substitutability ($\delta_j \rightarrow -1$) lead to higher informality.

$$\begin{aligned} \frac{\theta_L^{i*}}{\partial \delta_2} > 0 & \quad \frac{e^*}{\partial \delta_2} > 0 & \quad \frac{\kappa^*}{\partial \delta_2} < 0 \\ \text{if } B_2 = \left[\frac{\psi_2}{1 - \psi_2} \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} \frac{1 + \phi_1 q(e)}{1 + \tau} \right] > 1 & & \\ \frac{\theta_L^{i*}}{\partial \delta_2} < 0 & \quad \frac{e^*}{\partial \delta_2} < 0 & \quad \frac{\kappa^*}{\partial \delta_2} > 0 \quad \text{if } 0 < B_2 < 1 \end{aligned} \quad (27)$$

$$\begin{aligned}
& \frac{\theta_H^{i*}}{\partial \delta_1} > 0 \quad \frac{e^*}{\partial \delta_1} > 0 \quad \frac{\kappa^*}{\partial \delta_1} < 0 \\
& \text{if } B_1 = \left[\frac{\psi_1}{1 - \psi_1} \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} \frac{1 + \phi_1 q(e)}{1 + \tau} \right] > 1 \quad (28) \\
& \frac{\theta_H^{i*}}{\partial \delta_1} < 0 \quad \frac{e^*}{\partial \delta_1} < 0 \quad \frac{\kappa^*}{\partial \delta_1} > 0 \quad \text{if } 0 < B_1 < 1
\end{aligned}$$

In the first two panels of Figure C.2, I observe how the share of informality changes with the level of substitutability between formal and informal workers in the case of the educated workers and the non educated ones (δ_1 and δ_2) and the shares in the production function (ψ_1 and ψ_2). If these shares were lower, B_j is lower than 1, then the sign of the comparative statics of the informality would be the opposite.

4 Results

The empirical strategy is to estimate both the production function and the quality and the cost functions. However, Botero García (2010) calibrates the elasticity of substitution of a CES production function with informal and formal workers, the level of substitutability for low educated worker is -0.5, and between educated and non educated workers it is 3.3. Ulyssea (2010) also calibrates the parameters of the general level of substitutability between formal and informal production at -0.3. My empirical strategy is to estimate the parameters of the production function from the data. The production function is estimated by an approximation of a linear regression. The other parameters of the model are estimated with the Method of Moments.

4.1 Estimation of the production function

In order to estimate the CES production function (and the nested production function) I use the linear Taylor-series approximation, which was first developed by Kmenta (1967).

$$y = \gamma [\psi_1 x_1^{-\alpha} + (1 - \psi_1) x_2^{-\alpha}]^{-\frac{v}{\alpha}}$$

The second order Taylor Approximation at $\rho = 0$ ¹³

$$y = \gamma x_1^{v\psi_1} x_2^{v(1-\psi_1)} \exp(-0.5\alpha v \psi_1 (1 - \psi_1) (\ln x_1 - \ln x_2)^2)$$

¹³Kmenta (1967) justify this only by mathematical convenience and in order to estimate around the Cobb Douglass shape.

In the case of my paper the production function is defined as follows, $y = \gamma l_H^{\rho_1} l_L^{\rho_2}$,

$$l_H = \left[\psi_1 (l_H^f)^{-\delta_1} + (1 - \psi_1) (l_H^i)^{-\delta_1} \right]^{\frac{-(v_1)}{\delta_1}} \quad \text{and}$$

$$l_L = \left[\psi_2 (l_L^f)^{-\delta_2} + (1 - \psi_2) (l_L^i)^{-\delta_2} \right]^{\frac{-(v_2)}{\delta_2}},$$

using the same methodology to estimate it we obtain that:

$$\begin{aligned} \ln y \approx & \ln \gamma + \rho_1 v_1 \psi_1 \ln l_H^f + \rho_1 v_1 (1 - \psi_1) \ln l_H^i - \frac{1}{2} \rho_1 v_1 \psi_1 (1 - \psi_1) \delta_1 (\ln l_H^f - \ln l_H^i)^2 \\ & + \rho_2 v_2 \psi_2 \ln l_L^f + \rho_2 v_2 (1 - \psi_2) \ln l_L^i - \frac{1}{2} \rho_2 v_2 \psi_2 (1 - \psi_2) \delta_2 (\ln l_L^f - \ln l_L^i)^2 \end{aligned}$$

To estimate:

$$\ln y = \beta_0 + \beta_1 \ln l_H^f + \beta_2 \ln l_H^i + \beta_3 (\ln l_H^f - \ln l_H^i)^2 + \beta_4 \ln l_L^f + \beta_5 \ln l_L^i + \beta_6 (\ln l_L^f - \ln l_L^i)^2 + \epsilon$$

The main parameters of the production function are estimated:

$$\begin{aligned} \psi_1 &= \frac{\beta_1}{\beta_1 + \beta_2} \\ \delta_1 &= \frac{-2\beta_3}{\beta_2 \psi_1} \\ \psi_2 &= \frac{\beta_4}{\beta_4 + \beta_5} \\ \delta_2 &= \frac{-2\beta_6}{\beta_5 \psi_2} \end{aligned}$$

The results of the production function estimation appear in Table 4, the dependent variable is the logarithm of the aggregate value by sector and the independent variables are the logarithm of the informality shares. In Table 4, I estimate the parameters for all the countries together. The level of substitutability (δ 's) is slightly higher in the case of the non educated workers, but even for the educated, it is relatively high. The shares of workers are lower than 0.5, so the terms B_1 and B_2 are closer to 1, if these terms are lower than 1, the informality is decreasing respect to δ_1 and δ_2 .

The second and third columns present the IV estimation instrumented by the lags. In the second column, the estimation is through the inclusion of the error term of the first steps. Note that the joint test of these error terms is significative¹⁴. In the third column, the estimation is instrumented by the

¹⁴F(4, 775) = 9.41 Prob > F = 0.00

	OLS	IV(1)	IV(2)
$\ln l_H^f$	2.608*** (0.57)	3.227*** (0.55)	3.687*** (0.57)
$\ln l_H^i$	2.641*** (0.47)	3.493*** (0.42)	3.875*** (0.51)
$\ln l_L^f$	2.031*** (0.44)	2.180*** (0.45)	1.649*** (0.39)
$\ln l_L^i$	2.189*** (0.46)	2.312*** (0.48)	2.267*** (0.42)
$(\ln l_H^f - \ln l_H^i)^2$	0.346*** (0.08)	0.472*** (0.07)	0.615*** (0.09)
$(\ln l_L^f - \ln l_L^i)^2$	0.329*** (0.08)	0.375*** (0.08)	0.354*** (0.07)
Constant	16.97*** (0.79)	17.67*** (0.71)	17.25*** (0.77)
Year	yes	yes	yes
Sector	yes	yes	yes
R^2	0.573	0.689	0.667
N	987	807	865
ψ_1	0.497	0.48	0.488
δ_1	-0.528	-0.563	-0.651
ψ_2	0.481	0.485	0.421
δ_2	-0.624	-0.669	-0.741
$\rho_1 v_1$	5.25	6.72	7.56
$\rho_2 v_2$	4.22	4.49	3.92

Table 4: Estimation of the production function

predicted estimation from the first step. In the next section, I will use the estimation from the second column.

4.2 Estimation of the quality and cost parameters by country

In the estimation of the model's parameters through the Methods of Moments, I match the quality of the government enforcement $q(e)$, the quality of the benefits that the formal workers receive from the government $K(\kappa)$, the level of informality in both levels of education, the level of lump sum taxes, and the share of public expenditures.

As it is discussed in the introduction there are no clear variables to match in the case of the quality functions, so I want to compare how the fit is in the different countries¹⁵. In the case of the quality of the government enforcement I match with the indicator *compliance with the law*, and the quality of the benefits is matched with the indicator *taxes well spent*. Informality shares are

¹⁵The variables to fit are taken by surveys which are published for the IADB

estimated with the National Household Surveys, the public expenditures are matched with the figure from the National Accounts, and the level of the lump sum tax is matched at zero¹⁶.

Firstly, I present the result of the estimation for each country independently. The model fits the data quite well in general and in particular the ranking of the countries is respected, as is presented in Table 5. This latter point is somewhat important given the comparative objective of this paper, principally, if the comparison is between the countries with better performance, such as Brazil and Uruguay, with those with low performance as in the case of Peru. However, there are some features that I want to highlight: the informality for the low educated workers is underestimated, and probably the main problem is the overestimation in the high educated ones. The quality of government enforcement and quality of benefits are well estimated with the exception of the government enforcement in Brazil. The share of public expenditure is well estimated, and the taxes, which is a residual variable, fit well at zero.

	Argentina		Brasil		Colombia		Peru		Uruguay	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
θ_L^{i*}	0.2616	0.338	0.2193	0.228	0.2343	0.349	0.4134	0.485	0.2266	0.240
θ_H^{i*}	0.1244	0.079	0.0668	0.026	0.1151	0.057	0.1740	0.117	0.0849	0.024
$q(e^*)$	0.2002	0.232	0.2907	0.431	0.3165	0.317	0.1574	0.162	0.5080	0.508
$K(\kappa^*)$	0.1884	0.209	0.1458	0.161	0.1602	0.167	0.0922	0.101	0.3575	0.358
T	0.0001	0.000	-0.0003	0.000	-0.0002	0.000	0.0003	0.000	0.0001	0.000
SCP	0.1770	0.162	0.2018	0.246	0.1708	0.213	0.1237	0.110	0.1505	0.151

Table 5: Moment matching with the country by country estimation.

In Table 6 the estimation of the parameters for all five countries can be observed. Given that the effect of the quality parameters and the enforcement go in the same direction, it is not possible to observe a clear ranking in them. However, comparing a group of parameters as the term $(\frac{a(\phi_1 - \phi_2)}{4b_1})$ (which multiply the government enforcement e) is higher in countries with a lower level of informality, especially when the extreme countries (Uruguay or Brazil and Peru) are compared. Considering the countries with a low level of informality, the parameters of Uruguay and Brazil are 6.5 and 8.9 respectively and the Peruvian is 3.3. The order when the parameters that multiply the level of benefits are considered $(\frac{d}{4b_2})$ is not so clear. Again, the parameters of Peru is the lowest one (1.08), while the parameters are quite similar for Argentina, Colombia and Uruguay (2.45, 2.14 and 2.22 respectively).

¹⁶The wages are not considered as moments because there is no data about work hours in Colombia, and there is a lack of information to construct (comparatively) the formal wages including benefits for all the countries.

	Argentina	Brazil	Colombia	Peru	Uruguay
a	0.8951 (0.000)	0.9180 (0.096)	1.5336 (0.000)	0.6329 (0.002)	1.9675 (0.002)
d	0.8426 (0.000)	0.6521 (0.001)	0.8087 (0.001)	0.6247 (0.037)	1.6775 (0.007)
b_1	0.0944 (0.186)	0.0645 (0.050)	0.1138 (0.001)	0.0001 (0.002)	0.0209 (0.001)
b_2	0.0861 (0.053)	0.1159 (0.311)	0.0943 (0.000)	0.1442 (0.027)	0.1888 (0.001)
b_3	0.001 (-)	0.001 (-)	0.001 (-)	0.001 (-)	0.001 (-)
ϕ_1	1.1710 (0.327)	1.6423 (0.131)	1.2326 (0.065)	0.1319 (0.005)	0.3776 (0.142)
ϕ_2	0.1878 (0.192)	0.1547 (0.004)	0.0000 (0.000)	0.1298 (0.127)	0.1016 (0.002)

Table 6: Estimation of all quality and cost parameters with the production function's parameters $\delta_1 = -0.563$, $\delta_2 = -0.669$, $\psi_1 = 0.480$, $\psi_2 = 0.485$, $\rho_1 v_1 = 6.72$ and $\rho_2 v_2 = 4.92$. The standard deviations are estimated by the gradient of the moments vector.

4.3 Estimation of the quality and cost parameters by country and parameter

In order to disentangle how each parameter can differentiate the optimal level of the quality functions and the level of the informality for these five countries, I will estimate the parameters for all countries together, leaving only one parameter being different country by country. Tables A.1 - A.5 show the moment matching when the fines ϕ_1 , the market imperfection ϕ_2 , quality of the government enforcement a , the quality of the benefits d , and the cost parameters b_1 and b_2 are different country by country using the levels of informality, the quality of the enforcement and benefits, the lump sum tax and the public expenditures as moments. Obviously, the model fits worse than when the countries all have different parameters.

Table A.1 shows the model matching when the fines ϕ_1 is different country by country, the high informality is the moment which has most problems to fit with a clear overestimation (about the double of the data). Regarding the fit with the other moments, there is no clear tendency (neither underestimation nor overestimation for all countries).

Table A.2 shows the moment matching when the market imperfection ϕ_2 is different country by country, the high informality is again overestimated, and

the low informality is well estimated for those countries with low levels (Brazil and Uruguay) but there is an underestimation for the other ones.

Parameters	Value	Std. Dev.	Parameters	Value	Std. Dev.
a	1.1045	(0.031)	a	0.2581	(0.008)
d	0.6185	(0.123)	d	0.3625	(0.124)
b_1	0.0109	(0.0001)	b_1	0.0000	(0.000)
b_2	0.0297	(0.005)	b_2	0.0070	(0.001)
b_3	0.001	(-)	b_3	0.001	(-)
ϕ_1^a	0.2012	(0.008)	ϕ_1	0.8716	(0.037)
ϕ_1^b	1.3991	(0.027)	ϕ_2^a	0.8528	(0.041)
ϕ_1^c	0.9332	(0.017)	ϕ_2^b	0.1104	(0.293)
ϕ_1^p	0.0250	(0.001)	ϕ_2^c	0.0122	(0.003)
ϕ_1^u	0.8836	(0.020)	ϕ_2^p	0.8546	(0.035)
ϕ_2	0.0000	(0.0004)	ϕ_2^u	0.1421	(0.297)

Table 7: Estimation of fines (ϕ_1) and market imperfection (ϕ_2) parameters with the production function's parameters $\delta_1 = -0.563$, $\delta_2 = -0.669$, $\psi_1 = 0.480$, $\psi_2 = 0.485$, $\rho_1 v_1 = 6.72$ and $\rho_2 v_2 = 4.92$. The standard deviations are estimated by the gradient of the moments vector.

In the left panel of Table 7, I observe the performance of ϕ_1 , which is the amount of fines that the firm would have to pay for to have workers in informality, leaving the other parameters constant country by country. There is a clear and direct relation between the level of fines and level of informality, in particular it is higher in Brazil and Uruguay, which are 1.4 and 0.88, than in Argentina and Peru, which are 0.2 and 0.025 respectively. In the case of Colombia, it has a relatively high level of fines that is reflected in lower levels of informality in low educated workers (see Table A.1). The ability of ϕ_1 to capture heterogeneity in the model is a feature which is in line with the empirical evidence of the literature.

In the right panel of Table 7, there is the parameter estimation when ϕ_2 (the market imperfection) is different country by country. Countries with high levels of informality, such as Argentina and Peru present high values of ϕ_2 and Brazil and Uruguay have lower values. In this case, Colombia is again the country in which the order does not fit, and it is reflected in the underestimation in the informality of low educated workers (see Table A.2). The market imperfection parameter is not able to capture the heterogeneity as the fines parameter was able to.

In Table 8, I present the estimation when the parameters of the quality functions change. The moment matching of these estimation are presented

Parameters	Value	Std. Dev.	Parameters	Value	Std. Dev.
a^a	0.3226	(0.030)	a	0.3280	(0.088)
a^b	0.7757	(0.242)	d^a	0.2866	(0.003)
a^c	0.2688	(0.010)	d^b	0.3640	(0.004)
a^p	0.0581	(0.093)	d^c	0.1418	(0.001)
a^u	0.8792	(0.039)	d^p	0.0220	(0.0003)
d	0.6679	(0.019)	d^u	0.5059	(0.01)
b_1	0.0012	(0.00002)	b_1	0.0002	(0.00001)
b_2	0.0449	(0.0002)	b_2	0.0007	(0.00001)
b_3	0.001	(-)	b_3	0.001	(-)
ϕ_1	1.132	(0.065)	ϕ_1	0.6368	(0.015)
ϕ_2	0.0152	(0.007)	ϕ_2	0.0898	(0.0061)

Table 8: Estimation of quality function parameters with the production function's parameters $\delta_1 = -0.563$, $\delta_2 = -0.669$, $\psi_1 = 0.480$, $\psi_2 = 0.485$, $\rho_1 v_1 = 6.72$ and $\rho_2 v_2 = 4.92$. The standard deviations are estimated by the gradient of the moments vector.

in Tables A.3 and A.4. The matching which corresponds to the a estimation (Table A.3) shows the overestimation of the high educated informality and the underestimation of the low educated informality for Brazil and Uruguay, but a good fit for the other countries. The matching of d estimation (Table A.4) is similar to the previous one. Both quality parameters allow the model to capture the heterogeneity in the moments.

The left panel of Table 8 shows the estimation of the quality of the government enforcement, and the right panel the quality of the benefits. The estimations of the quality parameters show the same order as the informality and not the order of the quality moments among countries. These estimations show better quality parameters means lower informality.

Table 9 shows the estimation of the cost function, and Table A.5 is the moment matching of this estimation. The cost function is able to reproduce the heterogeneity, the model matches the order well but it has some problems in the case of Argentina. Lower cost function parameters lead to low levels of informality.

5 Conclusion

I consider informality using a loose form of dualism and it is formalized through the definition of the production function. This is one of the main contributions of this paper.

Parameters	Value	Std Dev
a	0.8707	(0.019)
d	0.8911	(0.049)
b_1^a	0.0023	(0.002)
b_2^a	0.1286	(0.017)
b_1^b	0.0019	(0.001)
b_2^b	0.0192	(0.000)
b_1^c	0.3458	(0.080)
b_2^c	0.4641	(0.212)
b_1^p	1.7367	(0.167)
b_2^p	0.1863	(0.028)
b_1^u	0.0000	(0.000)
b_2^u	0.0087	(0.000)
b_3	0.001	(-)
ϕ_1	1.2232	(0.192)
ϕ_2	0.0002	(0.006)

Table 9: Estimation of cost function parameters with the production function's parameters $\delta_1 = -0.563$, $\delta_2 = -0.669$, $\psi_1 = 0.480$, $\psi_2 = 0.485$, $\rho_1 v_1 = 6.72$ and $\rho_2 v_2 = 4.92$. The standard deviations are estimated by the gradient of the moments vector.

The model captures the quality of government enforcement and benefits, and the informality for two level of education in five different Latin American countries which have a great heterogeneity among them.

The first result is the estimation of a production function where formal and informal workers coexist when there are two level of education. I estimate the production function using data from all countries, and the level of substitutability of the low educated workers is higher than level of the high educated workers.

The second result is the estimation of the parameters in the quality of the government enforcement, the quality of the benefits that the workers receive in formal employment, the fines, the market imperfection and the cost function by the Method of Moments. Through this estimation, the model can capture the ranking of countries as well as the informality for different educative levels. However, the model has some difficulties in capturing the level of informality for high educative levels.

The third result is the estimation leaving one parameter free, only the market imperfection parameter ϕ_2 does not allow the model to reproduce the heterogeneity. The fines (ϕ_1), quality parameters (a and d) and cost parameters

(b_1 and b_2) allow the model to generate heterogeneity, and the moments are matched quite well.

In short, the model capture the main features of these economies and produces a good estimation for the parameters describing the countries' heterogeneity.

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A Appendix: Tables and figures.

The Tables A.1 - A.4 show the model matching when the estimation is done leaving only one parameters (ϕ_1 , ϕ_2 , a , d , b_1 and b_2) be different among countries.

Table A.1: Moment matching leaving free the fine parameter ϕ_1 .

	Argentina		Brasil		Colombia		Peru		Uruguay	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
θ_L^{i*}	0.3731	0.338	0.1817	0.228	0.2363	0.349	0.4040	0.485	0.2436	0.240
θ_H^{i*}	0.1609	0.079	0.0964	0.026	0.1158	0.057	0.1710	0.117	0.1182	0.024
$q(e^*)$	0.3075	0.232	0.4041	0.431	0.4069	0.317	0.1589	0.162	0.4061	0.508
$K(\kappa^*)$	0.1582	0.209	0.1613	0.161	0.1639	0.167	0.1546	0.101	0.1641	0.358
T	0.0003	0.000	-0.0002	0.000	-0.0002	0.000	0.0005	0.000	-0.0002	0.000
SCP	0.1338	0.162	0.1948	0.246	0.1718	0.213	0.1267	0.110	0.1692	0.151

Table A.2: Moment matching leaving free the market imperfection ϕ_2 .

	Argentina		Brasil		Colombia		Peru		Uruguay	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
θ_L^{i*}	0.2784	0.338	0.2214	0.228	0.2599	0.349	0.2822	0.485	0.2176	0.240
θ_H^{i*}	0.1300	0.079	0.1106	0.026	0.1238	0.057	0.1312	0.117	0.1092	0.024
$q(e^*)$	0.1532	0.232	0.4521	0.431	0.3964	0.317	0.1488	0.162	0.4436	0.508
$K(\kappa^*)$	0.1305	0.209	0.1302	0.161	0.1310	0.167	0.1305	0.101	0.1301	0.358
T	-0.0000	0.000	-0.0002	0.000	-0.0004	0.000	-0.0000	0.000	-0.0002	0.000
SCP	0.1419	0.162	0.1737	0.246	0.1505	0.213	0.1410	0.110	0.1741	0.151

Table A.3: Moment matching leaving free the quality of the government enforcement parameter a .

	Argentina		Brasil		Colombia		Peru		Uruguay	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
θ_L^{i*}	0.2983	0.338	0.1842	0.228	0.3173	0.349	0.3946	0.485	0.1674	0.240
θ_H^{i*}	0.1366	0.079	0.0973	0.026	0.1428	0.057	0.1679	0.117	0.0910	0.024
$q(e^*)$	0.1952	0.232	0.4871	0.431	0.1578	0.317	0.0227	0.162	0.5467	0.508
$K(\kappa^*)$	0.1578	0.209	0.1559	0.161	0.1570	0.167	0.1504	0.101	0.1545	0.358
T	-0.0001	0.000	-0.0002	0.000	-0.0000	0.000	0.0004	0.000	-0.0001	0.000
SCP	0.1524	0.162	0.1931	0.246	0.1474	0.213	0.1291	0.110	0.2020	0.151

Table A.4: Moment matching leaving free the quality of the benefits d .

	Argentina		Brasil		Colombia		Peru		Uruguay	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
θ_L^{i*}	0.2759	0.338	0.2481	0.228	0.3284	0.349	0.3630	0.485	0.2021	0.240
θ_H^{i*}	0.1291	0.079	0.1198	0.026	0.1464	0.057	0.1577	0.117	0.1038	0.024
$q(e^*)$	0.3032	0.232	0.2926	0.431	0.3195	0.317	0.3278	0.162	0.2721	0.508
$K(\kappa^*)$	0.2079	0.209	0.2861	0.161	0.0803	0.167	0.0066	0.101	0.4398	0.358
T	-0.0000	0.000	0.0001	0.000	-0.0002	0.000	-0.0003	0.000	0.0004	0.000
SCP	0.1544	0.162	0.1621	0.246	0.1444	0.213	0.1408	0.110	0.1789	0.151

Table A.5: Moment matching leaving free the cost function b_1 and b_2 .

	Argentina		Brasil		Colombia		Peru		Uruguay	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
θ_L^{i*}	0.1795	0.338	0.1420	0.228	0.3553	0.349	0.3670	0.485	0.1140	0.240
θ_H^{i*}	0.0956	0.079	0.0812	0.026	0.1552	0.057	0.1590	0.117	0.0699	0.024
$q(e^*)$	0.4694	0.232	0.4572	0.431	0.1243	0.317	0.0745	0.162	0.4933	0.508
$K(\kappa^*)$	0.1609	0.209	0.2961	0.161	0.1041	0.167	0.1402	0.101	0.3766	0.358
T	-0.0002	0.000	0.0001	0.000	-0.0000	0.000	0.0002	0.000	0.0001	0.000
SCP	0.1959	0.162	0.2162	0.246	0.1411	0.213	0.1363	0.110	0.2152	0.151

B Appendix: Household problem

The complete F.O.C. of the Representative household problem are:

$$C \frac{\partial L}{\partial C} = 0 : \quad C \left(\frac{\partial U(C)}{\partial C} + \lambda_1^{HH} \right) = 0 \quad (\text{B.1})$$

$$\theta_L^i \frac{\partial L}{\partial \theta_L^i} = 0 : \quad \theta_L^i \left(-\lambda_1^{HH} \left(\omega_L^i (1 - \phi_2 q(e)) \right) + \lambda_2^{HH} \right) = 0 \quad (\text{B.2})$$

$$\theta_L^f \frac{\partial L}{\partial \theta_L^f} = 0 : \quad \theta_L^f \left(-\lambda_1^{HH} \left(\omega_L^f (1 + K(\kappa)) \right) + \lambda_2^{HH} \right) = 0 \quad (\text{B.3})$$

$$\theta_H^i \frac{\partial L}{\partial \theta_H^i} = 0 : \quad \theta_H^i \left(-\lambda_1^{HH} \left(\omega_H^i (1 - \phi_2 q(e)) \right) + \lambda_3^{HH} \right) = 0 \quad (\text{B.4})$$

$$\theta_H^f \frac{\partial L}{\partial \theta_H^f} = 0 : \quad \theta_H^f \left(-\lambda_1^{HH} \left(\omega_H^f (1 + K(\kappa)) \right) + \lambda_3^{HH} \right) = 0 \quad (\text{B.5})$$

$$\lambda_1^{HH} \frac{\partial L}{\partial \lambda_1^{HH}} = 0 : \quad \lambda_1^{HH} \left(C - \sum_s \left[\omega_s^i \theta_s^i (1 - \phi_2 q(e)) - \omega_s^f \theta_s^f (1 + K(\kappa)) \right] + T - \Pi \right) = 0 \quad (\text{B.6})$$

$$\lambda_2^{HH} \frac{\partial L}{\partial \lambda_2^{HH}} = 0 : \quad \lambda_2^{HH} \left(-x + \theta_L^i + \theta_L^f \right) = 0 \quad (\text{B.7})$$

$$\lambda_3^{HH} \frac{\partial L}{\partial \lambda_3^{HH}} = 0 : \quad \lambda_3^{HH} \left(x - 1 + \theta_H^i + \theta_H^f \right) = 0 \quad (\text{B.8})$$

C Appendix: Comparative Statics of the equilibrium variables

In this section I show the comparative statics of the informality shares, the level of government enforcement and the benefits (θ_L^i , θ_H^i , e and κ) and the main exogenous variables (ϕ_1 , ϕ_2 , δ_j , ψ_j , a and d).

Some terms for $j = \{1, 2\}$:

$$\begin{aligned}
 A_j &= \frac{1}{\delta_j + 1} \left[\frac{\psi_j}{1 - \psi_j} \frac{1 + \mathbb{K}(\kappa)}{1 - \phi_2 q(e)} \frac{1 + \phi_1 q(e)}{1 + \tau} \right]^{-\frac{\delta_j + 2}{\delta_j + 1}} \frac{\psi_j}{1 - \psi_j} \frac{1}{(1 - \phi_2 q(e))(1 + \tau)} > 0 \\
 E_1 &= (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) (\phi_1 - \phi_2) \frac{\partial^2 q}{\partial e^2} - \frac{\partial^2 B}{\partial e^2} < 0 \\
 K_1 &= (\omega_L^f (x - \theta_L^i) + \omega_H^f (1 - x - \theta_H^i)) \frac{\partial^2 K}{\partial \kappa^2} - \frac{\partial^2 B}{\partial \kappa^2} < 0 \\
 B_j &= \left[\frac{\psi_j}{1 - \psi_j} \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} \frac{1 + \phi_1 q(e)}{1 + \tau} \right] > 0 \\
 C_j &= \frac{1}{\delta_j + 1} \left[\frac{\psi_j}{1 - \psi_j} \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} \frac{1 + \phi_1 q(e)}{1 + \tau} \right]^{-\frac{\delta_j + 2}{\delta_j + 1}} \frac{1 + K(\kappa)}{(1 - \phi_2 q(e))} \frac{1 + \phi_1 q(e)}{(1 + \tau)} > 0 \\
 j &= \{1, 2\}
 \end{aligned}$$

Denominator D :

$$\begin{aligned}
 D &= \overbrace{\frac{x}{(x - \theta_L^i)^2} \left[\frac{1 - x}{(1 - x - \theta_H^i)^2} \right] E_1 K_1}^{\geq 0} + \left[\frac{A_1 x}{(x - \theta_L^i)^2} + \frac{A_2 (1 - x)}{(1 - x - \theta_H^i)^2} \right] * \\
 &\quad \left[\underbrace{(\omega_L^f + \omega_H^f) (1 + \phi_1 q(e)) \left[\frac{\partial K}{\partial \kappa} \right]^2}_{\leq 0} E_1 - \underbrace{(\omega_L^i + \omega_H^i) \left(\frac{1 + K(\kappa)}{1 - \phi_2 q(e)} \right) (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2}_{\geq 0} K_1 \right] \\
 &\hspace{15em} (C.1)
 \end{aligned}$$

In the interval of interest of the parameters the simulation shows the positive terms are higher than the negative one, then:

$$D > 0$$

C.1 Comparative statics of θ_L^* and θ_H^*

In this section I show the comparative statics of θ_L^i and θ_H^i respect to the main exogenous parameters of the model. The change of θ_L^i and θ_H^i respect to ϕ_1 is negative, the amount of fines impact negatively in the equilibrium informal shares:

Numerator N_{11} :

$$\frac{\partial \theta_L^i}{\partial \phi_1} = \frac{N_{11}}{D} < 0 \quad (\text{C.2})$$

$$N_{11} = A_2 \left(1 + K(\kappa)\right) \frac{1-x}{(1-x-\theta_H^i)^2} K_1 \left[\frac{(\phi_1 + \phi_2)}{1 - \phi_2 q(e)} (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \left[\frac{\partial q}{\partial e} \right]^2 - q(e) E_1 \right] < 0 \quad (\text{C.3})$$

Numerator N_{21} :

$$\frac{\partial \theta_H^i}{\partial \phi_1} = \frac{N_{21}}{D} < 0 \quad (\text{C.4})$$

$$N_{21} = A_1 \left(1 + K(\kappa)\right) \frac{x}{(x - \theta_L^i)^2} K_1 \left[\frac{(\phi_1 + \phi_2)}{1 - \phi_2 q(e)} (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \left[\frac{\partial q}{\partial e} \right]^2 - q(e) E_1 \right] < 0 \quad (\text{C.5})$$

The impact is higher in the lower (higher) skilled informal worker if:

$$\frac{\partial \theta_L^i}{\partial \phi_1} \geq \frac{\partial \theta_H^i}{\partial \phi_1} \quad \text{iff} \quad \frac{A_2(1-x)}{(1-x-\theta_H^i)^2} \geq \frac{A_1 x}{(x-\theta_L^i)^2}$$

In the case of the comparative statics of θ_L^i and θ_H^i respect to ϕ_2 the sign is uncertain.

Numerator N_{12} :

$$\frac{\partial \theta_L^i}{\partial \phi_2} = \frac{N_{12} ??}{D} \quad (\text{C.6})$$

$$N_{12} = -A_2 \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} \frac{1-x}{(1-x-\theta_H^i)^2} K_1 \left[(1 + \phi_1 q(e)) q(e) E_1 + (\phi_1 + \phi_2) (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \left[\frac{\partial q}{\partial e} \right]^2 \right] ?? \quad (\text{C.7})$$

Numerator N_{22} :

$$\frac{\partial \theta_H^i}{\partial \phi_2} = \frac{N_{22} ??}{D} \quad (\text{C.8})$$

$$N_{22} = -A_1 \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} \frac{x}{(x - \theta_L^i)^2} K_1 \left[(1 + \phi_1 q(e)) q(e) E_1 + (\phi_1 + \phi_2) (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \left[\frac{\partial q}{\partial e} \right]^2 \right] ?? \quad (C.9)$$

Analyzing the comparative statics of θ_L^i respect to δ_2 , the sign depend on B_1 . If B_1 is higher than 1, the effect of a higher level of substitutability impact positively on the informality. in the case of θ_H^i the sign is uncertain.

Numerator N_{13} :

$$\begin{aligned} \frac{\partial \theta_L^i}{\partial \delta_2} &= \frac{N_{13}}{D} > 0 \quad \text{if } B_2 > 1 \\ \frac{\partial \theta_L^i}{\partial \delta_2} &= \frac{N_{13}}{D} < 0 \quad \text{if } B_2 < 1 \end{aligned} \quad (C.10)$$

$$N_{13} = \frac{\ln B_2 B_2^{\frac{-1}{\delta_2+1}}}{(\delta_2 + 1)^2} * \underbrace{\left[\frac{1-x}{(1-x-\theta_H^i)^2} E_1 K_1 - A_1 \frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2 \omega_H^i K_1 + A_1 (1 + \phi_1 q(e)) \left[\frac{\partial K}{\partial \kappa} \right]^2 \omega_H^f E_1 \right]}_{>0 \quad (\text{by simulation})}$$

$$\text{if } B_2 > 1 \quad (C.11)$$

Numerator N_{23} :

$$\frac{\partial \theta_H^i}{\partial \delta_2} = \frac{N_{23}}{D} ? \quad (C.12)$$

$$N_{23} = A_1 \frac{\ln B_2 B_2^{\frac{-1}{\delta_2+1}}}{(\delta_2 + 1)^2} \left[\frac{1 + K(\kappa)}{1 - \phi_2 q(e)} (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2 \omega_L^i K_1 - (1 + \phi_1 q(e)) \omega_L^f \left[\frac{\partial K}{\partial \kappa} \right]^2 E_1 \right] ? \quad (C.13)$$

When ψ_2 changes, impact negatively on the low skilled informality and has an uncertain effect on the higher ones:

Numerator N_{14} :

$$\frac{\partial \theta_L^i}{\partial \psi_2} = \frac{N_{14}}{D} < 0 \quad (C.14)$$

$$N_{14} = -\frac{C_2}{(1-\psi_2)^2} * \left[\frac{1-x}{(1-x-\theta_H^i)^2} E_1 K_1 - A_1 \frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2 \omega_H^i K_1 + A_1 (1+\phi_1 q(e)) \left[\frac{\partial K}{\partial \kappa} \right]^2 \omega_H^f E_1 \right] < 0 \quad (C.15)$$

Numerator N_{24} :

$$\frac{\partial \theta_H^i}{\partial \psi_2} = \frac{N_{24}}{D} ? \quad (C.16)$$

$$N_{24} = -\frac{C_2}{(1-\psi_2)^2} \left[\frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2 \omega_L^i K_1 - (1+\phi_1 q(e)) \omega_L^f \left[\frac{\partial K}{\partial \kappa} \right]^2 E_1 \right] ? \quad (C.17)$$

The sign of θ_H^i when δ_2 , changes depend on B_2 . If B_2 is higher than 1, the effect of a higher level of substitutability impact positively on the informality. in the case of θ_L^i the sign is uncertain.

Numerator N_{15} :

$$\frac{\partial \theta_L^i}{\partial \delta_1} = \frac{N_{15}}{D} ? \quad (C.18)$$

$$N_{15} = \frac{\ln B_1 B_1^{\frac{-1}{\delta_1+1}}}{(\delta_1+1)^2} \left[A_2 \frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2 \omega_L^i K_1 - A_2 (1+\phi_1 q(e)) \omega_H^f \left[\frac{\partial K}{\partial \kappa} \right]^2 E_1 \right] ? \quad (C.19)$$

Numerator N_{25} :

$$\begin{aligned} \frac{\partial \theta_H^i}{\partial \delta_1} &= \frac{N_{25}}{D} > 0 \quad \text{if } B_1 > 1 \\ \frac{\partial \theta_H^i}{\partial \delta_1} &= \frac{N_{25}}{D} < 0 \quad \text{if } B_1 < 1 \end{aligned} \quad (C.20)$$

$$N_{25} = \frac{\ln B_1 B_1^{\frac{-1}{\delta_1+1}}}{(\delta_1+1)^2} *$$

$$\left[\frac{1-x}{(1-x-\theta_H^i)^2} E_1 K_1 - A_2 \frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2 \omega_L^i K_1 + A_2 (1+\phi_1 q(e)) \left[\frac{\partial K}{\partial \kappa} \right]^2 \omega_L^f E_1 \right] > 0$$

if $B_1 > 1$

$$(C.21)$$

The change of ψ_1 has a uncertain effect on θ_L^i , and a negative effect on θ_H^i :

Numerator N_{16} :

$$\frac{\partial \theta_L^i}{\partial \psi_1} = \frac{N_{16}}{D} ? \quad (\text{C.22})$$

$$N_{16} = -\frac{C_1}{(1-\psi_1)^2} \left[A_2 \frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2 \omega_L^i K_1 - A_2 (1+\phi_1 q(e)) \omega_H^f \left[\frac{\partial K}{\partial \kappa} \right]^2 E_1 \right] ? \quad (\text{C.23})$$

Numerator N_{26} :

$$\frac{\partial \theta_H^i}{\partial \psi_1} = \frac{N_{26}}{D} < 0 \quad (\text{C.24})$$

$$N_{26} = -\frac{C_1}{(1-\psi_1)^2} * \left[\frac{1-x}{(1-x-\theta_H^i)^2} E_1 K_1 - A_2 \frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1^2 - \phi_2^2) \left[\frac{\partial q}{\partial e} \right]^2 \omega_L^i K_1 + A_2 (1+\phi_1 q(e)) \left[\frac{\partial K}{\partial \kappa} \right]^2 \omega_L^f E_1 \right] < 0 \quad (\text{C.25})$$

The change of the quality parameters a and d on θ_L^i and θ_H^i is negative. If the quality parameters are higher the informal shares reduce:

Numerator N_{17} :

$$\frac{\partial \theta_L^i}{\partial a} = \frac{N_{17}}{D} < 0 \quad (\text{C.26})$$

$$N_{17} = A_2 \frac{1+K(\kappa)}{1-\phi_2 q(e)} \frac{1-x}{(1-x-\theta_H^i)^2} (\phi_1 + \phi_2) \left[-\frac{\partial q}{\partial a} + (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) (\phi_1 - \phi_2) \frac{\partial^2 q}{\partial e \partial a} K_1 \right] < 0 \quad (\text{C.27})$$

Numerator N_{27} :

$$\frac{\partial \theta_H^i}{\partial a} = \frac{N_{27}}{D} < 0 \quad (\text{C.28})$$

$$N_{27} = A_1 \frac{1+K(\kappa)}{1-\phi_2 q(e)} \frac{x}{(x-\theta_L^i)^2} (\phi_1 + \phi_2) K_1 \left[(\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) (\phi_1 - \phi_2) \frac{\partial q}{\partial e} \frac{\partial^2 q}{\partial e \partial a} - \frac{\partial q}{\partial a} E_1 \right] < 0 \quad (\text{C.29})$$

Numerator N_{18} :

$$\frac{\partial \theta_L^i}{\partial d} = \frac{N_{18}}{D} < 0 \quad (\text{C.30})$$

$$N_{18} = A_2 (1+\phi_1 q(e)) \frac{1-x}{(1-x-\theta_H^i)^2} (\phi_1 + \phi_2) \left[-\frac{\partial K}{\partial \kappa} E_1 K_1 - (\omega_L^f (x-\theta_L^i) + \omega_H^f (1-x-\theta_H^i)) (\phi_1 - \phi_2) \frac{\partial^2 K}{\partial \kappa \partial d} \right] < 0 \quad (\text{C.31})$$

Numerator N_{28} :

$$\frac{\partial \theta_H^i}{\partial d} = \frac{N_{28}}{D} < 0 \quad (\text{C.32})$$

$$N_{28} = A_1(1 + \phi_1 q(e)) \frac{x}{(x - \theta_H^i)^2} \left[-\frac{\partial K}{\partial d} E_1 K_1 + (\omega_L^f(x - \theta_L^i) + \omega_H^f(1 - x - \theta_H^i)) \frac{\partial^2 K}{\partial \kappa \partial d} \frac{\partial K}{\partial \kappa} E_1 \right] < 0 \quad (\text{C.33})$$

The impact of the taxes τ on θ_L^i and θ_H^i is positive. If there are higher taxes (contributions in the model) the informality is higher:

Numerator N_{19} :

$$\frac{\partial \theta_L^i}{\partial \tau} = \frac{N_{19}}{D} > 0 \quad (\text{C.34})$$

$$N_{19} = A_2 \frac{1 - x}{(1 - x - \theta_H^i)^2} \frac{1 + K(\kappa)}{1 + \tau} (1 + \phi_1 q(e)) E_1 K_1 > 0 \quad (\text{C.35})$$

Numerator N_{29} :

$$\frac{\partial \theta_H^i}{\partial \tau} = \frac{N_{29}}{D} > 0 \quad (\text{C.36})$$

$$N_{29} = A_1 \frac{x}{(x - \theta_L^i)^2} \frac{1 + K(\kappa)}{1 + \tau} (1 + \phi_1 q(e)) E_1 K_1 > 0 \quad (\text{C.37})$$

The effect of the cost parameters (b_1 and b_2) on the informality is positive in all the cases. If the enforcement and the benefits are more expensive, the informality goes up.

Numerator N_{110} :

$$\frac{\partial \theta_L^i}{\partial b_1} = \frac{N_{110}}{D} > 0 \quad (\text{C.38})$$

$$N_{110} = -\frac{A_2(1 + K(\kappa))(\phi_1 + \phi_2)}{1 - \phi_2 q(e)} \frac{1 - x}{(1 - x - \theta_H^i)^2} \frac{\partial^2 B}{\partial e \partial b_1} \frac{\partial q}{\partial e} K_1 < 0 \quad (\text{C.39})$$

Numerator N_{210} :

$$\frac{\partial \theta_H^i}{\partial b_1} = \frac{N_{210}}{D} > 0 \quad (\text{C.40})$$

$$N_{210} = -\frac{A_1(1 + K(\kappa))(\phi_1 + \phi_2)}{1 - \phi_2 q(e)} \frac{x}{(x - \theta_L^i)^2} \frac{\partial^2 B}{\partial e \partial b_1} \frac{\partial q}{\partial e} K_1 < 0 \quad (\text{C.41})$$

Numerator N_{111} :

$$\frac{\partial \theta_L^i}{\partial b_2} = \frac{N_{111}}{D} > 0 \quad (\text{C.42})$$

$$N_{111} = -A_2 \frac{1-x}{(1-x-\theta_H^i)^2} \frac{\partial^2 B}{\partial \kappa \partial b_2} \frac{\partial K}{\partial \kappa} E_1 < 0 \quad (\text{C.43})$$

Numerator N_{211} :

$$\frac{\partial \theta_H^i}{\partial b_2} = \frac{N_{211}}{D} > 0 \quad (\text{C.44})$$

$$N_{211} = -A_1 \frac{x}{(x-\theta_L^i)^2} \frac{\partial^2 B}{\partial \kappa \partial b_2} \frac{\partial K}{\partial \kappa} E_1 < 0 \quad (\text{C.45})$$

C.2 Comparative statics of e^*

The effect of ϕ_1 on the equilibrium enforcement is negative, if there are more fines the quality of the enforcement is lower:

Numerator N_{31} :

$$\frac{\partial e}{\partial \phi_1} = \frac{N_{31}}{D} < 0 \quad (\text{C.46})$$

$$\begin{aligned} N_{31} = & A_2(1+K(\kappa))q(e) \frac{1-x}{(1-x-\theta_H^i)^2} (\phi_1 - \phi_2) \frac{\partial q}{\partial e} \omega_L^i K_1 \\ & - (1-\phi_2 q(e)) (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \frac{\partial q}{\partial e} \frac{\partial K}{\partial \kappa} \left(\frac{x}{x-\theta_L^i} A_1 (\phi_1 - \phi_2) \frac{\partial q}{\partial e} \omega_H^f + \frac{1-x}{1-x-\theta_H^i} A_2 \frac{\partial K}{\partial \kappa} \omega_L^f \right) < 0 \end{aligned} \quad (\text{C.47})$$

When ϕ_2 is higher the sign of the effect on e^* is uncertain.

Numerator N_{32} :

$$\frac{\partial e}{\partial \phi_2} = \frac{N_{32}}{D} ? \quad (\text{C.48})$$

$$\begin{aligned} N_{32} = & (1 + \phi_1 q(e)) \left[\frac{\partial K(\kappa)}{\partial \kappa} \right]^2 \left(\frac{\partial q}{\partial e} \right) (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \left(\frac{A_1 \omega_H^f x}{(x-\theta_L^i)^2} + \frac{A_2 \omega_L^f (1-x)}{(1-x-\theta_H^i)^2} \right) \\ & \frac{1+K(\kappa)}{1-\phi_2 q(e)} (1 + \phi_1 q(e)) (\phi_1 - \phi_2) q(e) \left(\frac{\partial q}{\partial e} \right) K_1 \left(\frac{A_1 \omega_H^i x}{(x-\theta_L^i)^2} + \frac{A_2 \omega_L^i (1-x)}{(1-x-\theta_H^i)^2} \right) \\ & \frac{1-x}{(1-x-\theta_H^i)^2} \frac{x}{(x-\theta_L^i)^2} (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \left(\frac{\partial q}{\partial e} \right) K_1 ?? \end{aligned} \quad (\text{C.49})$$

The comparative statics of e^* respect to δ_2 depend on B_2 . The sign goes in the same direction of the sign of the change of the informality.

Numerator N_{33} :

$$\begin{aligned}\frac{\partial e}{\partial \delta_2} &= \frac{N_{33}}{D} > 0 \quad \text{if } B_2 > 1 \\ \frac{\partial e}{\partial \delta_2} &= \frac{N_{33}}{D} < 0 \quad \text{if } B_2 < 1\end{aligned}\tag{C.50}$$

$$\begin{aligned}N_{33} &= -\frac{\ln B_2 B_2^{\frac{-1}{\delta_2+1}}}{(\delta_2+1)^2} \frac{1-x}{(1-x-\theta_H^i)^2} \left[\omega_L^i(\phi_1-\phi_2) \frac{\partial q}{\partial e} K_1 \right] > 0 \quad \text{if } B_2 > 1 \\ N_{33} &= -\frac{\ln B_2 B_2^{\frac{-1}{\delta_2+1}}}{(\delta_2+1)^2} \frac{1-x}{(1-x-\theta_H^i)^2} \left[\omega_L^i(\phi_1-\phi_2) \frac{\partial q}{\partial e} K_1 \right] < 0 \quad \text{if } B_2 < 1\end{aligned}\tag{C.51}$$

Comparative statics of e^* respect to ψ_2 :

Numerator N_{34} :

$$\frac{\partial e}{\partial \psi_2} = \frac{N_{34}}{D} < 0\tag{C.52}$$

$$N_{34} = \frac{C_2}{(1-\psi_2)^2} \frac{1-x}{(1-x-\theta_H^i)^2} \left[\omega_L^i(\phi_1-\phi_2) \frac{\partial q}{\partial e} K_1 \right] < 0\tag{C.53}$$

The comparative statics of e^* respect to δ_1 depend on B_1 . The sign goes in the same direction of the sign of the change of the informality.

Numerator N_{35} :

$$\begin{aligned}\frac{\partial e}{\partial \delta_1} &= \frac{N_{35}}{D} > 0 \quad \text{if } B_1 > 1 \\ \frac{\partial e}{\partial \delta_1} &= \frac{N_{35}}{D} < 0 \quad \text{if } B_1 < 1\end{aligned}\tag{C.54}$$

$$\begin{aligned}N_{35} &= -\frac{\ln B_1 B_1^{\frac{-1}{\delta_1+1}}}{(\delta_1+1)^2} \frac{x}{(x-\theta_L^i)^2} \left[\omega_H^i(\phi_1-\phi_2) \frac{\partial q}{\partial e} K_1 \right] > 0 \quad \text{if } B_1 > 1 \\ N_{35} &= -\frac{\ln B_1 B_1^{\frac{-1}{\delta_1+1}}}{(\delta_1+1)^2} \frac{x}{(x-\theta_L^i)^2} \left[\omega_H^i(\phi_1-\phi_2) \frac{\partial q}{\partial e} K_1 \right] < 0 \quad \text{if } B_1 < 1\end{aligned}\tag{C.55}$$

As in the case of ψ_2 , the effect on e^* of a positive change in ψ_1 is negative, when the informal share has lower weight in the production function, the equilibrium share is lower and also de level of enforcement.

Numerator N_{36} :

$$\frac{\partial e}{\partial \psi_2} = \frac{N_{34}}{D} < 0 \quad (\text{C.56})$$

$$N_{36} = \frac{C_1}{(1 - \psi_1)^2} \frac{x}{(x - \theta_L^i)^2} \left[\omega_H^i (\phi_1 - \phi_2) \frac{\partial q}{\partial e} K_1 \right] < 0 \quad (\text{C.57})$$

The effect of the quality parameters is uncertain in the case of a , which is one in the enforcement function, and negative in the one of the benefits function (d).

Numerator N_{37} :

$$\frac{\partial e}{\partial a} = \frac{N_{37}??}{D} \quad (\text{C.58})$$

$$\begin{aligned} N_{37} = & -(1 + \phi_1 q(e)) \left(\omega_L^i \theta_L^i + \omega_H^i \theta_H^i \right) (\phi_1 - \phi_2) \frac{\partial^2 q}{\partial e \partial a} \left[\frac{\partial K}{\partial \kappa} \right]^2 \left(\frac{A_1 \omega_H^f x}{(x - \theta_L^i)^2} + \frac{A_2 \omega_L^f (1 - x)}{(1 - x - \theta_H^i)^2} \right) \\ & \frac{1 + K(\kappa)}{1 - \phi_2 q(e)} (\phi_1^2 - \phi_2^2) \frac{\partial q}{\partial e} \frac{\partial q}{\partial a} K_1 \left(\frac{A_1 \omega_H^i x}{(x - \theta_L^i)^2} + \frac{A_2 \omega_L^i (1 - x)}{(1 - x - \theta_H^i)^2} \right) \\ & - \frac{x}{(x - \theta_L^i)^2} \frac{1 - x}{(1 - x - \theta_H^i)^2} \left(\omega_L^i \theta_L^i + \omega_H^i \theta_H^i \right) (\phi_1 - \phi_2) \frac{\partial^2 q}{\partial e \partial a} K_1?? \end{aligned} \quad (\text{C.59})$$

Numerator N_{38} :

$$\frac{\partial e}{\partial d} = \frac{N_{38}}{D} < 0 \quad (\text{C.60})$$

$$\begin{aligned} N_{38} = & \left[(1 + \phi_1 q(e)) (\phi_1 - \phi_2) \frac{\partial K}{\partial \kappa} \frac{\partial q}{\partial e} \right] \left[\left(K_1 - \left(\omega_L^f (x - \theta_L^i) + \omega_H^f (1 - x - \theta_H^i) \right) \frac{\partial^2 K}{\partial e \partial d} \right) \right. \\ & \left. \left(\frac{A_1 \omega_H^i x}{(x - \theta_L^i)^2} + \frac{A_2 \omega_L^i (1 - x)}{(1 - x - \theta_H^i)^2} \right) \right] < 0 \end{aligned} \quad (\text{C.61})$$

The effect of the taxes on the equilibrium enforcement is positive, there are more resources to spend and one of the ways to do it is through the level of enforcement.

Numerator N_{39} :

$$\frac{\partial e}{\partial \tau} = \frac{N_{39}}{D} > 0 \quad (\text{C.62})$$

$$N_{39} = -\frac{1+K(\kappa)}{1+\tau}(1+\phi_1 q(e))(\phi_1 - \phi_2) \frac{\partial q}{\partial e} K_1 \left[\frac{A_1 x \omega_H^i}{(x - \theta_L^i)^2} + \frac{A_2 (1-x) \omega_L^i}{(1-x - \theta_H^i)^2} \right] > 0 \quad (\text{C.63})$$

Comparative statics of e respect to b_1 and b_2 :

Numerator N_{310} :

$$\frac{\partial e}{\partial b_1} = \frac{N_{310}}{D} \quad (\text{C.64})$$

Comparative statics of e respect to b_1 and b_2 :

Numerator N_{311} :

$$\frac{\partial e}{\partial b_2} = \frac{N_{311}}{D} \quad (\text{C.65})$$

$$N_{311} = (1 + \phi_1 q(e))(\phi_1 - \phi_2) \frac{\partial K}{\partial \kappa} \frac{\partial q}{\partial e} \frac{\partial B}{\partial \kappa \partial b_2} \left[\frac{A_1 x \omega_H^i}{(x - \theta_L^i)^2} + \frac{A_2 (1-x) \omega_L^i}{(1-x - \theta_H^i)^2} \right] > 0 \quad (\text{C.66})$$

C.3 Comparative statics of κ^*

The effect of ϕ_1 on the equilibrium level of benefits is positive, more fines produce more resources to the formal workers:

Numerator N_{41} :

$$\frac{\partial \kappa}{\partial \phi_1} = \frac{N_{41}}{D} > 0 \quad (\text{C.67})$$

$$N_{41} = \left[A_1 \omega_H^f \frac{x}{(x - \theta_L^i)^2} + A_2 \omega_L^f \frac{1-x}{(1-x - \theta_H^i)^2} \right] * \left[\frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1 + \phi_2) (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \left[\frac{\partial q}{\partial e} \right]^2 \frac{\partial K}{\partial \kappa} - (1+K(\kappa)) q(e) \frac{\partial K}{\partial \kappa} E_1 \right] > 0 \quad (\text{C.68})$$

Comparative statics of e respect to ϕ_2 :

Numerator N_{42} :

$$\frac{\partial e}{\partial \phi_2} = \frac{N_{42}}{D} < 0 \quad (\text{C.69})$$

$$\begin{aligned}
N_{42} &= -\frac{1+K(\kappa)}{1-\phi_2 q(e)} (\phi_1 + \phi_2) \left[\frac{\partial K(\kappa)}{\partial \kappa} \right]^2 \left[\frac{\partial q}{\partial e} \right]^2 (\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) \left(\frac{A_1 \omega_H^f x}{(x - \theta_L^i)^2} + \frac{A_2 \omega_L^f (1-x)}{(1-x - \theta_H^i)^2} \right) \\
&\quad \frac{1+K(\kappa)}{1-\phi_2 q(e)} (1 + \phi_1 q(e)) q(e) \left(\frac{\partial K}{\partial \kappa} \right) E_1 \left(\frac{A_1 \omega_H^f x}{(x - \theta_L^i)^2} + \frac{A_2 \omega_L^f (1-x)}{(1-x - \theta_H^i)^2} \right) < 0
\end{aligned} \tag{C.70}$$

The effect of δ_2 in the κ^* depend on B_2 as in the other variables, but the sign is the opposite than in the case of the informal shares and the enforcement.

Numerator N_{43} :

$$\begin{aligned}
\frac{\partial \kappa}{\partial \delta_2} &= \frac{N_{43}}{D} < 0 \quad \text{if } B_2 > 1 \\
\frac{\partial \kappa}{\partial \delta_2} &= \frac{N_{43}}{D} > 0 \quad \text{if } B_2 < 1
\end{aligned} \tag{C.71}$$

$$\begin{aligned}
N_{43} &= \frac{\ln B_2 B_2^{\frac{-1}{\delta_2+1}}}{(\delta_2 + 1)^2} \frac{1-x}{(1-x - \theta_H^i)^2} \omega_L^f \frac{\partial K}{\partial \kappa} E_1 < 0 \quad \text{if } B_2 > 1 \\
N_{43} &= \frac{\ln B_2 B_2^{\frac{-1}{\delta_2+1}}}{(\delta_2 + 1)^2} \frac{1-x}{(1-x - \theta_H^i)^2} \omega_L^f \frac{\partial K}{\partial \kappa} E_1 > 0 \quad \text{if } B_2 < 1
\end{aligned} \tag{C.72}$$

The effect of ψ_1 and ψ_2 is positive on κ . When the weight of the formal workers in the production function is higher the level of benefits in equilibrium goes in the same direction:

Numerator N_{44} :

$$\frac{\partial K}{\partial \psi_2} = \frac{N_{44}}{D} > 0 \tag{C.73}$$

$$N_{44} = -\frac{C_2}{(1-\psi_2)^2} \frac{1-x}{(1-x - \theta_H^i)^2} \omega_L^f \frac{\partial K}{\partial \kappa} E_1 > 0 \tag{C.74}$$

The effect of δ_1 is similar as in the case of δ_2 :

Numerator N_{45} :

$$\begin{aligned}
\frac{\partial \kappa}{\partial \delta_1} &= \frac{N_{45}}{D} < 0 \quad \text{if } B_1 > 1 \\
\frac{\partial \kappa}{\partial \delta_1} &= \frac{N_{45}}{D} > 0 \quad \text{if } B_1 < 1
\end{aligned} \tag{C.75}$$

$$N_{45} = \frac{\ln B_1 B_1^{\frac{-1}{\delta_1+1}}}{(\delta_1+1)^2} \frac{x}{(x-\theta_L^i)^2} \omega_H^f \frac{\partial K}{\partial \kappa} E_1 < 0 \quad \text{if } B_2 > 1$$

$$N_{45} = \frac{\ln B_1 B_1^{\frac{-1}{\delta_1+1}}}{(\delta_1+1)^2} \frac{x}{(x-\theta_L^i)^2} \omega_H^f \frac{\partial K}{\partial \kappa} E_1 > 0 \quad \text{if } B_2 < 1$$
(C.76)

Comparative statics of κ respect to ψ_1 :

Numerator N_{46} :

$$\frac{\partial K}{\partial \psi_1} = \frac{N_{46}}{D} > 0$$
(C.77)

$$N_{46} = -\frac{C_1}{(1-\psi_1)^2} \frac{x}{(x-\theta_L^i)^2} \omega_H^f \frac{\partial K}{\partial \kappa} E_1 > 0$$
(C.78)

The effect of the quality parameters as is expected is positive in the level of benefits in equilibrium:

Numerator N_{47} :

$$\frac{\partial \kappa}{\partial a} = \frac{N_{47}}{D} > 0$$
(C.79)

$$N_{47} = \left(\frac{A_1 \omega_H^f x}{(x-\theta_L^i)^2} + \frac{A_2 \omega_L^f (1-x)}{(1-x-\theta_H^i)^2} \right) \frac{1+K(\kappa)}{1-\phi_2 q(e)} \frac{\partial K}{\partial \kappa} \left((\omega_L^i \theta_L^i + \omega_H^i \theta_H^i) (\phi_1^2 \phi_2^2) \frac{\partial^2 q}{\partial e \partial a} \frac{\partial q}{\partial e} - (\phi_1 + \phi_2) \frac{\partial q}{\partial a} E_1 \right) > 0$$
(C.80)

Numerator N_{48} :

$$\frac{\partial \kappa}{\partial d} = \frac{N_{48}}{D} > 0$$
(C.81)

$$N_{48} = \frac{1+K(\kappa)}{1-\phi_2 q(e)} \left(\omega_L^i \theta_L^i + \omega_H^i \theta_H^i \right) \frac{\partial^2 K}{\partial e \partial d} (\phi_1^2 - \phi_2^2) \left(\frac{A_1 \omega_H^i x}{(x-\theta_L^i)^2} + \frac{A_2 \omega_L^i (1-x)}{(1-x-\theta_H^i)^2} \right)$$

$$- (1 + \phi_1 q(e)) \frac{\partial K}{\partial \kappa} \frac{\partial K}{\partial d} E_1 \left(\frac{A_1 \omega_H^f x}{(x-\theta_L^i)^2} + \frac{A_2 \omega_L^f (1-x)}{(1-x-\theta_H^i)^2} \right)$$

$$- \frac{x}{(x-\theta_L^i)^2} \frac{1-x}{(1-x-\theta_H^i)^2} \left(\omega_L^i \theta_L^i + \omega_H^i \theta_H^i \right) \frac{\partial^2 K}{\partial e \partial d} E_1 > 0$$
(C.82)

The effect of the taxes (τ) is negative in the benefits, there are fewer formal workers and the government in equilibrium spends more money in the enforcement task:

Numerator N_{49} :

$$\frac{\partial \kappa}{\partial \tau} = \frac{N_{49}}{D} < 0 \quad (\text{C.83})$$

$$N_{49} = \frac{1 + K(\kappa)}{1 + \tau} (1 + \phi_1 q(e)) \frac{\partial K}{\partial \kappa} E_1 \left[\frac{A_1 x \omega_H^f}{(x - \theta_L^i)^2} + \frac{A_2 (1 - x) \omega_L^f}{(1 - x - \theta_H^i)^2} \right] < 0 \quad (\text{C.84})$$

Comparative statics of κ respect to b_1 :

Numerator N_{410} :

$$\frac{\partial \kappa}{\partial b_1} = \frac{N_{410}}{D} < 0 \quad (\text{C.85})$$

$$N_{410} = -\frac{1 + K(\kappa)}{1 - \phi_2 q(e)} (\phi_1 + \phi_2) \frac{\partial K}{\partial \kappa} \frac{\partial q}{\partial e} \frac{\partial^2 B}{\partial e \partial b_1} \left[\frac{A_1 x \omega_H^i}{(x - \theta_L^i)^2} + \frac{A_2 (1 - x) \omega_L^f}{(1 - x - \theta_H^i)^2} \right] < 0 \quad (\text{C.86})$$

C.4 Comparative statics respect of couple of parameters

Figure C.1: Changes in the equilibrium with the fines (ϕ_1) and the informal wage losses (ϕ_2)

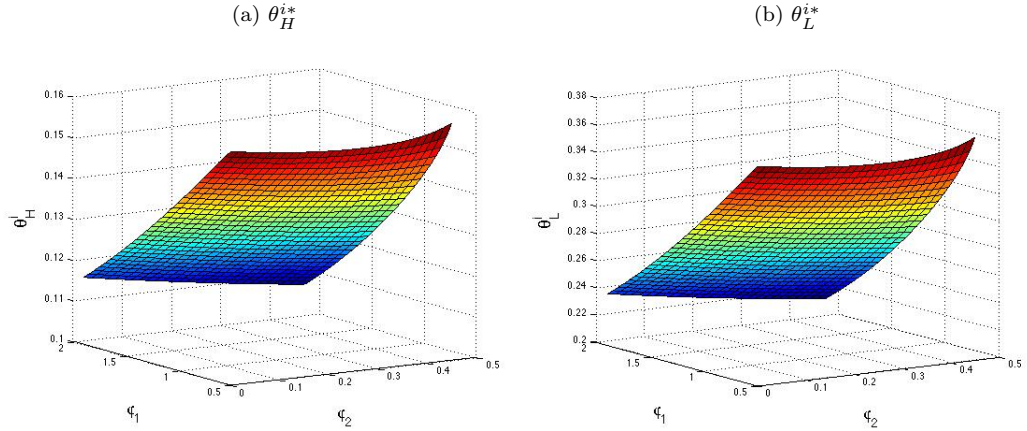
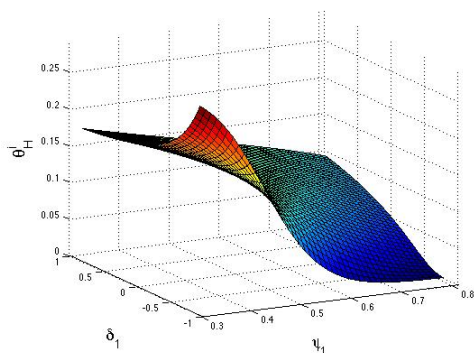
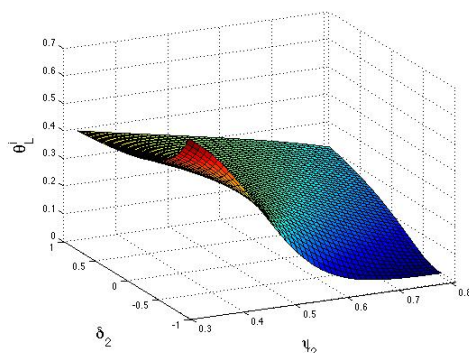


Figure C.2: Changes in the equilibrium with the level of substitutability and the shares (δ_j and ψ_j).

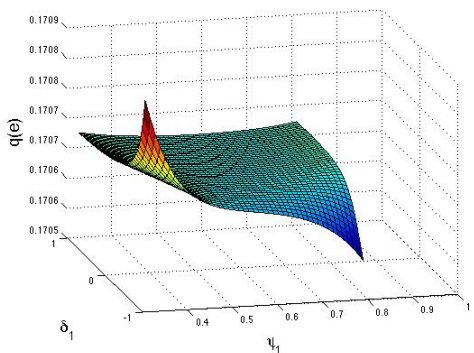
(a) $\theta_H^{i*}(\delta_1, \psi_1)$



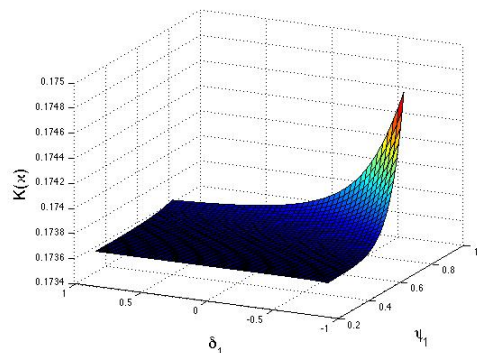
(b) $\theta_L^{i*}(\delta_2, \psi_2)$



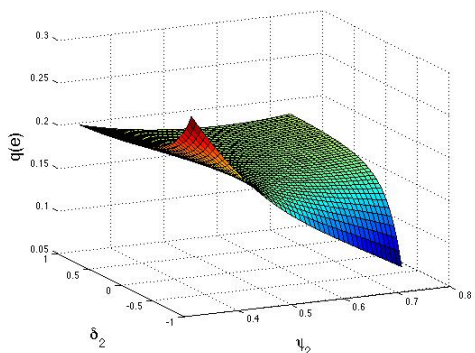
(c) $q(e)(\delta_1, \psi_1)$



(d) $K(\kappa)(\delta_1, \psi_1)$



(e) $q(e)(\delta_2, \psi_2)$



(f) $K(\kappa)(\delta_2, \psi_2)$

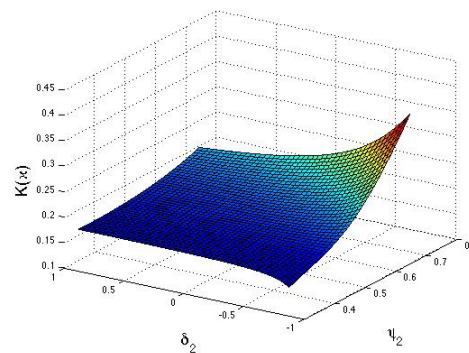


Figure C.3: Changes in the equilibrium with the quality parameters (a and d)

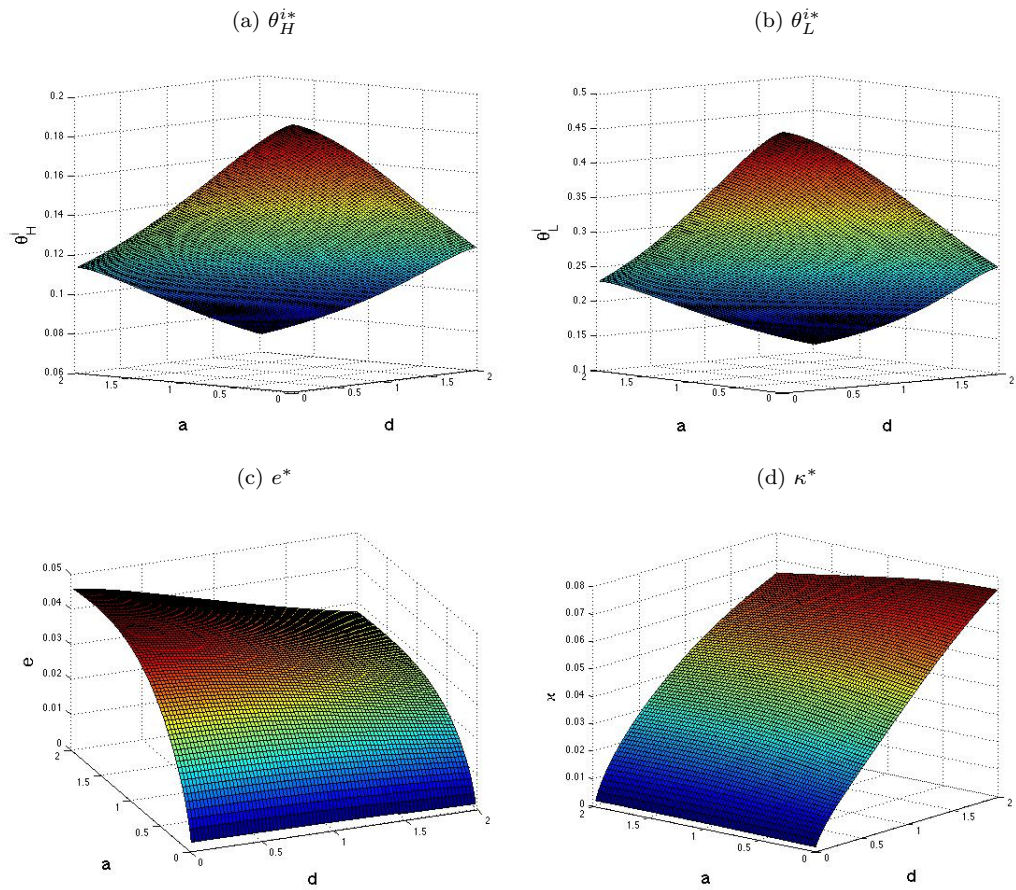


Figure C.4: Changes in the equilibrium with quality and cost of the enforcement
(a) and (b_1)

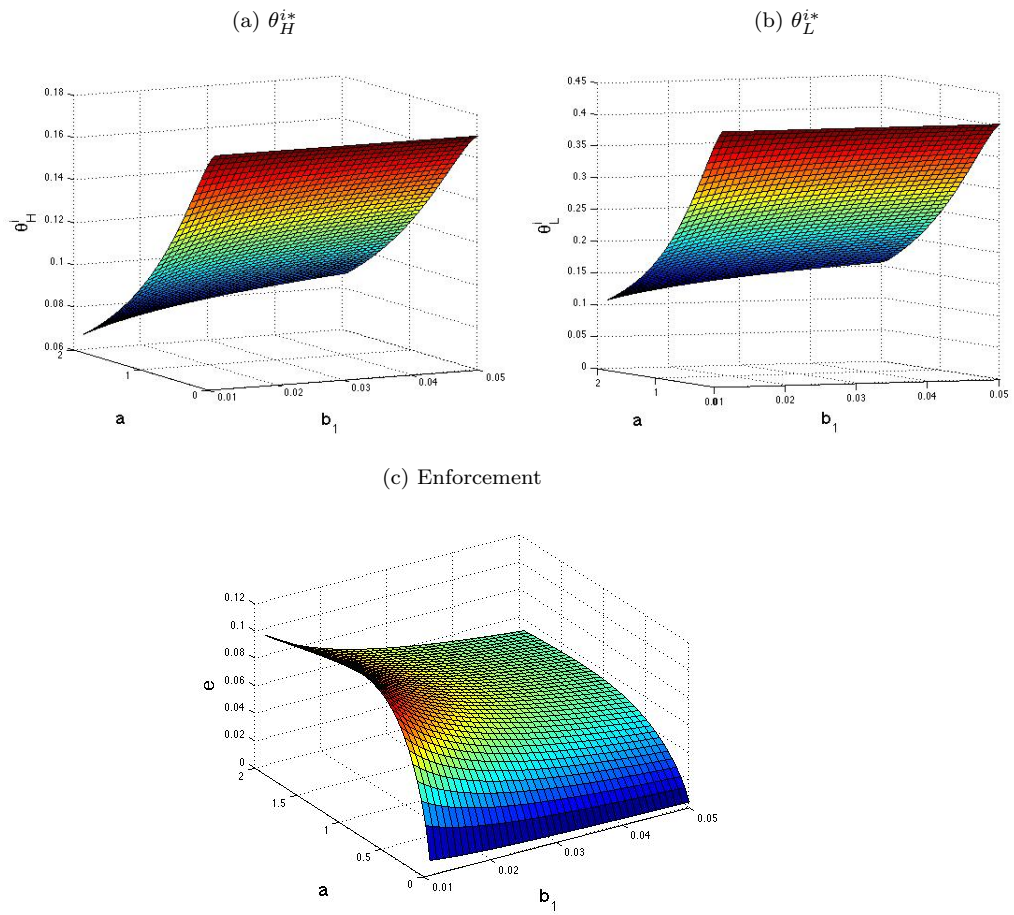


Figure C.5: Changes in the equilibrium with quality and cost of the benefits (d) and (b_2)

