

# On the Ambiguity of Job Search

Ying Tung, Chan and Chi Man, Yip  
McGill University

## Abstract

Unemployed workers and unfilled vacancies confront uncertainty about the distribution of the match specific productivity, and are averse to this ambiguity. However, prior search-and-matching models assume either complete information on the distribution or ambiguity neutrality. This paper constructs a search-and-matching model that features ambiguity aversion using the recursive specifications of Hansen and Sargent (2008). The model predicts that a robust unemployed worker tends to believe that higher match specific productivity is less likely to be realized, causing his/her outside option value and thus reservation wage to fall. Hence, an unemployment rate is lower under a higher degree of ambiguity aversion of workers. We propose a procedure to compute the “ambiguous” unemployment rate. Our calibration result indicates that the ambiguous unemployment could account for as large as 63.6% of the long-run average unemployment rate in the US. In addition, our results indicate that the ambiguity aversion amplifies the volatility of market tightness, potentially resolving the Shimer puzzle. Also, the impact of unemployment benefits on other labor market outcomes, such as market tightness, are shown to be larger under ambiguity aversion. This implication calls for a reexamination of the robust unemployment insurance scheme. Our analytical result shows that an efficient decentralized equilibrium is no longer guaranteed under Hosios (1990) condition.

# 1 Introduction

In existing search-and-matching models of unemployment (Rogerson et al., 2005), unemployed workers and vacancies do not know exactly match specific productivity before they meet, but they have complete information on a productivity distribution. In reality, how many workers can be confident about the productivity distribution, even though the Bureau of Labor Statistics updates the US labor market data monthly? If not, how do job finding behaviors under ambiguity aversion differ from the implications of the canonical job search model? To answer this question, the present paper proposes a search-and-matching model in which workers and vacancies are ambiguity averse: the productivity distribution is unknown, and agents are averse to this ambiguity. The objectives of this study are twofold: to explore the theoretical properties of the job search model under ambiguity aversion, and to assess, quantitatively, the role of the ambiguity aversion in the search model using US data.

The present model is not only theoretically appealing but also can make a significant difference for quantitative results. For example, the volatility of market tightness is much higher under ambiguity aversion in response to productivity shocks; therefore, our model potentially resolves the Shimer puzzle (Shimer, 2005). Furthermore, this paper demonstrates that the effect of unemployment benefits on labor market outcomes is subject to ambiguity aversion. Hence, the design of an unemployment benefit scheme should also consider ambiguity aversion of a worker. Moreover, we prove theoretically that unemployment rates are lower if workers are ambiguity averse, and this paper proposes a procedure to compute an “ambiguous unemployment” using US data.

Robust decision makers aim to guarantee a minimum level of an expected outcome conditional on the degree to which they fear the misspecification of their approximating model (Hansen and Sargent, 2008). If they are too conservative, they risk paying robustness premia. On the other hand, if they are not conservative enough, they risk being exposed to considerable misspecification errors. Understanding the implications of making robust choices is highly relevant for decision making during a job search process.

A more ambiguity averse worker tends to believe that average productivity of a potential job offer is lower, which causes his/her outside option value to decline. Therefore, lower match specific productivity is required for an unemployed worker and an unfilled vacancy to sign a contract. With a lower reservation productivity threshold, a lower wage is required to compensate a worker. Therefore, the expected match surplus of a vacancy is improved, which in turn provides an incentive to create vacancies. The lower reservation productivity level and the higher market tightness shorten an unemployment spell; as a result, the economy has a lower unemployment rate under a higher degree of ambiguity aversion.

As known, search unemployment, cyclical unemployment, and structural unemployment are

the main sources of unemployment.<sup>1</sup> Our model shows that ambiguity aversion naturally lowers unemployment rates. Such “ambiguous” unemployment is the increment in an unemployment rate if agents become ambiguity neutral. The present paper proposes the procedure to calculate an unemployment rate solely arising from ambiguity aversion. Our calibration result suggests that the “ambiguous” unemployment could account for as large as 63.6% of the long-run average unemployment rate in the US.

Our study also contributes to the literature on the impact of productivity shocks on a labor market. We show that ambiguity aversion leads to a milder growth in a reservation wage but a larger increase in market tightness in response to a positive productivity shock. When average productivity grows by a unit of standard derivation, the increase in market tightness under ambiguity aversion is about 1.5 times that of the standard model under ambiguity neutrality. While Shimer (2005) indicates that the volatility of market tightness predicted by the Mortensen and Pissarides (1994) model is too low to explain the US data, this result complements the literature (Shimer, 2005; Mortensen and Nagypal, 2007; Hagedorn and Manovskii, 2008; Pissarides, 2009; Fujita and Ramey, 2012) in understanding the cyclical properties of the labor market variables generated from ambiguity.

Another contribution of this analysis is the impact of unemployment benefits on labor market outcomes. For example, while a standard job search model predicts that an increase in unemployment benefits reduces market tightness, this study demonstrates that such reduction is much larger under ambiguity aversion. As shown in the literature<sup>2</sup>, market tightness plays an important role in a labor market. Therefore, the result implies that ambiguity aversion is one of the important factors to consider when designing an optimal unemployment insurance scheme.

Applications of robust control methods in Economics, pioneered by Hansen and Sargent, have been seen in Financial Economics (Cao et al., 2005), International Finance (Djeutem and Kasa, 2013), Macroeconomics (Ilut and Schneider, 2014), Monetary Economics (Adam and Woodford, 2012), Public Finance (Croce et al., 2012), etc. To the best of our knowledge, a job search model under ambiguity aversion is almost absent from the literature. Nishimura and Ozaki (2004) is an exception; their paper is the first to incorporate Knightian uncertainty into a job searcher problem. One of their key contributions is an association between a reservation wage and Knightian uncertainty. They find that a rise in Knightian uncertainty reduces a reservation wage under a general framework.

This paper complements the existing literature by considering ambiguity aversion from both worker and vacancy sides. The number of vacancies is endogenized so that the influence of am-

---

<sup>1</sup>Recent works discover another source of unemployment known as “rationing unemployment” and “mismatch unemployment”. Interested readers are referred to Michailat (2012) and Sahin et al. (2014) respectively.

<sup>2</sup>For example, Blanchard and Galí (2010) find that market tightness is a central variable in a New Keynesian model with search unemployment.

biguity on market tightness and unemployment rates can be explicitly examined. Our analytical results show that a reservation wage falls with the degree of ambiguity aversion of workers and vacancies; market tightness rises with the degree of ambiguity aversion of workers but falls with that of vacancies. In addition, we show that when vacancies have an extremely high degree of ambiguity aversion, they tend to have a belief that the realized match specific productivity will be too low to sign any contract. As a consequence, there exists no incentive to create any vacancy; thus, a labor market collapses.

This section serves as an introduction. Section 2 compares our preliminary model with the one under Knightian uncertainty in Nishimura and Ozaki (2004). Section 3 presents a basic model setting, followed by a discussion of its properties in the steady state equilibrium in Section 4. Comparative statics are shown analytically in Section 5. Section 6 presents a welfare analysis. We show that an efficient decentralized economics is not guaranteed in a search model with ambiguity under the Hosios condition. In Section 7, we calibrate the model to match the US data; in particular, we calibrate context-specific entropy penalty parameters. In addition, we introduce and compute the ambiguous unemployment using the US data. In the last part of Section 7, we investigate other roles of ambiguity in the search model. Section 8 concludes the paper.

## 2 Knightian Uncertainty

Consider an economy with infinitely lived workers, who are ambiguity averse, receiving wages when employed. A worker is assumed to be paid with an output  $\delta$ . A worker, once employed, will stick to the job forever. Denote  $J^E(\delta)$  and  $J^U$  as discounted present values of employment and unemployment, respectively. Hence,  $J^E(\delta) = \delta$ .

An unemployed worker enjoys an unemployment insurance  $b$ , and receives a job offer at the end of each period. When an unemployed worker and an unfilled vacancy meet, match specific productivity  $\delta$  is realized, where  $\delta$  follows a cumulative distribution function  $F(\delta)$ , and the corresponding probability density function is denoted by  $f(\delta)$ . The unemployed worker accepts a job offer only if  $J^E(\delta)$  is at least as large as his/her outside option  $J^U$ .

Nevertheless, an unemployed worker may not be strongly confident about the productivity distribution  $F(\delta)$ . When seeking a job, the unemployed worker is uncertain not only about the realized productivity but also the expected productivity. Meanwhile, the unemployed worker prefers a known productivity distribution; as a result, a robust worker maximizes the minimum expected outcome. Hence,  $J^U$  is written as follows:

$$J^U = b + \beta \min_m \left( E \left( m(\max\{J^E(x), J^U\}) - \frac{1}{\alpha} E m \ln m \right) \right) \quad (1)$$

subject to  $\int m(x)dF(x) = 1$ .  $\beta = 1/(1+r)$  is a discounted factor, where  $r$  is an interest rate. A

robust unemployed worker chooses  $m = \hat{f}(\delta)/f(\delta)$ , a likelihood ratio, to minimize the last term of the preceding equation.  $E m \ln m$ , known as a relative entropy, measures the Kullback Leibler distance between two distributions. In the present model, it measures the discrepancy between the approximated and the distorted model.  $\alpha \leq 0$  is a penalty parameter for the relative entropy (the last term of the preceding equation), which captures the degree of ambiguity aversion of a worker. If there is no fear of model misspecification,  $\alpha = 0$ . The more fear about the model misspecification the worker, the lower the  $\alpha$  will be. When  $\alpha$  approaches negative infinity, the last term of equation (1) vanishes. In this case, a worker chooses the distorted measure to minimize the value function, which is a well-known multiple prior preference. The optimal likelihood ratio that satisfies  $\int m(x)dF(x) = 1$  is given by

$$\frac{\hat{f}(\delta)}{f(\delta)} = \frac{e^{\alpha \max\{J^E(\delta), J^U\}}}{\int_0^\infty e^{\alpha \max\{J^E(x), J^U\}} f(x) dx} \quad (2)$$

It is interesting to note that  $\alpha = 0$  implies  $\hat{f}(\delta) = f(\delta)$ . When a worker does not worry about model misspecification, his/her belief of the probability density function is identical to the approximating one. If  $J^{E'}(\delta) > 0$  (which will be shown later), the likelihood ratio declines with  $\delta$ . Interestingly, the result implies that to minimize an expected outcome a robust unemployed worker chooses to believe that higher match specific productivity is less likely to be realized.

In Nishimura and Ozaki (2004), an unemployed worker, under Knightian uncertainty, minimizes his/her expected discounted future income as follows:

$$J^U = \min_{p \in \mathcal{P}_0} \left\{ \int_W I(w) dP(w) \right\} \quad (3)$$

where  $I(w)$ , a discounted future income, is a bounded measurable function of the observed offer  $w$ . The formula of  $I(w)$  is given by equation (18) of their paper.  $\mathcal{P}_0$  is a set of distributions; Knightian uncertainty increases with the size of the set  $\mathcal{P}_0$ . The key difference between Nishimura and Ozaki (2004) and our model lies on the specification of the probability set  $\mathcal{P}_0$ . In Nishimura and Ozaki (2004), the  $\mathcal{P}_0$  in the multiple prior expected utility is unrestricted.<sup>3</sup> In the present model, the size of the probability set depends on both the reference density  $f(\delta)$  and the penalty index  $\alpha$ . In fact, according to Hansen and Sargent (2001), the multiplier problem (1) is equivalent to the following constraint problem:

$$\min_{\hat{F}} \left\{ \int_W I(w) d\hat{F}(w) \right\} \text{ subject to } E_{\hat{F}}[\ln(d\hat{F}/dF)] < \gamma$$

which is equivalent to the problem (3) with  $\mathcal{P}_0$  restricted to the set of the probability distribution

---

<sup>3</sup>Since the current paper does not aim to investigate Knightian uncertainty, readers interested in its axiomatic foundation are referred to Gilboa and Schmeidler (1989).

$\hat{F}$  that satisfies  $E_{\hat{F}}[\ln(d\hat{F}/dF)] < \gamma$ .<sup>4</sup>  $\gamma$  is an upper bound of the discrepancy between the approximated distribution  $\hat{F}$  and the prior distribution  $F$ , and depends on the penalty index  $\alpha$ . The lower the  $\alpha$ , the higher will be the  $\gamma$ ; thus, a lower  $\alpha$  implies that a larger set of the distribution  $\hat{F}$  can be chosen. Therefore, the use of  $\alpha$  as a measure of the ambiguity aversion is in line with Nishimura and Ozaki (2004). In the extreme case where  $\alpha \rightarrow 0$ , we have  $\gamma \rightarrow 0$ ; as a result, only the prior distribution  $F$  satisfies  $E_{\hat{F}}[\ln(d\hat{F}/dF)] < \gamma$ . This is equivalent to the extreme case in Nishimura and Ozaki (2004) where the size of the probability set reduces to a singleton  $\mathcal{P}_0 = \{F\}$ .<sup>5</sup>

When  $\delta$  goes to zero,  $J^E = 0$  is less than  $J^U$ . Together with the fact that  $\partial J^E(\delta)/\partial \delta > 0$ , there exists a unique reservation productivity threshold  $\delta^R$ , above which workers will accept the job offer. Hence, we have  $\delta^R = J^U$ . Substituting the reservation productivity level and the optimal likelihood ratio (2) into equation (1), simple algebra yields

$$\frac{r\delta^R}{1+r} = b + \frac{\beta}{\alpha} \ln \left( F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha(x-\delta^R)} dF(x) \right)$$

Partially differentiating the preceding equation with respect to  $\alpha$ , it is straightforward to show that the higher the  $\alpha$ , the higher will be the threshold  $\delta^R$ . Nishimura and Ozaki (2004) build a similar model to demonstrate the impact of Knightian uncertainty on the reservation productivity threshold.<sup>6</sup> The implication of this model is in line with Nishimura and Ozaki (2004). They find that compared to ambiguity neutrality, the reservation productivity level is lower under Knightian uncertainty. In this case, a worker with a higher ambiguity aversion (lower  $\alpha$ ) tends to believe that a higher match specific productivity is less likely to be realized. Hence, his/her outside option value is lower, inducing the reservation productivity level to fall. Simple algebra shows that  $rJ^U/(1+r)$  equals an unemployment benefit  $b$  when  $\alpha$  approaches negative infinity.

As pointed out by Werner (2011), when  $\mathcal{P}_0$  in a multiple-prior expected utility expands to include all probability measures, the problem (3) reduces to the Wald minmax criterion. This yields the same value of  $J^U$  as in the present model when  $\alpha$  goes to negative infinity, even though the set  $\mathcal{P}_0$  in our model does not include all the probability measures as it depends on both the reference distribution  $F$  and the functional form of the instantaneous utility.

---

<sup>4</sup>Readers interested in its application to financial markets are referred to Cao et al. (2005).

<sup>5</sup>While the multiple-prior expected utility in Nishimura and Ozaki (2004) is not nested by the multiplier utility in our model, Werner (2011) points out that the multiple-prior preference is in the class of variational preference where cost function is given by  $C_{\mathcal{P}_0}\{P\} = \begin{cases} 0, & P \in \mathcal{P}_0; \\ \infty, & P \notin \mathcal{P}_0. \end{cases}$

<sup>6</sup>In section 2 of Nishimura and Ozaki (2004), a similar model is built for demonstration, followed by a more rigorous analysis under general preference and productivity distribution in section 4.

### 3 The Basic Model

In this section, we present a complete model to look into the mechanism through which ambiguity aversion of workers and vacancies influences wages, job offer acceptance rates and unemployment rates during the job search process. Consider an economy with a fixed labor force, without loss of generality normalized to unity. Workers are either employed or unemployed; they have infinite horizons and time is discrete.

Unlike the model in the previous section, an employed worker with a match specific productivity  $\delta$  receives a wage  $w(\delta)$  and, at the end of each period, faces a separation shock at a rate of  $\lambda$ . When the shock arrives, the worker becomes unemployed.  $J^E(\delta)$  can be written as follows:

$$J^E(\delta) = w(\delta) + \beta(\lambda J^U + (1 - \lambda)J^E) \quad (4)$$

An unemployed worker and an unfilled vacancy meet via a matching technology:  $M(u, v)$ , where  $u$  and  $v$  are the number of unemployed workers and the number of unfilled vacancies respectively. Assume that the matching technology is constant returns to scale. In each period, an unfilled vacancy contacts an unemployed worker at a rate  $q \equiv M(u, v)/v$ , where  $q$  is a differentiable decreasing function, and depends on market tightness  $\theta = v/u$ . It follows immediately from the constant returns to scale that  $M(u, v)/u = \theta q$ . From workers' perspective, the contact rate is given by  $p$  where  $p \equiv \theta q$  is an increasing function of  $\theta$ . (see Pissarides (2000)) We also make the standard Inada-type assumptions on  $M(u, v)$  so that  $\lim_{\theta \rightarrow 0} q = \infty$ ,  $\lim_{\theta \rightarrow \infty} q = 0$ ,  $\lim_{\theta \rightarrow 0} p = 0$ , and  $\lim_{\theta \rightarrow \infty} p = \infty$ .

The unemployed worker receives an unemployment benefit  $b$ , and meets an unfilled vacancy at a rate of  $p$ . When an unemployed worker and an unfilled vacancy meet, match specific productivity  $\delta$  is realized, where  $\delta$  follows a cumulative distribution function  $F(\delta)$  and the corresponding probability density function is denoted by  $f(\delta)$ . Again, the unemployed worker is ambiguity averse on the productivity distribution. Therefore, the unemployed worker chooses a likelihood ratio to minimize the expected outcome as follows.

$$J^U = b + \beta \min_m \left( E(m(p \max\{J^E(x), J^U\} + (1 - p)J^U)) - \frac{1}{\alpha} E m \ln m \right) \quad (5)$$

subject to  $\int m(x)dF(x) = 1$ , where  $m(\delta) = \hat{f}(\delta)/f(\delta)$  is a likelihood ratio. The corresponding optimal likelihood function is given by

$$\frac{\hat{f}(\delta)}{f(\delta)} = \frac{e^{\alpha \max\{J^E(\delta), J^U\}}}{\int_0^\infty e^{\alpha \max\{J^E(x), J^U\}} f(x) dx} \quad (6)$$

A filled vacancy generates a production value  $\delta$ , pays the worker wage  $w(\delta)$ , and faces the separation shock at the same rate  $\lambda$ . When the shock arrives, the vacancy becomes unfilled.

Denote  $J^F(\delta)$  and  $J^V$  as asset values of a filled vacancy and an unfilled vacancy respectively.  $J^F(\delta)$  can be written as follows:

$$J^F(\delta) = \delta - w(\delta) + \beta(\lambda J^V + (1 - \lambda)J^F) \quad (7)$$

When an unemployed worker and an unfilled worker meet, a match specific productivity  $\delta$  is realized. Understanding an entropy penalty parameter is also important in wage determination. If they do not have complete information on productivity distribution, an unemployed worker and an unfilled vacancy could obtain this kind of information during an interview. For example, an expected salary in a resume could reveal an unemployed worker's belief of his/her expected productivity. Another example is an aptitude test, which could serve multiple functions. In reality, this test enables vacancies to understand a worker's match-specific productivity and attitudes towards risk and ambiguity. For simplicity, we assume that entropy penalty parameters of workers and vacancies are common information. With the information, they bargain on a wage so that the wage maximizes the generalized Nash product as follows:

$$w(\delta) = \arg \max (J^E(\delta) - J^U)^\eta (J^F(\delta) - J^V)^{1-\eta}$$

where  $\eta$  is a bargaining power of a worker. Simple algebra gives the following sharing rule.

$$J^E(\delta) - J^U = \eta(J^E(\delta) - J^U + J^F(\delta) - J^V) \quad (8)$$

Intuitively, a match surplus of a worker is a fraction  $\eta$  of the total match surplus. An unfilled vacancy pays a maintenance cost  $c > 0$ , and faces a probability  $q$  of being filled. Denote  $\alpha_v \leq 0$  as a measure of the ambiguity aversion of a vacancy. Similar to workers, an unfilled vacancy is ambiguity averse and uncertain about the productivity distribution. Analogous to the unemployed, an unfilled vacancy maximizes the minimum expected outcome. To do so, a vacancy chooses  $m_v(\delta)$ , a likelihood ratio, to minimize the expected payoff. Hence, the discounted present value of being unfilled can be written as

$$J^V = -c + \beta \min_{m_v} \left( E (m_v(q \max\{J^F(x), J^V\} + (1 - q)J^V)) - \frac{1}{\alpha_v} E m_v \ln m \right) \quad (9)$$

subject to  $\int m_v(x) dF(x) = 1$ , where  $m_v(\delta) = \hat{f}_v(\delta)/f(\delta)$  is a likelihood ratio. The corresponding likelihood ratio is given by

$$\frac{\hat{f}_v(\delta)}{f(\delta)} = \frac{e^{\alpha_v \max\{J^F(\delta), J^V\}}}{\int_0^\infty e^{\alpha_v \max\{J^F(x), J^V\}} f(x) dx} \quad (10)$$



Assumption of free entry and exit is made; hence, rent is exhausted, and thus

$$J^V = 0 \quad (11)$$

Denote  $\delta_u^R$  as a reservation productivity level, below which an unemployed worker will not accept any job offer. Noting that the unemployed worker will accept the job offer only if  $J^E(\delta) \geq J^U$ ; hence,  $J^E(\delta_u^R) = J^U$ . A reservation productivity threshold for an unfilled vacancy  $\delta_v^R$  is defined in a similar way; therefore,  $J^F(\delta_v^R) = J^V$ . Using the sharing rule, the reservation productivity levels for a worker and a vacancy are identical. Hereafter, denote the reservation productivity level as  $\delta^R$ . Noting that  $J^F(\delta)$  strictly increases with  $\delta$  while other variables in the sharing rule (8) are independent of  $\delta$ .  $\delta^R$  is therefore unique. Hence, the unemployed and unfilled vacancies, when they meet, accept the offer for all  $\delta \geq \delta^R$ . Using equations (4), (7), (8), and (11), the wage equation is derived.

$$w(\delta) = (1 - \eta) \frac{rJ^U}{1 + r} + \eta\delta \quad (12)$$

Hence, a wage is a fraction of a productivity, plus a fraction of an outside option value. Using equations (4) and (12),

$$w(\delta^R) = \delta^R = \frac{rJ^U}{1 + r} \quad (13)$$

A reservation wage, a reservation productivity, and an outside option value are equal. With  $\delta^R$ , a worker is compensated with  $w(\delta^R)$ , which is equal to his/her outside option value. Hence, a match surplus of a worker with  $\delta^R$  is zero. From the vacancy's perspective, a filled vacancy generates an output  $\delta^R$ , and pays the worker with  $w(\delta^R)$ . Hence, a match surplus of a vacancy with  $\delta^R$  is also zero.

A steady state unemployment rate is determined by equating flows out and into an unemployment, and is given by

$$u = \frac{\lambda}{\lambda + (1 - F(\delta^R))p} \quad (14)$$

## 4 Characterization of Steady-State Equilibrium

**Definition 1.** A steady state equilibrium is defined as  $\{\hat{f}(\delta), \hat{f}_v(\delta), \delta^R, w(\delta), \theta, J^E(\delta), J^U, J^F(\delta), J^V\}$  such that equations (4)-(11) and (13) for all  $\delta \geq \delta^R$  are satisfied. The steady state unemployment rate is given by equation (14).

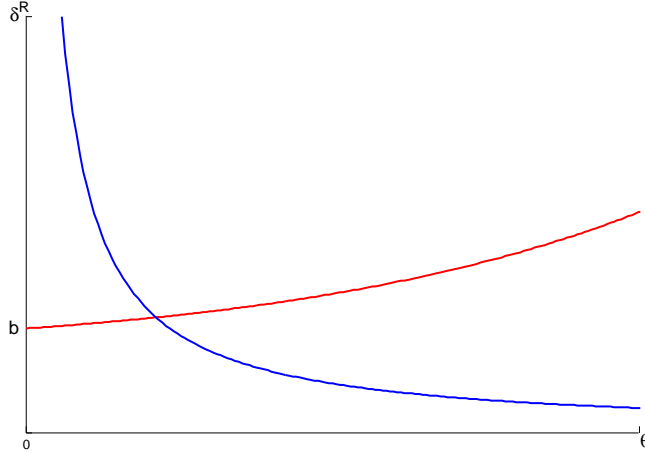


Figure 1: Equilibrium Determination

Substituting equations (4), (6), (12), and (13) into equation (5), simple algebra gives

$$\delta^R = b + \frac{\beta p}{\alpha} \ln \left( F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha \eta (1+r)}{r+\lambda} (x-\delta^R)} dF(x) \right) \quad (15)$$

Similarly, substituting equations (7), (10), (12), and (13) into equation (9) yields

$$c = \frac{\beta q}{\alpha_v} \ln \left( F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha_v (1-\eta)(1+r)}{r+\lambda} (x-\delta^R)} dF(x) \right) \quad (16)$$

A steady state equilibrium is characterized by the intersection of the above two loci: equations (15) and (16). The loci in the  $\delta^R$ - $\theta$  plane are shown in Figure 1. Locus (15), along which an outside option value of the unemployed equals the reservation productivity level, is upward sloping: a higher  $\theta$  increases the RHS of equation (15); thus,  $\delta^R$  has to increase. When  $\theta$  goes to zero,  $\delta^R$  approaches  $b$ . Intuitively, an increase in market tightness raises the transition rate from unemployment to employment, inducing an outside option value of the unemployed to rise. Hence, a higher reservation productivity level is required for them to accept a job offer.

A locus (16), along which an unfilled vacancy makes zero profits, is downward sloping. In contrast to the unemployed, a rise in market tightness reduces vacancies' probability of being filled, lowering  $J^V$ . Hence, a reservation productivity level falls so as to raise the transition rate of being filled to maintain zero profits. When  $\theta$  goes to zero,  $\delta^R$  goes to infinity. By the continuity of the two functions, the two loci intersect once, where  $\delta^R \in (b, \infty)$  and  $\theta \in (0, \infty)$ .

**Proposition 1.** *There exists a unique steady state equilibrium and is characterised by equations (15) and (16). In the equilibrium,  $\delta^R \in (b, \infty)$  and  $\theta \in (0, \infty)$ .*

Using equations (5), (6), (9), and (10), an outside option value of a worker and a vacancy can

be written as follows:

$$\begin{aligned} J^U &= b + \beta \left( \frac{p}{\alpha} \ln \int_0^\infty e^{\alpha \max\{J^E(x), J^U\}} dF(x) + (1-p)J^U \right) \\ J^V &= -c + \beta \left( \frac{q}{\alpha_v} \ln \int_0^\infty e^{\alpha_v \max\{J^F(x), J^V\}} dF(x) + (1-q)J^V \right) \end{aligned}$$

Two points merit comment. When  $\alpha = 0$  and  $\alpha_v = 0$ , the preceding equations reduce to

$$\begin{aligned} J^U &= b + \beta(p \int_0^\infty \max\{J^E(x), J^U\} dF(x) + (1-p)J^U) \\ J^V &= -c + \beta(q \int_0^\infty \max\{J^F(x), J^V\} dF(x) + (1-q)J^V) \end{aligned}$$

They are outside option values of a worker and a vacancy in a standard job search model. Therefore, our model nests the standard one. Another point is that the outside option values of a worker and a vacancy do not preserve the feature of linearity with  $J^E(\delta)$  and  $J^F(\delta)$  under ambiguity. The nice feature of linearity of the value functions enable economists to easily simplify terms using the sharing rule (8). For example, Hosios (1990) makes use of the linearity to simplify a welfare function to a weighted sum of flow values of workers and vacancies.<sup>7</sup> The linearity is noteworthy if one is concerned with an efficiency in a decentralized equilibrium. We will discuss this issue in Section 6.

**Proposition 2.** *In the steady state equilibrium, i)  $\theta$  falls with  $\alpha$  but increases with  $\alpha_v$ ; ii)  $\delta^R$  increases with both  $\alpha$  and  $\alpha_v$ ; and iii)  $w(\delta)$  increases with both  $\alpha$  and  $\alpha_v$ , for all  $\delta \geq \delta^R$ . The unemployment rate is lower in the economy under higher degree of ambiguity aversion of workers.*

*Proof.* See the Appendix. □

Figure 2 demonstrates the impact of the fall in  $\alpha$  and  $\alpha_v$  respectively. An increase in the degree of the ambiguity aversion (smaller  $\alpha$ ) rotates a locus (15) clockwise. Noting that a change in  $\alpha$  does not have any effect on the locus (16). Now, we have a new steady state equilibrium at the intersection of two loci, with  $\delta^R$  smaller and  $\theta$  larger than the old steady state level.

Intuitively, when workers have more fear about the uncertain productivity distribution, they tend to believe that they will match with a vacancy with a lower match specific productivity. Outside option values of workers fall; hence, a lower reservation productivity level and a smaller match surplus are required for the unemployed to accept a job offer. Noting that the expected match gain equals a transition probability from unemployment to employment times a match surplus. The effect of  $\alpha$  on the expected match gain is amplified by  $\theta q(\theta)$ , which increases with  $\theta$ . Therefore, the impact of  $\alpha$  on  $J^U$  and thus  $\delta^R$  rises with  $\theta$ . When  $\theta$  is zero, the expected match

---

<sup>7</sup>The computation can be found in the footnote 3 in Hosios (1990).

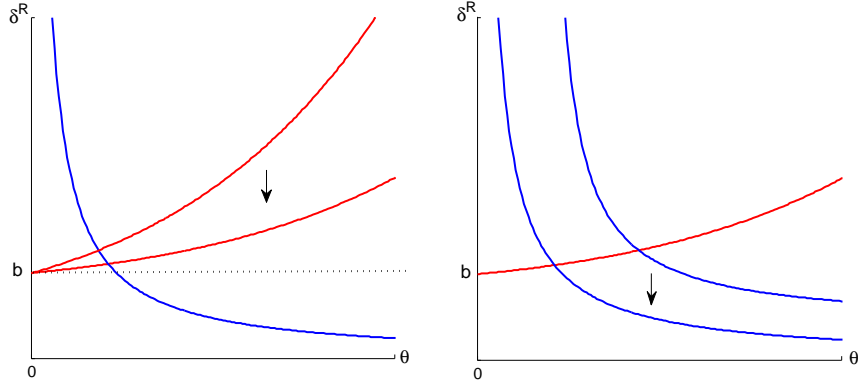


Figure 2: The left (right) panel shows the effect of stronger ambiguity aversion of worker (vacancy).

gain is zero regardless of  $\alpha$ . Hence, when  $\theta$  is zero,  $\delta^R$  remains unchanged even though workers are more ambiguity averse. Regarding the general equilibrium effect, a fall in  $J^U$  implies that for all  $\delta \geq \delta^R$ , a lower wage  $w(\delta)$  is required to compensate workers. Hence, vacancies, when filled, makes more profits, causing the expected match surplus of vacancy to climb up. More vacancies are thus created to exhaust the rent, inducing  $\theta$  to rise. Both the reduction in  $\delta^R$  and the increase in  $\theta$  raise the transition rate from unemployment to employment and thus shorten the unemployment spell.

Unfilled vacancies with stronger ambiguity aversion (smaller  $\alpha_v$ ) associate with a lower expected productivity level to match. With lower expected profits, some unfilled vacancies exit a market to raise  $q(\theta)$  so as to maintain zero profits in the steady state equilibrium. This shifts the locus (16) to left. Noting that a change of vacancies' attitude towards ambiguity does not affect the locus (15). In equilibrium, the fall in the number of vacancies reduces the matching rate of the unemployed, lowering their  $J^U$  and thus  $\delta^R$  and  $w(\delta)$ . As a result, the fall in  $\alpha_v$  reduces the reservation productivity level and the wage in the equilibrium. While the fall in  $\theta$  reduces a job contact rate, the fall in  $\delta^R$  increases a job acceptance rate. We are therefore uncertain about the effect of  $\alpha_v$  on the unemployment rate.

Denote  $u(\alpha, \alpha_v)$  as the steady state level of the unemployment rate. In the steady state, the unemployment rate arises from ambiguity aversion is equal to  $u(\alpha, \alpha_v) - u(0, 0)$ , where  $u(0, 0)$  is the steady state unemployment rate under ambiguity neutrality. Proposition 2 implies that  $du(\alpha, \alpha_v)/d\alpha > 0$ . In section 7, this "ambiguous" unemployment rate is computed numerically using US data.

**Proposition 3.** *When  $\alpha \rightarrow -\infty$ , the reservation productivity level  $\delta^R$  equals the unemployment benefit  $b$ .  $\hat{f}(\delta) = 0$  for all  $\delta > \delta^R$ , and  $\hat{f}(\delta) = f(\delta)/F(\delta^R)$  for all  $\delta \leq \delta^R$ . When  $\alpha_v \rightarrow -\infty$ , the steady state equilibrium  $\delta^R = b$  and  $\theta = 0$ .  $\hat{f}_v(\delta) = 0$  for all  $\delta > \delta^R$ , and  $\hat{f}_v(\delta) = f_v(\delta)/F(\delta^R)$*

for all  $\delta \leq \delta^R$ .

*Proof.* See the Appendix. □

When  $\alpha \rightarrow -\infty$ , the locus (15), as shown in Figure 2, is a horizontal line at  $\delta^R = b$ . An unemployed worker does not believe that s/he could find any vacancy with a realized productivity above the reservation threshold. As a result, their outside option values reduce to the unemployment benefit. Given  $\delta^R = b$ , equilibrium  $\theta$  is pinned down using equation (16). On the contrary, when  $\alpha_v$  goes to negative infinity, unfilled vacancies, to minimize expected outcome, believe that there will be no match with a realized productivity above the reservation threshold. All the vacancies leave a market; thus  $\theta$  equals zero. Hence, the locus (16) is a vertical line at  $\theta = 0$ . The two loci intersect at  $\delta^R = b$  and  $\theta = 0$  in the steady state equilibrium.

**Corollary 1.** *In the steady state equilibrium, a labor market persists if  $\alpha$  approaches  $-\infty$  but collapses if  $\alpha_v$  tends to  $-\infty$ .*

## 5 Comparative Statics

In this section, we investigate how changes in a productivity, a maintenance cost, an unemployment benefit and a job matching function influence other labor market outcomes under ambiguity aversion. Using equations (12) and (13), the wage equation is given by

$$w(\delta) = (1 - \eta)\delta^R + \eta\delta \tag{17}$$

A rise in a reservation productivity level increases a wage given  $\delta \geq \delta^R$ . This association is repeatedly used in following propositions. First, we prove that a productivity shock increases a reservation wage. Consider a productivity distribution  $G$  that first order stochastically dominates another productivity distribution  $F$ . The following proposition highlights its property in a steady state equilibrium.

**Proposition 4.** *If  $F$  and  $G$  are two productivity distributions where  $G \succeq_{FOSD} F$ ,  $w_G(\delta) > w_F(\delta)$  for all  $\delta \geq \delta_G^R$ .*

*Proof.* See the Appendix. □

With an improvement in the productivity distribution, a worker expects to draw a higher  $\delta$ . Hence, s/he is willing to wait longer for higher realized productivity, causing the reservation productivity level and thus the wage to rise. The impact of such improvement in productivity on market tightness is uncertain. The improvement leads to higher expected profits, which attracts more vacancies to enter the market. Meanwhile, it could lower the successful matching rate because unemployed workers become more ‘picky’, discouraging vacancies to stay in the market.

We are uncertain about the magnitude of the two effects and thus the total effect on the market tightness in the equilibrium.

**Proposition 5.** *In a steady state equilibrium, a rise in an unemployment benefit reduces  $\theta$ , increases  $\delta^R$  and thus  $u$ . A rise in a maintenance cost reduces  $\theta$  and  $\delta^R$ .*

*Proof.* See the Appendix. □

A rise in an unemployment benefit increases workers' outside option values, which in turn raises a reservation productivity level. As a result, an unemployed worker becomes more 'picky'. This lengthens an unemployment spell and thus worsen an unemployment rate. As indicated by the wage equation (17), higher wages and thus lower expected flow profits result. Therefore, vacancies leave a market until the free entry and exit condition holds, which causes market tightness to fall.

Similar to an unemployment benefit, a rise in a cost of posting a vacancy reduces market tightness. As  $c$  increases, it requires higher expected flow profits to maintain zero profits, discouraging vacancies to stay in a market. This causes  $\theta$  to fall. From workers' perspective, it becomes harder to seek a job, lowering their outside option values and thus  $\delta^R$ . A wage therefore declines in response to an increase in the posting cost.

**Proposition 6.** *If  $q_1(\theta) > q_2(\theta)$  for all  $\theta \in \mathcal{R}_{++}$ ,  $\delta_1^R > \delta_2^R$  and thus  $w_1(\delta) > w_2(\delta)$  in the equilibrium.*

*Proof.* See the Appendix. □

It is obvious that an improvement in a matching technology increases a job contact rate. A higher matching rate of both workers and vacancies allow them to re-search another offer more easily if either side of them reject the current offer. They become more 'picky' to the offer, driving up a reservation level of productivity. A natural consequence is that a wage increases in response to an increase in  $\delta^R$ . However, the impact of the improvement in a matching technology on  $\theta$  is uncertain. On the one hand, a rise in a contact rate increases an outside option value of a vacancy. On the other hand, a rise in a wage reduces flow profits and thus a match surplus of a vacancy. This general equilibrium effect deteriorates an outside option value of a vacancy. Therefore, we are uncertain about the effect of the improvement in a matching technology on  $J^V$  and thus  $\theta$ .

We show analytically how changes of a maintenance cost, an unemployment benefit, a job matching function, and a productivity affect the other labor market outcomes in our model. Since our model nests the standard one under ambiguity neutrality, the results hold under both ambiguity neutrality and ambiguity aversion. In section 7, numerical methods are used to investigate how ambiguity aversion influences these effects on labor market outcomes.

## 6 Welfare Analysis

Average productivity generated by employed workers is  $\int_{\delta^R}^{\infty} x dF(x)/(1 - F(\delta^R))$ ; benefits enjoyed by unemployed workers are  $b$ . Each vacancy incurs a maintenance cost  $c$ . The social planner of ambiguity neutrality chooses the measure of vacancies, a reservation productivity level, and the next period's employment level; hence, the social planner's problem for an infinitely lived economy can be written as

$$\max_{v_t, n_{t+1}, \delta^R} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\int_{\delta^R}^{\infty} x dF(x)}{1 - F(\delta^R)} n_t + b(1 - n_t) - cv_t \right\}$$

subject to

$$n_{t+1} = (1 - \lambda)n_t + q\left(\frac{v_t}{1 - n_t}\right)v_t(1 - F(\delta^R)) \quad (18)$$

Since a wage is the distribution of an output, the social planner is not interested in the wage. The optimal paths of the unemployment, the reservation productivity level, and the market tightness satisfy the constraint (18), and are given by

$$-\beta^t c + v_t q(\theta_t)(1 - \zeta(\theta_t))(1 - F(\delta^R)) = 0 \quad (19)$$

$$\beta^{t+1} \left( \frac{\int_{\delta^R}^{\infty} x dF(x)}{1 - F(\delta^R)} - b \right) - v_t + v_{t+1}((1 - \lambda) - \theta_{t+1} q(\theta_{t+1}) \zeta(\theta_{t+1})(1 - F(\delta^R))) = 0 \quad (20)$$

$$\beta^t \left( \frac{-\delta^R f(\delta^R)}{1 - F(\delta^R)} + \frac{f(\delta^R) \int_{\delta^R}^{\infty} x dF(x)}{(1 - F(\delta^R))^2} \right) n_t - v_t v_{t+1} q(\theta_t) f(\delta^R) = 0 \quad (21)$$

where  $v_t$  is a co-state variable. Denote  $\zeta(\theta_t) \in (0, 1)$  as the negative of the elasticity of  $q(\theta_t)$ . We substitute  $v_t$  from equation (19) into equation (20) and evaluate the outcome in the steady state to yield

$$\left( \int_{\delta^R}^{\infty} x dF(x) - b(1 - F(\delta^R)) \right) q(\theta)(1 - \zeta(\theta)) = c(r + \lambda + (1 - F(\delta^R))\theta q(\theta)\zeta(\theta)) \quad (22)$$

This efficiency condition is identical to the one in the standard job search model. (Pissarides, 2000) In a decentralized economy, the conventional search model could give the following equation:

$$\left( \int_{\delta^R}^{\infty} x dF(x) - b(1 - F(\delta^R)) \right) q(\theta)(1 - \eta) = c((r + \lambda) + (1 - F(\delta^R))\theta q(\theta)\eta) \quad (23)$$

A comparison of the social optimum (22) to the private outcome (23) shows that the decen-

tralized equilibrium is efficient only if the Hosios condition  $\zeta = \eta$  is satisfied. (Hosios, 1990) Noting that if the elasticity of  $q(\theta)$  is less than the bargaining power of worker, a vacancy supply is too low in the equilibrium. A rise in the elasticity of  $q(\theta)$  increases the negative impact of an additional vacancy on other unfilled vacancies' probability of being filled. As a result, the social planner prefers a lower bargaining power of a worker so as to induce more vacancies in the market.

To have the expression (23) in a decentralized equilibrium requires  $\alpha = 0$  and  $\alpha_v = 0$ . In this case, workers and vacancies are ambiguity neutral. In fact, to obtain equation (23),  $J^U$  is required to be linear with  $b$  and  $J^E(\delta)$  while  $J^V$  is required to be linear with  $c$  and  $J^F(\delta)$ . In the presence of ambiguity aversion of worker or vacancy,  $J^U$  and  $J^V$  are not linear with  $J^E(\delta)$  and  $J^F(\delta)$  respectively. Our model could by no means reach equation (23) if  $\alpha < 0$  or  $\alpha_v < 0$  under Hosios condition.

**Proposition 7.** *If  $\alpha < 0$  or  $\alpha_v < 0$ , an efficient decentralized equilibrium is not guaranteed under the Hosios condition. If  $\alpha = 0$  and  $\alpha_v = 0$ , the Hosios condition generates an efficient decentralized equilibrium.*

The proposition highlights that if  $\alpha = \alpha_v = 0$ , the Hosios condition will be efficient as shown in (Hosios, 1990). When  $\alpha < 0$ , workers tend to believe that lower match specific productivity is more likely to be realized. This belief reduces outside option values of unemployment and thus improves profits; therefore, a vacancy supply will be too high in the equilibrium. A rise in the negative of the elasticity of the probability of a vacancies with respect to  $\theta$  increases the negative impact of an additional vacancy on other unfilled vacancies' probability of being filled. The social planner therefore prefers a higher bargaining power of a worker to reduce the number of vacancies. Therefore, if  $\alpha < 0$  and  $\alpha_v = 0$ , a decentralized equilibrium is inefficient under the Hosios condition. The social planner prefers  $\zeta < \eta$ .

Similar argument can be applied to the case of  $\alpha = 0$  and  $\alpha_v < 0$ . Vacancies expect that the realization of higher productivity is less likely to happen, which causes their  $J^V$  to fall. Hence, the social planner prefers a lower bargaining power of a worker to provide an incentive to create vacancies. Therefore, if  $\alpha = 0$  and  $\alpha_v < 0$ , a decentralized equilibrium is inefficient under the Hosios condition. The social planner prefers  $\zeta > \eta$ .

## 7 Quantitative Assessment

To understand effects of a concern about the misspecification on the labor market outcomes, the model with no concern about robustness (that is,  $\alpha = \alpha_v = 0$ ) is used as a benchmark. Therefore, we first calibrate the model with  $\alpha = \alpha_v = 0$ , followed by a discussion of choices of  $\alpha$  and  $\alpha_v$ .



## 7.1 Calibrating Job Search Model Under Ambiguity Neutrality

In this calibration exercise, our strategy and parameter values mainly follow Hagedorn and Manovskii (2008). We calibrate all the parameters in the case of ambiguity neutrality (that is,  $\alpha = \alpha_v = 0$ ) to match the wage elasticity with respect to the productivity  $\varepsilon_{w,\delta}(\delta)$ , the market tightness  $\theta$ , and the job matching rate  $(1 - F(\delta^R))\theta q(\theta)$  in the US.

Following Zanetti (2011), we specify a match specific productivity to be log-normal distributed, in line with empirical evidence of Lydall (1968) and Heckman and Sedlacek (1985) in that a wage distribution has a unique interior mode with a log-normal like skewness. Hence,  $\delta \sim \ln N(\mu, \sigma^2)$ , where  $\mu$  and  $\sigma$  are two parameters we calibrate. We follow Michailat (2012) to specify a matching function as  $m(u, v) = u^\gamma v^{1-\gamma}$ , in line with empirical evidence documented in Petrongolo and Pissarides (2001). Another reason to adopt a Cobb-Douglas matching function is to ensure that the elasticity of  $q(\theta)$  with respect to  $\theta$  is constant. In this way, we can calibrate the elasticity equal to the workers' bargaining power  $\eta$  so that the Hosios condition holds. Hence, we match the three aforementioned targets by choosing the workers' bargaining power  $\eta$ , the mean  $\mu$  and the standard derivation  $\sigma$  of the match-specific productivity distribution. We restrict the parameter  $\gamma$  in the matching function to be the same as  $\eta$ ; therefore, one degree of freedom is lost.

We choose the model period to be one-twelfth of a quarter, which is approximately a week. In Shimer (2005), a quarterly interest rate is 0.012, which is equivalent to a weekly interest rate  $r=1.012^{1/12}-1 \approx 0.000995$ . Shimer (2005) estimates the average monthly job finding rate during 1951-2003 to be 0.45 while Hagedorn and Manovskii (2008) estimate the separation rate to be 0.026. The probability of not matching a job in a week is  $(1-0.45)^{1/4}=0.861$ ; thus, the job matching rate within a week is  $1-0.861=0.139$ . The job separation rate is then 0.0081.<sup>8</sup>

Shimer (2005) sets an unemployment insurance  $b=0.4$ . Hagedorn and Manovskii (2008) argue that the value is too low as it does not include the leisure or the home production forgone. They calibrate the parameters to match the cyclical properties of wages observed in post WWII BLS data; however, their result  $b=0.955$  is implausibly large. Mortensen and Nagypal (2007) argue that if  $b=0.955$ , the flow surplus of the employed will be too low. We follow Hall and Milgrom (2008) to set  $b=0.71$ . This value is larger than 0.4 because it also includes the consumption difference between states of employment and unemployment. In Hagedorn and Manovskii (2008), the cost of posting a vacancy includes a non-capital hiring cost of 0.110 and an idle capital cost 0.474; thus, we follow their model to set costs of posting a vacancy  $c$  to be 0.584.

Hagedorn and Manovskii (2008) calibrate the value of  $b$  and workers' bargaining power  $\eta$  to match the market tightness  $\theta=0.634$  and the elasticity of the wage to the productivity  $\varepsilon_{w,\delta} = 0.449$ .

---

<sup>8</sup>Following Hagedorn and Manovskii (2008), given that a worker was employed one month ago, the probability that the worker is unemployed now is equal to  $\lambda[(1-f)(f\lambda + (1-f)^2) + f(\lambda(1-f) + (1-\lambda)\lambda)] + (1-\lambda)[\lambda(f\lambda + (1-f)^2) + (1-\lambda)(\lambda(1-f) + (1-\lambda)\lambda)]=0.026$ , where  $f=0.139$  is a job finding rate. Hence, solving the equation gives  $\lambda=0.0081$ .

Table 1: Endogenous Variables for Calibration and Calibrated Parameters

Panel A: Values of Endogenous Variables for Calibration			
Variable	Value	Description	
$\delta^R$	1.015	Reservation Productivity Level	
$\theta$	0.634	Market Tightness	
Panel B: Values of Calibrated Parameters			
Parameter	Value	Description	Target
$\mu$	0.9484	Mean of Productivity Distribution	$\theta=0.634$
$\sigma$	0.0736	Variance of Productivity Distribution	$\varepsilon_{w,\delta}(\bar{\delta})=0.449$
$\eta$	0.4521	Worker's Bargaining Power	$(1 - F(\delta^R))\theta q(\theta)=0.139$
Parameter	Value	Description	Source
$r$	0.000995	Weekly Interest Rate	Shimer (2005)
$\lambda$	0.0081	Separation Rate	Hagedorn and Manovskii (2008)
$b$	0.71	Value of Leisure	Hall and Milgrom (2008)
$c$	0.584	Cost of Posting a Vacancy	Hagedorn and Manovskii (2008)

Using equations 17,  $\varepsilon_{w,\delta}(\bar{\delta}) = \eta\bar{\delta}/(\eta\bar{\delta} + (1 - \eta)\delta^R)$  in our model. Noting that  $\bar{\delta}$  is the mean of the productivity distribution of the accepted offers, which follows a log-normal distribution truncated from below at  $\delta^R$ . Since workers and vacancies accept all the offers in Hagedorn and Manovskii (2008), the job finding rate is simply  $\theta q(\theta)$ . However, in our model, workers and vacancies will reject an offer if a match-specific productivity level is too low. Hence, the job finding rate is  $(1 - F(\delta^R))\theta q(\theta)$  in our model, where  $F(\cdot)$  is a cdf of log-normal distribution, and the reservation productivity threshold  $\delta^R$  is to be solved in the model. Since the job matching rate is calibrated to the same target as in Hagedorn and Manovskii (2008), the unemployment rate, given by  $\lambda/(\lambda + (1 - F(\delta^R))\theta q(\theta)) = 0.055$ , is identical to the unemployment rate in Hagedorn and Manovskii (2008).

Indeed, all of the targets  $\varepsilon_{w,\delta}(\bar{\delta})$ ,  $\theta$  and  $(1 - F(\delta^R))\theta q(\theta)$  are dependent on the equilibrium level of  $\theta$  and  $\delta^R$  obtained from equations (15) and (16). Meanwhile,  $\mu$ ,  $\sigma$  and  $\eta$  also affect the equilibrium level of  $\theta$  and  $\delta^R$ . Therefore, the two equilibrium conditions (15) and (16) and the three constraints:  $\varepsilon_{w,\delta}(\bar{\delta}) = 0.449$ ,  $\theta = 0.634$  and  $(1 - F(\delta^R))\theta q(\theta) = 0.139$  are solved simultaneously by choosing  $(\mu, \sigma, \eta, \theta, \delta^R)$ . Table (1) summarizes the results.

## 7.2 Calibrating Misspecification Fears

To calibrate two entropy penalty parameters  $\alpha$  and  $\alpha_v$ , we follow a calibration strategy suggested by Hansen and Sargent (2008). We first map the parameters to detection error probabilities for discriminating between the approximating model and an endogenous worst model associated with

the corresponding parameter. To compute the detection error probabilities, other parameter values are frozen at the values from Table 1. The detection error probability is then used to determine the parameters  $\alpha$  and  $\alpha_v$ .

We first brief the procedure to estimate a detection error probability.<sup>9</sup> A likelihood ratio test is used to compute a detection error probability. Consider two alternative models: model A is an approximating model and model B is a distorted model. The test suggests that a worker will pick model A iff  $L_A > L_B$ ; otherwise, model B is selected. Noting that  $L_j$  is a likelihood function of the corresponding model  $j$ . Given that model  $j$  generates the data, a detection error probability is  $\Pr(L_j < L_{-j}|j)$ . Intuitively, it is a probability of choosing a wrong model  $-j$  when the underlying model is  $j$ . Set the prior probability of model A and B as 1/2. A detection error probability is given by

$$\Pr(\alpha) = \frac{1}{2} (\Pr(L_A < L_B|A) + \Pr(L_A > L_B|B))$$

In this calibration exercise, we generate 10,000 samples for each  $\alpha$ . In each sample, 200 observations of wage are generated from the approximated model  $\delta \sim \ln N(\mu, \sigma)$ . Noting that  $\mu=0.9484$  and  $\sigma=0.0736$  are acquired from section 7.1, and the distorted model is described in equation (6). Since the model period is about a week, 200 observations of wage are equivalent to data of four years. Likelihood functions  $L_A^i$  and  $L_B^i$  are computed for each sample. With  $N(\mu, \sigma)$  and  $w(\delta) = (1 - \eta)\delta^R + \eta\delta$ , the likelihood function of the approximated model is given by

$$L_A^i = \frac{1}{1 - F(\delta^R)} \left( \prod_{t=1}^{200} \frac{\eta}{(w_t^i - (1 - \eta)\delta^R)\sqrt{2\pi\sigma}} e^{-\frac{(\ln(w_t^i - (1 - \eta)\delta^R) - \ln \eta - \mu)^2}{2\sigma^2}} \right)$$

where  $F(\cdot)$  is a cdf of  $\ln N(\mu, \sigma)$  and  $w_t^i$  is the  $t$ -th observation of wage in sample  $i$ .  $L_B^i$  can be calculated in a similar way based on the conditional density given by equation (6), in which  $\hat{f}(\delta|\delta > \delta^R) = \hat{f}(\delta)/(1 - \hat{F}(\delta^R))$ . Noting that  $\hat{F}(\cdot)$  is a cdf of  $\hat{f}(\cdot)$ , and  $\hat{F}(\delta^R) = e^{\alpha J^U} F(\delta^R) / \int_0^\infty e^{\alpha \max\{J^E(x), J^U\}} dx$ .  $\Pr(L_j < L_{-j}|j)$  equals  $\sum_{i=1}^N I(L_j^i < L_{-j}^i)/N$ , where  $I(\cdot)$  is an indicator function to count the number of samples in which a worker picks a wrong model.

In our model, workers and vacancies are ambiguity averse; hence, we compute detection error probabilities  $p(\alpha)$  and  $p(\alpha_v)$  in Figure 3. When the entropy penalty parameter is zero, the approximating model and the distorted model are identical. Since the two models cannot be distinguished, the detection error probability is 0.5. As the entropy penalty parameter falls, the two models differ more from each other. An agent is more capable to distinguish the two models; as a result, the detection error probability declines. Notice that both the  $p(\alpha)$  and the  $p(\alpha_v)$  decay sharply to zero.

---

<sup>9</sup>Readers, who are interested in the details, are referred to Chapter 9 and 10 in Hansen and Sargent (2008).

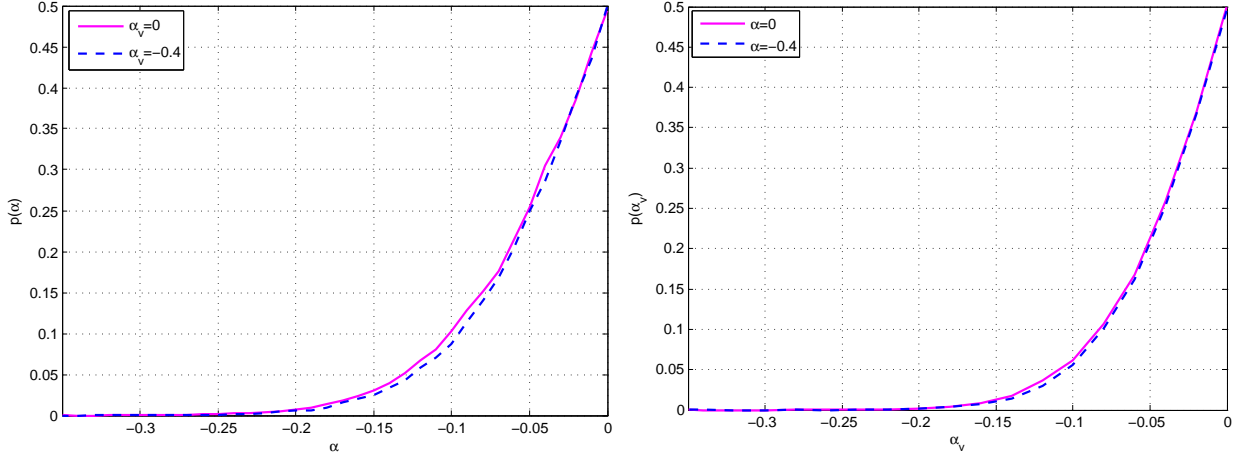


Figure 3: The left (right) panel shows the detection error probability as a function of ambiguity aversion of worker (vacancy).

Here,  $\alpha$  can also be interpreted as workers' concern about robustness of the approximating model with the detection error probability  $p(\alpha)$ . According to Figure 3, workers with  $\alpha = -0.05$  are those who pick the worst case model such that the probability of the misspecification error is about 10%. If a reasonable preference for robustness is the rules that function well for alternative models with detection error probabilities 10% or more,  $\alpha = -0.05$  will be a choice of the parameter.

According to the analysis, we pick the values of  $\alpha$  to be 0, -0.05, -0.1 and -0.3, which correspond to the detection error probabilities about 50%, 25%, 10% and 0%. Corresponding to these detection error probabilities,  $\alpha_v$  equal to 0, -0.04, -0.08 and -0.25 are selected for further analysis. Notice that  $p(\alpha)$  does not change much for  $\alpha_v = 0$  and  $\alpha_v = -0.4$ . We can see that  $p(\alpha)$  is not affected much by  $\alpha_v$  no matter whether the value of  $\alpha_v$  with the corresponding detection error probability equal to 0% or 50% is used. Such conclusion also holds for  $p(\alpha_v)$ .

With various combinations of  $\alpha$  and  $\alpha_v$ , the steady state equilibrium levels of the market tightness  $\theta$  and the reservation productivity  $\delta^R$  are summarized in Table 2. Obviously, the associations are consistent with Proposition 2. For example,  $\theta$  increases with  $\alpha$  and  $\alpha_v$ .  $\theta$  is 0.634 under ambiguity neutrality; in fact, it could be smaller when ambiguity aversion is considered.

### 7.3 Ambiguous Unemployment

We have two objectives in this section. First, we would like to highlight the importance of ambiguity in unemployment. Second, we aim to quantify the ambiguous unemployment specifically to this model using US data. In the existing literature, there are four types of unemployment: search unemployment, cyclical unemployment, structural unemployment, and rationing unemploymen-

Table 2: Equilibrium  $\theta$  and  $\delta^R$  under Various Combinations of  $\alpha$  and  $\alpha_v$

$\alpha$	$\theta$				$\delta^R$			
	$\alpha_v$				$\alpha_v$			
	0	-0.04	-0.08	-0.25	0	-0.04	-0.08	-0.25
0	0.634	0.581	0.535	0.394	1.015	1.014	1.012	1.005
-0.005	0.644	0.590	0.544	0.401	1.015	1.013	1.012	1.005
-0.1	0.686	0.628	0.579	0.426	1.014	1.012	1.010	1.004
-0.3	0.954	0.874	0.805	0.592	1.007	1.005	1.003	0.996

t.<sup>10</sup> This paper demonstrates that unemployment could also be attributable to ambiguity aversion. Since workers' and vacancies' pessimistic beliefs of a productivity density result in an earlier acceptance of job offers, unemployment spells are shortened. This reduces an unemployment rate in a steady state.

As known, a long-run average unemployment in large part consists of search unemployment. However, Proposition 2 indicates that unemployment rates fall with the degree of ambiguity aversion of workers. If workers are ambiguity neutral, the search unemployment will have been larger. In fact, the ambiguity could vary with time. If workers have more concern about the uncertainty during recession, ambiguity could explain unemployment more in the economic downturn. A negative productivity shock to an economy in fact creates less cyclical unemployment than the one economists estimate in the model under ambiguity neutrality. Similarly, it takes time for workers and vacancies to learn about the structural change. The lack of information could create ambiguity aversion, which could in turn create inefficient job matches that reduce structural unemployment. The unemployment under ambiguity neutrality would in principle be larger than what we observe in reality.

We now proceed to quantify the ambiguous unemployment in our model. Consider that  $(\alpha, \alpha_v)$  equals  $(x, z)$ . Noting that  $(x, z)$  could be  $(0,0)$ . We follow the same procedure in section 7.1 except that we use  $(\alpha, \alpha_v) = (x, z)$  to calibrate the model. Using all the calibrated parameters, we assume  $(x, z)=(0,0)$  to compute the equilibrium  $\theta$  and  $\delta^R$  using equations (15) and (16). The corresponding unemployment is given by equation (14).

The same procedure is conducted with various combinations of  $(x, z)$ . Our calibration exercise matches so well that all the targets summarized in Table 1 are met. Therefore, the unemployment rates  $u(x, z)$  are 5.5% as demonstrated by a horizontal line in Figure 4. The two lines represent the case that  $\alpha_v = 0$  and  $\alpha_v = -0.25$ . Unemployment rates are measured by assuming  $\alpha = 0$

<sup>10</sup>Michaillat (2012) decomposes the US unemployment rate into search unemployment and rationing unemployment. Interested readers are refer to Table 5 of his paper.

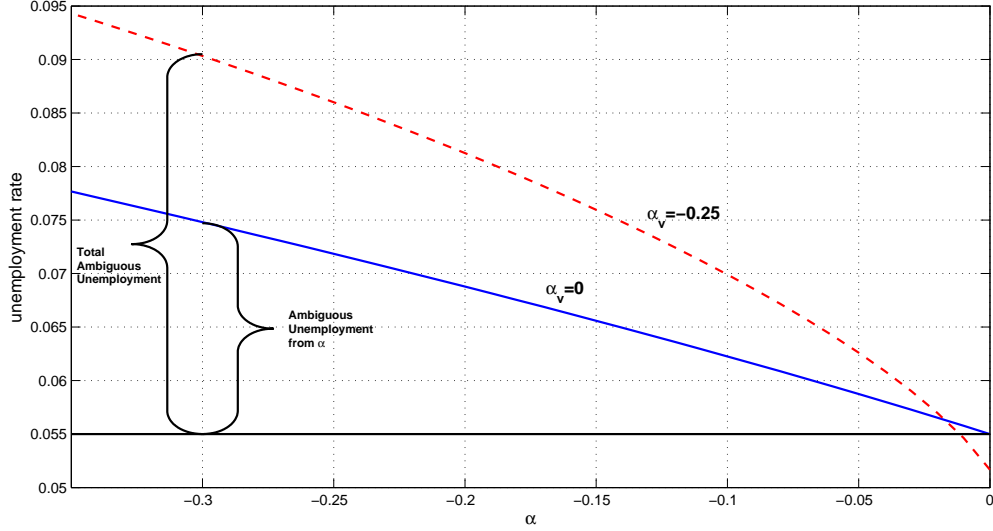


Figure 4: Ambiguous Unemployment as a Function of Ambiguity Aversion of Workers.

Table 3: Ambiguous Unemployment Rate

$u=5.5\%$	$\alpha_v=-0.25$	$\alpha_v=0$
$\alpha=-0.3$	3.5%	2%
$\alpha=-0.1$	1.5%	0.75%

and  $\alpha_v = 0$  along these two curves.

Suppose  $\alpha = -0.3$  and  $\alpha_v = -0.25$  in reality. If workers and vacancies were all ambiguity neutral, the unemployment rate would reach about 9.0% as shown by the dash line at  $\alpha = -0.3$ . As observed in the data, the US unemployment rate is about 5.5%. The difference in the unemployment rate, equal to 3.5%, results from the ambiguity aversion. Therefore, the distance between the dash curve and the horizontal line captures the ambiguous unemployment at  $\alpha_v = -0.25$  and the corresponding  $\alpha$  on the x-axis. This is the total ambiguous unemployment when  $\alpha_v = -0.25$ . Similarly, the distance between the solid line and the horizontal line captures the ambiguous unemployment that solely arises from ambiguity aversion of workers  $\alpha$ .

As summarized in Table 3, the ambiguous unemployment could be as large as 3.5% when  $\alpha = -0.3$  and  $\alpha_v = -0.25$ . This amounts to 63.6% of the unemployment rate. Also, the ambiguous unemployment solely from ambiguity aversion of workers reaches 2% when  $\alpha = -0.3$ . The ambiguous unemployment solely derived from workers' misbelief could be as large as 36.4% of the unemployment.

## 7.4 Other Roles of Ambiguity in Job Search Model

### 7.4.1 Effect of Productivity Shocks

Figure 5 demonstrates the effects of technological advancement on  $\delta^R$  and  $\theta$ . We characterize a new productivity distribution  $G$  that is first order stochastically dominate  $F$  such that  $G$  has a higher mean than and the same standard derivation as  $F$ . In particular, we pick the mean of  $G$  to be one standard derivation larger than that of  $F$ . Recall that  $\mu=0.9484$  and  $\sigma=0.0736$  in our calibration exercise. Hence,  $F \sim \ln N(0.9484, 0.0736)$  and  $G \sim \ln N(1.0220, 0.0736)$ .

It is not surprising that changes in  $\delta^R$  are positive, as stated in Proposition 4. The changes are smaller under a higher degree of ambiguity aversion of workers and/or vacancies. With lower  $\alpha$  and/or  $\alpha_v$ , agents' pessimistic beliefs undermine an increase in the expected productivity from a technological improvement. This lowers the increase in their outside option values and thus  $\delta^R$ . Noting that the RHS of equation (16) falls with  $\delta^R$ . With an identical productivity growth, the expected match surplus grows more with a smaller  $\alpha$ . This leads to a larger growth rate in the market tightness. In addition, these results imply that the technological improvement reduces an unemployment rate more under an economy of lower  $\alpha$ . On the other hand, vacancies under ambiguity aversion believe that the productivity growth is not as much as it actually is. The growth in their expected flow profit is lower than the actual one, discouraging potential vacancies to enter a market. Therefore, a technological improvement does raise  $\theta$  but the increase is milder with a smaller  $\alpha_v$ .

Shimer (2005) points out that the volatility of market tightness observed in the postwar US data is larger than that generated from Mortensen and Pissarides (1994) model with productivity shocks. Hagedorn and Manovskii (2008) successfully generate a larger volatility; nevertheless, as argued by Mortensen and Nagypal (2007), the replacement rate of unemployment benefits 95.5% in Hagedorn and Manovskii (2008) is probably too high. Following Hall and Milgrom (2008), this paper adopts a more reasonable replacement rate of 71%. In response to a unit of standard deviation increase in the mean productivity, this model predicts that the change in  $\theta$  in an economy of  $\alpha=-0.3$  and  $\alpha_v=0$  is more than 1.5 times of the change in  $\theta$  under ambiguity neutrality. Despite that our model is not an endogenous job destruction model as in Mortensen and Pissarides (1994), the calibration result indicates that a higher degree of ambiguity aversion of workers could amplify the volatility of  $\theta$  in response to productivity shocks. Therefore, the feature of ambiguity aversion potentially fills the gap in the literature. Since research on the cyclical behavior of equilibrium unemployment is beyond the scope of this paper, we leave this to future research.

### 7.4.2 Variations in Unemployment Benefit

As stated in Proposition 2, the higher the degree of ambiguity aversion of workers for any given  $b$ , the higher will be the  $\theta$  and the lower will be the  $\delta^R$ . Figure 6 complements the proposition

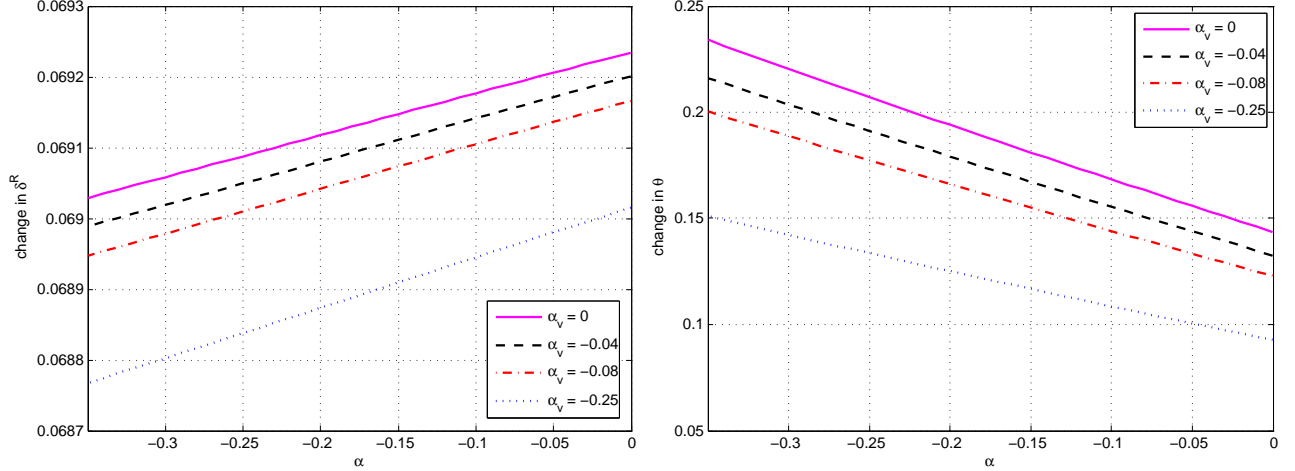


Figure 5: The left (right) panel shows the impact of productivity growth on reservation productivity level  $\delta^R$  (market tightness  $\theta$ ) as a function of  $\alpha$ .

by illustrating  $\theta$  and  $\delta^R$  as a function of  $b$  for various levels of  $\alpha$  and  $\alpha_v$ . Since  $J^U$  consists of an unemployment benefit  $b$  and a discounted present value of expected future income. A higher degree of ambiguity aversion decreases expected search gain and thus shrinks the proportion of the expected search gain in  $J^U$ . Therefore,  $b$  will have a larger proportional impact on  $\delta^R$  and thus  $\theta$  under the higher degree of ambiguity aversion.

The figure indicates that given  $\alpha_v$ , there could exist a large discrepancy in  $\theta$  between economies of various  $\alpha$ . When an unemployment benefit is low,  $\theta$  in an economy of  $\alpha=-0.3$  could be 60% more than the one in the ambiguity neutral economy irrespective of  $\alpha_v$ . Such discrepancy in  $\theta$  and  $\delta^R$  could lead to a difference in a reservation wage and an unemployment rate. Furthermore, the slope  $d\theta/db$  is much steeper for  $\alpha < 0$  in Figure 6. While a standard job search model only predicts that an increase in an unemployment benefit reduces market tightness, this study shows that the reduction is much larger under ambiguity aversion.

The results indicate that ambiguity aversion does play an important role in effects of unemployment benefits in a labor market. As known, researchers are eager to seek for an optimal unemployment benefit. Despite that the ambiguity aversion is an important determinant of effects of unemployment benefits, it is rare to see existing job search literature to address this issue under ambiguity aversion. (Cahuc and Lehmann, 2000; Fredriksson and Holmlund, 2001; Coles and Masters, 2006; Fredriksson and Holmlund, 2006; Boone et al., 2007) Our numerical exercise therefore provides a new direction for future research avenues.



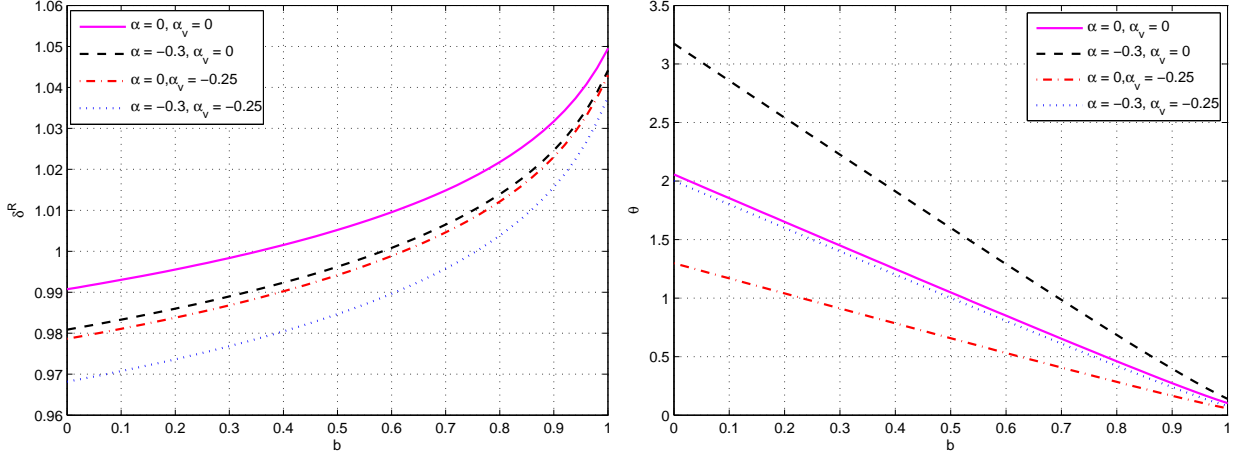


Figure 6: The left (right) panel shows reservation productivity level (market tightness) as a function of unemployment benefit. ( $\alpha_v = 0$ )

## 8 Conclusion

The present paper constructs a search-and-matching model under the ambiguity aversion of workers and vacancies. The equilibrium is analytically tractable, preserves most (if not all) of the intuitive comparative statics as in the canonical job search model, and is ready to be adapted to investigate a number of labor market policy issues. For example, we analytically show that a rise in unemployment benefits reduces market tightness and increases a reservation wage, which in turn worsens an unemployment rate. Our calibration further demonstrates that the ambiguity aversion amplifies the impact of the unemployment benefits on the market tightness, which is one of the key variables in determining other labor market outcomes. Thus, the result calls for the reexamination of the robust optimal unemployment benefits.

More importantly, this paper illustrates how the ambiguity aversion influences a job search and a job creation decision. In particular, an unemployment rate is shown to be lower under a higher degree of the workers' ambiguity aversion. We propose a method to compute this "ambiguous" unemployment and find that it could account for as large as 63.6% of the long-run average unemployment rate in the US. Moreover, our calibration results indicate that the market tightness is much more volatile under a higher degree of the ambiguity aversion in response to productivity shocks, potentially resolving the Shimer puzzle.

Lastly, this paper demonstrates that a decentralized equilibrium is no longer efficient even under Hosios (1990) condition. Future researches could pay attention on whether the uniqueness of such efficient decentralized equilibrium exists. In sum, this paper puts forward promising research avenues focusing on the influence of the ambiguity aversion on labor market outcomes via a job search and a job creation decision and other decisions made during a job search process.

## 9 Appendix

### 9.1 Proof of Proposition 2

Rearranging equation (15), we have

$$\delta^R = b + \beta p \ln \left( F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha \eta (1+r)}{r+\lambda} (x-\delta^R)} dF(x) \right)^{\frac{1}{\alpha}}$$

Noting that

$$\frac{\partial B^{\frac{1}{\alpha}}}{\partial \alpha} = \frac{-1}{\alpha^2} \int_{\delta^R}^{\infty} \frac{\eta(1+r)}{r+\lambda} (x-\delta^R) e^{\frac{\alpha \eta (1+r)}{r+\lambda} (x-\delta^R)} dF(x) B^{\frac{1}{\alpha}} \ln B > 0$$

where  $B \equiv F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha \eta (1+r)}{r+\lambda} (x-\delta^R)} dF(x)$

Also, noting that  $\ln B < 0$ . Hence, the partial derivative of the RHS of equation (15) with respect to  $\alpha$  is positive. Similar argument can be used to show that the partial derivative of the RHS of equation (16) with respect to  $\alpha_v$  is positive. Applying Cramer's rule to equations (15) and (16),

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -d\alpha_v \\ -d\alpha \end{pmatrix}$$

Denote  $A \equiv \det \begin{pmatrix} - & - \\ + & - \end{pmatrix}$ .

$$\frac{d\theta}{d\alpha_v} = \frac{\det \begin{pmatrix} - & - \\ 0 & - \end{pmatrix}}{A} > 0, \quad \frac{d\delta^R}{d\alpha_v} = \frac{\det \begin{pmatrix} - & - \\ + & 0 \end{pmatrix}}{A} > 0$$

$$\frac{d\theta}{d\alpha} = \frac{\det \begin{pmatrix} 0 & - \\ - & - \end{pmatrix}}{A} < 0, \quad \frac{d\delta^R}{d\alpha} = \frac{\det \begin{pmatrix} - & 0 \\ + & - \end{pmatrix}}{A} > 0$$

Using wage equation (12), simple algebra gives

$$\frac{dw(\delta)}{d\alpha} = (1-\eta) \frac{d\delta^R}{d\alpha} > 0, \quad \frac{dw(\delta)}{d\alpha_v} = (1-\eta) \frac{d\delta^R}{d\alpha_v} > 0$$

## 9.2 Proof of Proposition 3

When  $\alpha$  goes to negative infinity, the second term of the RHS of equation (15) vanishes, and thus  $\delta^R = b$ . Let  $\delta' > \delta^R$ . Then, for all  $\delta > \delta^R$ , the likelihood ratio (6) is as follows:

$$\begin{aligned} \frac{\hat{f}(\delta')}{f(\delta')} &= \frac{e^{\alpha J^E(\delta')}}{\int_0^{\delta^R} e^{\alpha J^U} dF(x) + \int_{\delta^R}^{\infty} e^{\alpha J^E(x)} dF(x)} \\ &= \frac{1}{\int_0^{\delta^R} e^{\alpha(J^U - J^E(\delta'))} dF(x) + \int_{\delta^R}^{\infty} e^{\alpha(J^E(x) - J^E(\delta'))} dF(x)} \end{aligned}$$

Noting that  $J^U - J^E(\delta') < 0$ . When  $\alpha$  tends to negative infinity, the RHS of the above equation goes to zero. Hence, for all  $\delta > \delta^R$ ,  $\hat{f}(\delta) = 0$ . For all  $\delta'' \leq \delta^R$ , the likelihood ratio is as follows:

$$\begin{aligned} \frac{\hat{f}(\delta'')}{f(\delta'')} &= \frac{e^{\alpha J^U}}{\int_0^{\delta^R} e^{\alpha J^U} dF(x) + \int_{\delta^R}^{\infty} e^{\alpha J^E(x)} dF(x)} \\ &= \frac{1}{F(\delta^R) + \int_{\delta^R}^{\infty} e^{\alpha(J^E(x) - J^U)} dF(x)} \end{aligned}$$

When  $\alpha$  approaches  $-\infty$ ,  $\hat{f}(\delta'')$  goes to  $f(\delta'')/F(\delta^R)$ . Similar procedure can be applied to equation (16) to yield Proposition 3.

## 9.3 Proof of Proposition 4

$G \succeq_{FOSD} F$  iff  $\int h(x)dG(x) \leq \int h(x)dF(x)$  for any non-increasing function  $h(x)$ . For any  $\delta^R \in \mathcal{R}_+$ , we define two non-increasing functions:

$$h(x) = \begin{cases} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)}, & \text{if } \delta > \delta^R; \\ 1, & \text{otherwise.} \end{cases}, \quad \tilde{h}(x) = \begin{cases} e^{\frac{\alpha\nu(1-\eta)(1+r)}{r+\lambda}(x-\delta^R)}, & \text{if } \delta > \delta^R; \\ 1, & \text{otherwise.} \end{cases}$$

Hence, equations (15) and (16) can be written as follows:

$$\begin{aligned} \delta^R &= b + \frac{\beta p}{\alpha} \ln \left( \int_0^{\infty} h(x) dF(x) \right) \\ c &= \frac{\beta q}{\alpha \nu} \ln \left( \int_0^{\infty} \tilde{h}(x) dF(x) \right) \end{aligned}$$

Define a surjective function  $A(\delta^R; a) : \mathcal{R}_+ \times \mathcal{R} \rightarrow \mathcal{R}$ , with  $\partial A(\delta^R; a)/\partial a > 0$ . Notice that for any distribution  $G$ , there exist  $a_G$  and  $\tilde{a}_G$  that satisfy the following equations.

$$\begin{aligned} \int_0^{\infty} h(x) dG(x) + A(\delta_F^R; a_G) &= \int_0^{\infty} h(x) dF(x) \\ \int_0^{\infty} \tilde{h}(x) dG(x) + A(\delta_F^R; \tilde{a}_G) &= \int_0^{\infty} \tilde{h}(x) dF(x) \end{aligned}$$

where  $\delta_F^R$  is the reservation productivity under the distribution  $F$ . Notice that  $a_G$  and  $\tilde{a}_G$  are unique due the monotonicity of  $A(\delta^R; a)$  with respect to  $a$ . Without loss of generality, we set  $a_F = \tilde{a}_F = 0$  so that  $A(\delta^R; 0) = 0$ . For any  $G \succeq_{FOSD} F$  where  $G \neq F$ , either  $a_G > 0$  or  $\tilde{a}_G > 0$  (or both). Consider a distribution  $G_0 \succeq_{FOSD} F$  with  $a_{G_0} = \tilde{a}_{G_0}$  slightly above zero, the preceding two equations can be written as follows:

$$\begin{aligned}\delta^R &= b + \frac{\beta p}{\alpha} \ln \left( \int_0^\infty h(x) dF(x) - A(\delta^R; a_{G_0}) \right) \\ c &= \frac{\beta q}{\alpha_v} \ln \left( \int_0^\infty \tilde{h}(x) dF(x) - A(\delta^R; \tilde{a}_{G_0}) \right)\end{aligned}$$

To investigate the impact of changing the distribution from  $F$  to  $G_0$ , it is equivalent to see the impact of changing  $a$  from 0 to  $a_{G_0} = \tilde{a}_{G_0}$ . Hence, we apply Cramer's rule to the above two equations,

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -da \\ -da \end{pmatrix}$$

$$\frac{d\delta^R}{da} = \frac{\det \begin{pmatrix} - & - \\ + & - \end{pmatrix}}{A} > 0, \quad \frac{d\theta}{da} = \frac{\det \begin{pmatrix} - & - \\ - & - \end{pmatrix}}{A}$$

Thus,  $\delta_G^R > \delta_F^R$  if  $G \succeq_{FOSD} F$ . Hence,  $w_G(\delta) = (1 - \eta)\delta_G^R + \eta\delta > (1 - \eta)\delta_F^R + \eta\delta = w_F(\delta)$  for all  $\delta \geq \delta_G^R$ . Similarly, applying Cramer's Rule to the cases  $(a_G > 0, \tilde{a}_G = 0)$  and  $(a_G = 0, \tilde{a}_G > 0)$  gives the same result.

## 9.4 Proof of Proposition 5

Applying Cramer's rule to equations (15) and (16),

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} +dc \\ -db \end{pmatrix}$$

Denote  $A = \det \begin{pmatrix} - & - \\ + & - \end{pmatrix} > 0$ .

$$\frac{d\theta}{dc} = \frac{\det \begin{pmatrix} + & - \\ 0 & - \end{pmatrix}}{A} < 0, \quad \frac{d\delta^R}{dc} = \frac{\det \begin{pmatrix} - & + \\ + & 0 \end{pmatrix}}{A} < 0$$

$$\frac{d\theta}{db} = \frac{\det \begin{pmatrix} 0 & - \\ - & - \end{pmatrix}}{A} < 0, \quad \frac{d\delta^R}{db} = \frac{\det \begin{pmatrix} - & 0 \\ + & - \end{pmatrix}}{A} > 0$$

## 9.5 Proof of Proposition 6

If  $q_1 = \varphi q$  where  $\varphi \in (0, 1)$ , then  $p_1 = \varphi p$ . With the new matching technology, equations (15) and (16) are written as

$$\begin{aligned} \delta^R &= b + \frac{\beta\varphi p}{\alpha} \ln \left( F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha\eta(1+r)}{r+\lambda}(x-\delta^R)} dF(x) \right) \\ c &= \frac{\beta\varphi q}{\alpha_v} \ln \left( F(\delta^R) + \int_{\delta^R}^{\infty} e^{\frac{\alpha_v(1-\eta)(1+r)}{r+\lambda}(x-\delta^R)} dF(x) \right) \end{aligned}$$

Applying Cramer's rule to the above two equations,

$$\begin{pmatrix} - & - \\ + & - \end{pmatrix} \begin{pmatrix} d\theta \\ d\delta^R \end{pmatrix} = \begin{pmatrix} -d\varphi \\ -d\varphi \end{pmatrix}$$

$$\frac{d\delta^R}{d\varphi} = \frac{\det \begin{pmatrix} - & - \\ + & - \end{pmatrix}}{A} > 0, \quad \frac{d\theta}{d\varphi} = \frac{\det \begin{pmatrix} - & - \\ - & - \end{pmatrix}}{A}$$

Thus,  $dw(\delta)/d\varphi > 0$  for all  $\delta \geq \delta^R$ .

## References

- Adam, K. and M. Woodford (2012). Robustly optimal monetary policy in a microfounded new keynesian model. *Journal of Monetary Economics* 59(5), 468–487.
- Blanchard, O. and J. Galí (2010). Labor markets and monetary policy: a new keynesian model with unemployment. *American Economic Journal: Macroeconomics* 2(2), 1–30.
- Boone, J., P. Fredriksson, B. Holmlund, and J. C. Van Ours (2007). Optimal unemployment insurance with monitoring and sanctions. *The Economic Journal* 117(518), 399–421.
- Cahuc, P. and E. Lehmann (2000). Should unemployment benefits decrease with the unemployment spell? *Journal of Public Economics* 77(1), 135–153.
- Cao, H. H., T. Wang, and H. H. Zhang (2005). Model uncertainty, limited market participation, and asset prices. *Review of Financial Studies* 18(4), 1219–1251.

- Coles, M. and A. Masters (2006). Optimal unemployment insurance in a matching equilibrium. *Journal of Labor Economics* 24(1), 109–138.
- Croce, M. M., T. T. Nguyen, and L. Schmid (2012). The market price of fiscal uncertainty. *Journal of Monetary Economics* 59(5), 401–416.
- Djeutem, E. and K. Kasa (2013). Robustness and exchange rate volatility. *Journal of International Economics* 91(1), 27–39.
- Fredriksson, P. and B. Holmlund (2001). Optimal unemployment insurance in search equilibrium. *Journal of Labor Economics* 19(2), 370–399.
- Fredriksson, P. and B. Holmlund (2006). Improving incentives in unemployment insurance: A review of recent research. *Journal of Economic Surveys* 20(3), 357–386.
- Fujita, S. and G. Ramey (2012). Exogenous versus endogenous separation. *American Economic Journal: Macroeconomics* 4(4), 68–93.
- Gilboa, I. and D. Schmeidler (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics* 18(2), 141–153.
- Hagedorn, M. and I. Manovskii (2008). The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review* 98(4), 1692–1706.
- Hall, R. E. and P. R. Milgrom (2008). The limited influence of unemployment on the wage bargain. *American Economic Review* 98(4), 1653–1674.
- Hansen, L. P. and T. J. Sargent (2001). Robust control and model uncertainty. *American Economic Review* 91(2), 60–66.
- Hansen, L. P. and T. J. Sargent (2008). *Robustness*. Princeton University Press.
- Heckman, J. J. and G. Sedlacek (1985). Heterogeneity, aggregation, and market wage functions: an empirical model of self-selection in the labor market. *Journal of Political Economy* 93(6), 1077–1125.
- Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies* 57(2), 279–298.
- Ilut, C. L. and M. Schneider (2014). Ambiguous business cycles. *American Economic Review* 104(8), 2368–99.
- Lydall, H. (1968). *The Structure of Earnings*. Clarendon Press.

- Michaillat, P. (2012). Do matching frictions explain unemployment? not in bad times. *American Economic Review* 102(4), 1721–1750.
- Mortensen, D. T. and E. Nagypal (2007). More on unemployment and vacancy fluctuations. *Review of Economic Dynamics* 10(3), 327–347.
- Mortensen, D. T. and C. A. Pissarides (1994). Job creation and job destruction in the theory of unemployment. *Review of Economic Studies* 61(3), 397–415.
- Nishimura, K. G. and H. Ozaki (2004). Search and knightian uncertainty. *Journal of Economic Theory* 119(2), 299–333.
- Petrongolo, B. and C. A. Pissarides (2001). Looking into the black box: A survey of the matching function. *Journal of Economic Literature* 39(2), 390–431.
- Pissarides, C. A. (2000). Equilibrium unemployment theory. *MIT Press Books 1*.
- Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica* 77(5), 1339–1369.
- Rogerson, R., R. Shimer, and R. Wright (2005). Search-theoretic models of the labor market: A survey. *Journal of Economic Literature* 43(4), 959–988.
- Sahin, A., J. Song, G. Topa, and G. L. Violante (2014). Mismatch unemployment. *American Economic Review* 104(11), 3529–64.
- Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review* 95(1), 25–49.
- Werner, J. (2011). Risk aversion for variational and multiple-prior preferences. *Journal of Mathematical Economics* 47(3), 382–390.
- Zanetti, F. (2011). Labor market institutions and aggregate fluctuations in a search and matching model. *European Economic Review* 55(5), 644–658.