# Taking Selection to Task: Bounds on Trends in Occupational Task Prices for the U.S., 1984-2013 ${ }^{1}$ 

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## 1. Introduction

A long and growing list of papers characterize labour markets in developed economies as undergoing a polarization of their employment structures and an increase in wage inequality in the last few decades. Explanations for these trends are wide-ranging and varied but all of them are judged ultimately on how they accord with core patterns in the data. For example, Katz and Murphy (1992)'s claim that the U.S. economy underwent a skill-biased demand shift in the 1980s rested on the observation that both the average wages of the more educated and their employment levels were rising relative to those of the less educated. Many different models of what was happening in the U.S. labour market have emerged since that initial paper but all of them have had to include some element of a shift in relative demand. Similarly, work by David Autor and co-authors has argued that polarization in the U.S. labour market since the early 1990s reflects increased demand

[^0]for both cognitive and service tasks related directly or indirectly to the computer revolution (Autor et al. 2003, 2007; Acemoglu and Autor 2011). Again, the claim that we are witnessing demand shifts of this type comes about because both employment and wages for service and cognitive workers increased in the U.S. in the 1990s. Beaudry et al. (2012), on the other hand, argue that the 2000s in the U.S. can be characterized, relative to the 1990s, as a period of declining demand for cognitive tasks and a large positive supply shock in the labour market for service workers. Again, support for the argument comes down ultimately to relative movements in employment rates and wages in the various occupation groups.

Given the way these arguments are built, it is obviously important to get appropriate measures of wage movements for workers of different skill types. This, though, is complicated by two main issues. First, there have been substantial compositional shifts in the workforce coinciding with the increases in inequality and polarization. Both female labour force participation and the average level of education in the U.S. workforce has increased substantially, and these have been accompanied by the movement of the baby-boom through the age structure. Any one of these is large enough to affect the average wages we see in the economy; together they form a compositional tidal wave.

The second issue is indicated by the economic models used to interpret the rise in polarization. In models addressing polarization patterns, in particular, workers must be allocated to different sectors in the economy. By far the most common approach taken in these models is to incorporate some version of a Roy model in which workers have heterogeneous abilities in the various sectors and each worker chooses a sector to maximize his or her earnings. More specifically, workers are endowed with a vector of abilities corresponding to the various tasks demanded in each sector. The combination of firm demand for labour and the supply decisions of workers determines a price for each of these tasks in equilibrium. The wage a worker would get in a sector then reflects a combination of his or her abilities in the tasks specific to that sector and the task prices in the economy. If demand for the tasks in one sector increases (as, for example, computerization is argued to have done for cognitive sector tasks) then the price for tasks in that sector will increase but movements in the observed average wage in the sector are less clear. This is true because the increased task price will draw more workers into the sector and, in a classic Roy model, those new workers will have lower task abilities in that sector than those who were already working there. Thus, the average observed wage will be affected by offsetting effects in the form of an increased task price but a declining average ability of workers in the sector. Importantly, what is needed for evaluating the competing models of changes in the labour market is task prices not average wages. It is these prices that reflect the deeper forces in terms of shifts in production technology, etc., in which we are interested. Our goal in this paper is to provide estimates of trends in task prices that could prove useful in helping to adjudicate among the various models of polarization.

The idea that wages need to be adjusted for shifts in observable and unobservable characteristics is not a new one. A considerable literature exists that assesses the importance of compositional shifts over the business cycle in order to establish whether the actual price of labour moves cyclically or counter-cyclically. We follow that literature in the way we address shifts in composition in
terms of observable characteristics.
The idea that it is task prices, rather than wages, that are the relevant construct in a Roy model context goes back at least to the work of Willis and Rosen (1979) and Heckman (1979) in the late 1970s and, at least implicitly, back as far as the Roy (1951) model itself. More recently, Acemoglu and Autor (2011) raised the point in the context of the recent polarization literature. Yet, in spite of the models of polarization being about the shifting of workers across sectors, we know of only one other paper that has attempted to address the impact of composition and selectivity on wages (or, in other words, to get the task prices) in this context: Cortes (2012). We build on Cortes (2012) by providing new estimates of trends in task prices using a more general approach.

Given that we are interested in providing evidence relevant to analysis of polarization patterns, we divide our data into what has become a commonly used set of sectors in the polarization literature. In particular, we divide occupations into Cognitive, Routine, and Manual occupations, following Acemoglu and Autor (2011), and seek to establish the trends in the task prices associated with each of these occupation groups. We begin by using a standard shift-share analysis in order to control for the effects of composition in terms of observable characteristics on observed wage patterns. We show that moving from using the mean wage to the median wage (which downplays composition adjustments in the tails of the distribution) and adjusting for education, age and gender composition has substantial impacts on the trend in the central tendencies of the overall wage distribution and the wage distributions in each of the three sectors. The importance of shifts in observable characteristics may point to substantial impacts from selectivity in terms of unobservable abilities. One possible approach to cutting through selectivity effects is to use a Heckman-correction for selection in sector specific wage regressions. However, the believability of the results from such an exercise rests on the validity of exclusion restrictions in the form of variables that affect sector selection but do not enter the wage determination process directly. We do not have good candidates for such variables and so do not pursue this approach.

A second candidate for obtaining task prices is the one pursued in Cortes (2012). This approach, which has also been used in the cyclical wage movements literature, follows individual workers using panel data. The idea is that if a worker's task specific abilities do not change over time then within-sector changes in that worker's wage over time must reflect changes in the sectoral task price. While this is a potentially promising approach, its usefulness is challenged by the results from the literature that establishes that implicit contracting models are a useful lens through which to examine the labour market (Beaudry and DiNardo 1995, for example). The results in that literature indicate that movements in the wages of higher tenure workers reflect contracting issues that divorce them from movements in contemporaneous productivity changes and the task price movements that are based on them. While the implicit contract model in the empirical literature is almost always written as being in a stationary environment, we show that with trending productivity, changes in the wages of workers at time of entry to a job are directly reflective of changes in current productivity. Based on this, we work with wages for new job entrants.

Working with new entrants solves potential problems related to the stickiness of wages for workers within a firm over time but it does not addres the problem of selection on unobservables that arises because successive cohorts of new entrants face potentially different task prices and make different occupation choices as a consequence. To address the selection problem, we form bounds on the movements in task prices using an approach that is in the spirit of Manski (1990) and Horowitz and Manski (2014), and motivated by recent papers by Lee (2009) and Blundell et al. (2007) in other contexts. In particular, we work with variants of the Roy model stemming from different assumptions about the correlations of sector specific tasks to develop different sets of bounds on the task price changes. ${ }^{2}$

In the end, we obtain sets of bounds on true task price movements for each of the Cognitive, Manual and Routine sectors, and for all workers combined. We find ....

## 2. Theory

Our goal in this paper is to present price trends that can be used to distinguish among a wide set of competing models of inequality and polarization. Given that we want to impose as few restrictions as possible on the data, we start with a very general characterization of wage determination that nests key models. We then discuss sector selection within the context of this model.

### 2.1. A Model of Implicit Contracts with Trending Productivity

The first point of concern is what wage measure to use. In thinking about this, we are influenced by the literature suggesting that implicit contract considerations are important for understanding wage movements (e.g. Beaudry and DiNardo (1995); Ham and Reilly (2002)). However, while much of that literature works with a stationary macro environment, we are interested in examining trends. For that reason, we will set out a rudimentary implicit contract model with a stochastic trend as a framework for discussing what wages we want to focus on.

Begin by considering firms and workers in sector, $k$. Firms hire workers, indexed by $j$, with output generated by a specific worker given by,

$$
\begin{equation*}
z_{k j t}=\eta_{k j} \cdot \exp \left(e_{k j t}\right) \tag{1}
\end{equation*}
$$

That is, a worker has a time invariant component of sector specific productivity given by $\eta_{k j}$ and an idiosyncratic component, $\exp \left(e_{k j t}\right)$. The two components are independent across workers and

[^1]of each other for the same worker. The idiosyncratic component is drawn from a distribution with $E\left(\exp \left(e_{k j t}\right)\right)=1$.

The value of marginal product of the worker is given by,

$$
\begin{equation*}
\Phi_{j k t}=p_{k t} z_{k j t} \tag{2}
\end{equation*}
$$

where, $p_{k t}$ is the price of output in sector $k$, movements in which will partly reflect sector wide productivity shocks. In what follows, we will interpret $z_{k j t}$ as the amount of the sector specific task that worker $j$ supplies to sector $k$ (expressed in units of output) and will refer to $p_{k t}$ as the task price. For the purposes of the current discussion, we assume that the task price has a stochastic trend (though, in our estimation, we will not impose any assumption about the form of any trend in the price):

$$
\begin{equation*}
p_{k t}=p_{k t-1}+\xi_{k t} \tag{3}
\end{equation*}
$$

where $\xi_{k t}$ is a disturbance that is independent over time and with respect to the other random variables in the model.

To keep the discussion as simple as possible, we will assume that contracts are fully enforceable and that jobs are infinitely lived apart from the fact that workers die with probability $1-\delta$ in any period. Workers inelastically supply a fixed number of hours of work per period and have utility equal to the log of their wage. There is free entry of firms so that the present value of profits is driven to zero. Firms and workers face a common discount factor, $\beta$.

The solution to the contract involving worker $j$ in sector $k$ and starting in period $t_{0}$ is the solution to the problem:

$$
\begin{equation*}
\max _{w_{k j t}} \sum_{\tau=0}^{\infty}(\beta \delta)^{t} E_{t_{0}}\left[\ln w_{k j t_{0}+\tau}\right] \tag{4}
\end{equation*}
$$

st.

$$
\sum_{\tau=0}^{\infty}(\beta \delta)^{t} E_{t_{0}}\left[w_{k j t_{0}+\tau}\right]=\sum_{\tau=0}^{\infty}(\beta \delta)^{t} E_{t_{0}}\left[p_{k t_{0}+\tau} \eta_{k j} \cdot \exp \left(e_{k j t_{0}+\tau}\right)\right]
$$

where, $E_{t_{0}}$ refers to expectations with respect to information available at time, $t_{0}$.
The solution to this problem is a fixed wage for all future periods for person $j$ in sector $k$ given by:

$$
\begin{equation*}
w_{k j t, t_{0}}=\eta_{k j} p_{k t_{0}}, \forall t>t_{0} \tag{5}
\end{equation*}
$$

Several points follow from this result. First, while every person starting a job in period $t_{0}$ has a sector $k$ potential wage given by (5), only workers for whom this wage is better than their alternatives in other sectors will actually be observed with a sector $k$ wage. Thus, the observed average wage for individuals starting a job in sector $k$ in period 0 is given by,

$$
\begin{equation*}
E\left(w_{k j t, t_{0}} \mid D_{j k t_{0}}=1\right)=p_{k t_{0}} E\left(\eta_{k j} \mid D_{j k t_{0}}=1\right) \tag{6}
\end{equation*}
$$

where, $D_{j k t_{0}}$ is a dummy variable equalling 1 if person $j$ starts a job in sector $k$ in period $t_{0}$. Based on this equation, one can see that the average starting wage in a sector will potentially move both because of movements in the task price and because of changes in the composition of who works in the sector. Our goal will be to try to address the selection issues in order to get estimates of trends in the task price.

Holding the selection issues aside for the moment, the other main issue arising from (5) is that the overall average wage in a sector at a point in time will reflect a weighted average of previous period sectoral productivity shocks. In particular, assume for simplicity that only workers of type $l^{*}$ choose to work in sector $k$. Then the average sector $k \log$ wage in period $t_{1}$ would be given by (again, ignoring any other selectivity),

$$
\begin{gather*}
\ln \bar{w}_{k t_{1}}=(1-\delta) \sum_{\tau=0}^{\infty} \delta^{\tau} \ln w_{l^{*} t_{1}, t_{1}-\tau}  \tag{7}\\
=\ln \eta_{k l^{*}}+\ln p_{k t_{1}}-(1-\delta) \sum_{\tau=0}^{\infty} \delta^{\tau} \sum_{m=0}^{\tau} \xi_{k, t-m}
\end{gather*}
$$

where, we have assumed that a set of measure 1 of workers enter employment in the sector in each period so that at time $t_{1}$ there will be $\frac{1}{(1-\delta)}$ workers alive. ${ }^{3}$ The key point is that the wage reflects an average of past productivity shocks with declining weights for periods farther in the past. The difference in average wages between periods $t_{1}$ and $t_{2}$ would then equal $\ln p_{k t_{2}}-\ln p_{k t_{1}}$ plus a term reflecting the fact that different weights would be put on the past shocks in the average wage for each period. This is true even in our simple specification with a constant death rate of workers. In a more realistic scenario with different rates of entry to and exit from jobs over time because of macro events, the weighting put on past shocks could be even more different in any two periods.

While the latter point is straightforward (and follows simply from assuming an implicit contract framework) it is important in helping to determine how we approach trying to isolate movements in the task price. In particular, one potential approach to the selection problem described earlier is to use panel data and follow individual workers in sector $k$ through time. Since this would, by definition, imply following a set of workers with time invariant ability held constant, the resulting changes should capture changes in the task price. This is the approach taken in Cortes (2012). However, in the implicit contract model used here, we would see zero change in wages for a given set of workers even though there are productivity differences driving differences in wages across

[^2]new entrant cohorts. Simply following average wages in an occupations combines this "stickiness" problem with the selection problem described earlier.

The alternative we pursue in this paper is to examine the wages of new job entrants. Using equation (5), we can write the difference in the average log wage between new entrants in two periods as,

$$
\begin{align*}
E\left(\ln w_{k j t_{1}, t_{1}} \mid D_{j k t_{1}}=1\right)-E\left(\ln w_{j t_{0}, t_{0}} \mid D_{j k t_{0}}=1\right) & =\ln p_{t_{1}}-\ln p_{t_{0}} \\
& +E\left(\ln \eta_{k j} \mid D_{j k t_{1}}=1\right)-E\left(\ln \eta_{k j} \mid D_{j k t_{0}}=1\right) \tag{9}
\end{align*}
$$

Thus, examining differences in new entrant wages allows us to focus on contemporaneous movements in task prices but still involves a problem in terms of changes in selection over time. ${ }^{4}$ We will address the latter through a combination of re-weighting and bounding techniques.

Finally, it is worth noting that in a spot market model, log wages for a worker will equal,

$$
\begin{equation*}
\ln w_{k j t}=\ln p_{k t}+\ln \eta_{k j}+e_{k j t} \tag{10}
\end{equation*}
$$

In that case, following new entrant wages will still capture contemporaneous movements in the task price but will again reflect selection effects that need to be addressed. Thus, the results we provide are of interest under the alternative assumption that wages are set in a spot market.

### 2.2. A Roy Model with Three Sectors

We turn now to the problem of allocation of workers to occupational sectors. Following many other papers, we will discuss the allocation as determined by individual choice in the context of a Roy model. We assume that workers can choose among three occupations: $C$ (cognitive); $R$ (routine); and $M$ (manual). As in the earlier discussion, workers are endowed with time invariant abilities (expressed in terms of the number of task units they provide) in sector $k, \eta_{k j}$. They take the price per task unit, $p_{k t}$, as given. Then, the $\log$ wage faced by worker $j$ in sector $k$ in period $t$ is given by:

$$
\begin{equation*}
\ln w_{k j t}=\ln p_{k t}+\ln \eta_{k j}+e_{k j t} \text { for } k \in\{C, R, M\} \tag{11}
\end{equation*}
$$

where $\ln p_{k t}$ is the log of the price per unit of the sector specific task, and $e_{k j t}$ is a mean zero, idiosyncratic error whose properties we will discuss below. In a contracting model in which $p_{k t}$

[^3]follows a random walk, this would be the wage for a person starting a job in this sector at time $t$, $e_{k j t}$ would correspond to measurement error, and the wage (apart from measurement error) would be constant for all subsequent years on that job. In a spot market model, this would be the wage in any period, $t$, and $e_{k j t}$ would correspond to to a combination of an idiosyncratic productivity shock and measurement error.

We will consider the problem of an unemployed person choosing among the three sectors, abstracting from the possibility of workers currently employed in a sector deciding whether to remain in that sector or move to another. A person chooses the sector in which to work based on a comparison of the present value of expected wages plus other sector-specific, work related costs. In the contracting model, the starting wage in period $t$ in a sector fully captures the expected value of future wages associated with choosing that sector at time $t$. Thus, we can write that a person prefers sector $C$ to sector $R$ iff:

$$
\begin{equation*}
\ln w_{C j t}-u_{C j t}>\ln w_{R j t}-u_{R j t} \tag{12}
\end{equation*}
$$

where, $u_{k j t}$ is an idiosyncratic cost of working in sector $k$. Substituting from (11), we can re-write (12) as:

$$
\begin{equation*}
z_{C R j t}=\ln \eta_{C j}-\ln \eta_{R j}+e_{C j t}-e_{R j t}-u_{C j t}+u_{R j t}-\left(\ln p_{R t}-\ln p_{C t}\right), \tag{13}
\end{equation*}
$$

where person $j$ prefers sector $C$ to sector $R$ iff $z_{C R j t}>0$ and prefers sector $R$ to sector $C$ otherwise. Similar equations can be written for comparing sector $C$ to sector $M$, and for comparing sector $M$ to sector $R$. Given a joint distribution on the disturbances (the $\eta$ 's, $e$ 's, and $u$ 's), one can get the proportion of workers in each occupation as a function of the set of task prices.

## 3. Addressing Selection Using Bounding

Our goal is to obtain estimates of movements in the occupation specific task prices (the $p_{k t}$ 's). While we can see from (11) that movements in observed wages in each sector will partly reflect such price movements, the selection processes imply that the price movements will induce changes in worker selection across occupations that will also potentially impact on the sectoral distributions of observed wages. Our approach to solving this problem is in the spirit of the Blundell et al. (2007) and Lee (2009) and involves bounding the price movements. While this bounding can be done using purely statistical arguments, we attempt to provide sharper bounds by employing a minimal amount of theory embodied in the Roy-type selection model just described. As we will see, the specifics of the bounding will depend crucially on assumptions about the error terms in equations (11) and (13). We consider a set of specific cases that correspond broadly to the Roy model sub-cases described in Willis (1986). We begin with the most straightforward and common case: an hierarchical ability model with three sectors and no observable covariates. This allows us to establish the main intuition and mechanics of our approach. We then discuss the addition of covariates and a fourth sector (non-employment) before moving to a more heuristic discussion of implications of other ability structures. We present complete discussions of how to implement the
bounds in the text only for the initial cases we examine. Details on implementation for all other cases are presented in Appendix ??.

### 3.1. Hierarchical Ability Model

The first model is one of the most commonly used in analyses of this problem. For example, it is the case that implicitly underlies many discussions of returns to education. In this model, we assume that there is only one type of ability affecting wages and sector choice, but allow it to be differentially productive in generating task units in each sector. That is, assume we can write:

$$
\begin{equation*}
\ln \eta_{k j}=\gamma_{k} \ln \eta_{j} . \tag{14}
\end{equation*}
$$

We also assume that the $e_{k j t}$ 's correspond to classical measurement error and, so, do not enter the sectoral choice processes. Finally, there are no idiosyncratic costs ( $u$ 's) in the selection process. Given these assumptions, the wage equation becomes:

$$
\begin{equation*}
\ln w_{k j t}=\ln p_{k t}+\gamma_{k} \ln \eta_{j}+e_{k j t} \text { for } k \in\{C, R, M\} \tag{15}
\end{equation*}
$$

and the equation determining preference of sector $C$ over sector $R$ becomes:

$$
\begin{equation*}
z_{C R j t}=\left(\gamma_{C}-\gamma_{R}\right) \ln \eta_{j}-\left(\ln p_{R t}-\ln p_{C t}\right) . \tag{16}
\end{equation*}
$$

The $\eta$ 's are independent across people and the $e$ 's are independent across people, time and sectors. We also assume that $\eta$ and $e$ are independent of each other, with the latter being mean zero, and that both are drawn from stationary distributions.

We follow Blundell et al. (2007) in focusing on estimation of median wages because, as we will see, this provides a natural way of using the implications of the selection model. To begin, we can write the median log wage in the $C$ sector as: ${ }^{5}$

$$
\begin{equation*}
\operatorname{Med}\left[\ln W_{C t} \mid Z_{C R t}>0 \& Z_{C M t}>0\right]=\ln p_{C t}+\gamma_{C} \operatorname{Med}\left[\ln \eta+e_{C t} \mid \ln \eta>A_{C R t} \& \ln \eta>A_{C M t}\right] \tag{17}
\end{equation*}
$$

where $A_{C R t}=\frac{\left(\ln p_{R t}-\ln p_{C t}\right)}{\left(\gamma_{C}-\gamma_{R}\right)}$ and $A_{C S t}$ is defined analogously. Capital letters correspond to random variables and small letters correspond to specific realizations of those random variables. As Cortes (2012) shows, $A_{C R t}$ corresponds to a threshold value for $\ln \eta$ such that individuals with $\ln \eta>A_{C R t}$ work in the $C$ sector; individuals with $\ln \eta \leq A_{C R t}$ and above another threshold work in the $R$ sector. Given this, only one of the inequalities in (17) binds and we can re-write it as:

$$
\begin{equation*}
\operatorname{Med}\left[\ln W_{C t} \mid Z_{C R t}>0 \& Z_{C M t}>0\right]=\ln p_{C t}+\gamma_{C} \operatorname{Med}\left[\ln \eta+e_{C t} \mid \ln \eta>A_{C R t}\right] . \tag{18}
\end{equation*}
$$

[^4]In period $t+1$, we can, similarly, write:

$$
\begin{equation*}
\operatorname{Med}\left[\ln W_{C t+1} \mid Z_{C R t+1}>0 \& Z_{C M t+1}>0\right]=\ln p_{C t+1}+\gamma_{C} \operatorname{Med}\left[\ln \eta+\epsilon_{C t+1} \mid \ln \eta>A_{C R t+1}\right] . \tag{19}
\end{equation*}
$$

Inspection of (18) and (19) reveals that changes in the median log wage observed in $C$ over time will partly reflect the changes in $\ln p_{C t+1}$, which is what we are interested in, but also changes in the median of the distribution of abilities that are selected into sector $C$.

We address the problem of selection on unobservables by using an approach that is related to the approaches described in Blundell et al. (2007) and Lee (2009). To see how to implement this, it is useful to write out the expression that implicitly defines the median $\log$ wage in sector $C$ in time $t$ (18), which we will call $m_{C t}$ as:

$$
\begin{equation*}
\operatorname{Pr}\left(\ln W_{C t}<m_{C t} \mid \ln \eta>A_{C R t}\right)=0.5 . \tag{20}
\end{equation*}
$$

Next, assume there has been an increase in $\left(\ln p_{C t}-\ln p_{R t}\right)$. Under the standard selection model set out earlier, such a price change implies that some workers will move into $C$ but none will move out. Because of this, we can write:

$$
\begin{align*}
\operatorname{Pr}\left(\ln W_{C t+1}<\tilde{m}_{C t+1} \mid\right. & \left.\ln \eta>A_{C R t+1}\right)=\pi_{C t+1} \operatorname{Pr}\left(\ln W_{C t+1}<\tilde{m}_{C t+1} \mid \ln \eta>A_{C R t}\right) \\
& +\left(1-\pi_{C t+1}\right) \operatorname{Pr}\left(\ln W_{C t+1}<\tilde{m}_{C t+1} \mid A_{C R t} \geq \ln \eta>A_{C R t+1}\right), \tag{21}
\end{align*}
$$

where $\left(1-\pi_{C t+1}\right)$ is the proportion of people in $C$ in period $t+1$ who entered $C$ because of the price change and $\tilde{m}_{C t+1}$ is the median log wage in $t+1$ for the workers who would have been in sector $C$ under period $t$ prices. That is, $\tilde{m}_{C t+1}$ is defined by:

$$
\begin{equation*}
\operatorname{Pr}\left(\ln W_{C t+1}<\tilde{m}_{C t+1} \mid \ln \eta>A_{C R t}\right)=0.5 \tag{22}
\end{equation*}
$$

and can be written as:

$$
\begin{equation*}
\tilde{m}_{C t+1}=\ln p_{C t+1}+\gamma_{C} \cdot \operatorname{Med}\left[\ln \eta+e_{C t+1} \mid \ln \eta>A_{C R t}\right] . \tag{23}
\end{equation*}
$$

If we could get an estimate of this then subtracting (18) from (23) would provide an estimate of the change in the $C$ sector task price. ${ }^{6}$ In fact, we provide bounds for this change. With some abuse of terminology, we call the set of people in the conditioning set in these equations the "stayers," i.e., the set of people who would choose sector $C$ under prices in either period. They are not truly stayers since, based on the logic from the previous section, we are looking at entry wages for new job starters and so they are actually different people in each period. But, given stationarity in the $\eta_{j}$ distribution, they correspond to people with the same set of values for $\eta_{j}$ in each period.

[^5]Similarly, we will call the set of people who would choose one sector under the period $t$ prices and another sector under the period $t+1$ prices "movers."

To generate bounds, return to (21) and note that the probability on the left hand side is based on a distribution we observe (the $C$ sector wage distribution for workers observed in the sector in $t+1$ ). Under the selection model assumptions, we also know that an increase in the relative price of the $C$ task will imply an inflow of people from sector $R$ to $C$ but no flows out of $C$. Thus, we can get $\pi_{C t+1}$ as the change in the number of people in sector $C$ divided by the total number of people in sector $C$ in period $t+1$. This leaves two probabilities on the right hand side of (21) that are unknown. Under the bounding approach, we adopt extreme values for one of these probabilities in order to bound the possible values of the other. In our case, since we are interested in the median wage for the "stayers," we will form bounds on it by adopting extreme values for the the probability associated with the "movers" (workers with $\eta_{j}$ values such that they would choose sector $R$ in period $t$ but would choose sector $C$ in response to the price increase in period $t+1$ ).

The widest bounds we can form are associated with the fact that $\operatorname{Pr}\left(\ln W_{C t+1}<\tilde{m}_{C t+1} \mid A_{C R t} \geq\right.$ $\left.\ln \eta>A_{C R t+1}\right)$ is a probability and so must lie between 0 and 1 . In particular, if this probability equals zero then we can write:

$$
\begin{equation*}
\operatorname{Pr}\left[\ln W_{C t+1}<\tilde{m}_{C t+1} \mid \eta>A_{R C t}\right] \leq \frac{1}{\pi_{C t+1}} \operatorname{Pr}\left[\ln w_{C t+1}<\tilde{m}_{C t+1} \mid \eta>A_{R C t+1}\right] \tag{24}
\end{equation*}
$$

and, if it equals 1 then we obtain,

$$
\begin{align*}
\operatorname{Pr}\left[\ln W_{C t+1}<\tilde{m}_{C t+1} \mid \eta>A_{R C t}\right]>\frac{1}{\pi_{C t+1}} & \operatorname{Pr}\left[\ln w_{C t+1}<\tilde{m}_{C t+1} \mid \eta>A_{R C t+1}\right] \\
& -\frac{\left(1-\pi_{C t+1}\right)}{\pi_{C t+1}} . \tag{25}
\end{align*}
$$

Then setting the left hand sides of (24) and (25) to 0.5 (since that defines $\tilde{m}_{C t+1}$ ), and solving them as equalities yields estimates of the lower bound on $\tilde{m}_{C t+1}, \tilde{m}_{C t+1}^{L}$, and the upper bound, $\tilde{m}_{C t+1}^{U}$, respectively. Once we have these, we can form a lower bound on the increase in the sector $C$ task price as $\left(\tilde{m}_{C t+1}^{L}-m_{C t}\right)$ and an upper bound as, $\left(\tilde{m}_{C t+1}^{U}-m_{C t}\right)$. One can show that the solutions to (24) and (25) are the equivalent of trimming $\pi_{t+1}$ proportion of observations from the top and bottom of the period $t+1 C$ sector wage distribution, respectively, and then obtaining medians for these trimmed distributions. This is similar to the implementation in Lee (2009) and is the way we obtain the bounds. We will call the bounds formed this way (trimming from either the top or the bottom of the distribution) the Extreme Bounds. We will describe ways of using economic theory to tighten the bounds as we proceed.

We derived the formulae for the bounds assuming that the relative price of $C$ had increased. If, instead, it had decreased then workers would exit sector $C$ between periods $t$ and $t+1$. In that case, we would form bounds by trimming a number of workers equivalent to the net decrease in the size of the $C$ sector from the top and bottom of the period $t$ distribution of $C$ sector wages and
comparing the medians of the resulting distributions to the median wage for all workers in sector $C$ in period $t+1$. Given the logic of the model, we can determine which type of trimming to do based on the sign of the net change in the number of workers in $C .{ }^{7}$ That is, if we observe a net increase in the number of workers in $C$ between two periods then we know there must have been a relative increase in the price of $C$ tasks and we trim from the period $t+1$ distribution, accordingly. If we observe a net decrease in number of workers in $C$ then we form bounds by trimming the period $t$ distribution.

### 3.1.1. Generating Bounds for the $R$ and $M$ Sectors

The approach just outlined for generating bounds for the $C$ sector task prices can be used in a similar manner to construct bounds for the $R$ and $M$ sector prices. For the $M$ sector, in fact, the approach is identical. If the size of the $M$ sector expands this implies that $\ln p_{M t}$ has increased relative to $\ln p_{R t}$ (recalling that with the single factor ability model, workers moving in or out of $M$ are only moving from and to the $R$ sector). As with the $C$ sector analysis, bounds can then be formed by trimming the number of workers equal to the net change from either the top or the bottom of the $S$ wage distribution in period $t+1$ and obtaining medians of the trimmed sample. If $M$ declines in size then, again as with the $C$ sector, the trimming would be done on the period $t$ sample.

For the $R$ sector, the analysis is somewhat more complicated because changes in $\ln p_{R t}$ have to be compared to movements in both $\ln p_{C t}$ and $\ln p_{S t}$. This does not cause problems if $\ln p_{R t}$ increases with respect to both of the other prices or decreases with respect to both. In either case, the change in the number of workers in the $R$ sector can be used to trim distributions and form bounds in the same way as just described for the other two sectors. However, if $\ln p_{R t}$ rises relative to the task price in one sector but falls relative to the task price in the other sector then there will be both flows in and out of the $R$ sector. In that case, the net change in the size of the $R$ sector is not informative for forming bounds. Fortunately, within the context of the single ability model we know the size of the gross flows into and out of the $R$ sector. In particular, both the changes in the number of workers in $C$ and the change in the number of workers in $S$ must imply equal and opposite changes in the numbers in $R$ since there are no flows between the $M$ and $C$ sectors. Suppose, then, that $N_{R C t}\left(=\Delta N_{C t}\right)$ workers move from $R$ to $C$ and $N_{M R t}\left(=\left|\Delta N_{S t}\right|\right)$ move from $M$ to $R$. We can form an upper bound on the movement in the price in $R$ by trimming $N_{R C t}$ observations from the top of the period $t R$ sector wage distribution and trimming $N_{M R t}$ observations from the bottom of the period $t+1 R$ sector wage distribution. We then get the medians from each of the trimmed samples and take the difference between those medians. The

[^6]lower bound would be obtained by trimming $N_{R C t}$ observations from the bottom of the period $t R$ sector wage distribution and $\left|N_{M R t}\right|$ from the top of the period $t+1$ distribution.

### 3.1.2. Identifying Assumptions

The procedure in the previous subsection identifies bounds on movements in a composition constant median wage if two conditions are met:

1. Monotonicity. A non-randomly selected (with respect to $\eta$ ) set of workers can be added or withdrawn from a sector in a period, but both cannot happen at the same time.
2. Time invariance of the $\eta$ distribution, i.e., $F_{t}(\eta)=F(\eta)$

Under the Monotonicity assumption, the observed change in the proportion of workers in the $C$ sector corresponds to all the relevant movement in workers into or out of the sector and allows us to use that proportion to form bounds. If, instead, non-random selections of workers both entered and left the sector at the same time then trimming according to the net change will not result in a composition constant set of workers. The basic selection model, in fact, imposes the monotonicity assumption: people will only move one way or the other in response to a price change or a change in the cost of entering the sector given the linear index determining the sector the person is in. This is a key point from Vytlacil (2002), which demonstrates formally that standard selection models are equivalent to imposing monotonicity. ${ }^{8}$

The time invariance assumption says that the overall distribution of abilities in the population does not change and, so, the distribution of abilities in a sector changes only because of worker movements in response to price or cost changes; movements that we address using the bounding approach. Without the assumption, trimming the number of workers added to the sector between period $t$ and $t+1$ will not result in a trimmed period $t+1$ sample with the same ability distribution as the period $t$ sample and we would not have formed bounds on a composition constant median wage. ${ }^{9}$ As a specific instance of this assumption, it requires that changes in the task prices do not induce changes in the ability distribution, i.e., $f\left(\eta \mid P_{t}\right)=f(\eta)$. Such a change could arise if workers obtained sector specific training in response to the price changes.

[^7]Under these two assumptions, changes in the median and all other percentiles for a composition constant set of individuals are identified. Like other identifying assumptions, these cannot be tested.

Our approach can be seen as an application of the approaches in Blundell et al. (2007) and Lee (2009), which are themselves built on the work in Manski (1990). Blundell et al. (2007) obtains bounds on the median wage unconditional on working, addressing the problem that workers are not selected randomly. This would be equivalent to our bounding the median wage that would hold if all workers were employed in one sector. Since we are ultimately interested in bounding changes in task prices, we only require a bound on movements in wages for some composition constant subset of workers. This is the approach in Lee (2009) and the one we follow here. ${ }^{10}$ It provides bounds that are less erratic than those on the unconditional median obtained under the Blundell et al. (2007) approach. Indeed, bounds on the unconditional median wage in a sector often do not exist if the size of the sector is sufficiently small.

The main difference between our work and the existing literature is that previous papers bound the median or mean wage. We want to identify not just wage movements but the movements in the task prices that underlie these wage changes. This requires a second set of assumptions under which we can obtain the price changes from the changes in the composition constant median wage movements. To understand those assumptions, it is useful to write the wage determination equation in a more general form:

$$
\begin{equation*}
w_{k j t}^{*}=f_{k}\left(x_{j t}, \eta_{k j}, p_{k t}, e_{k j t} ; \theta_{t}\right) \tag{26}
\end{equation*}
$$

where $x_{j t}$ is a time varying observable covariate, $\eta_{k j}, p_{k t}, e_{k j t}$ are unobservable covariates, and $\theta_{t}$ is a vector of parameters that may be time varying, including $\gamma_{k t}$ in the earlier specification. For the moment, assume that we stratify on the $x_{j t}$ vector and so ignore that part of the function. We return to considering covariates in the next section. Our objective here is to ascertain the minimal set of restrictions on $f_{k}(\cdot)$ that would allow us to argue that movements in the composition constant median wage reveal movements in the underlying task prices.

Leaving aside covariates, two main issues arise in determining the wage functions that can be used to identify the change in sector prices. The first relates to other time varying arguments in $f_{k}\left(x_{j t}, \eta_{k j}, p_{k t}, e_{k j t} ; \theta_{t}\right)$. If, in particular, any of the parameters defining the $f_{k}(\cdot)$ function are time varying then part of what is picked up in changes in the median wage will be changes in those parameters. For example, in the simple wage specification that we have used so far, if the sector specific factor loadings on ability (the $\gamma_{k}$ 's) are time varying then the change in the median log wage will equal $\Delta p_{k t}+\Delta \gamma_{k t} \cdot \operatorname{Med}\left[\eta_{j} \mid \eta_{j}>A_{C R t}\right]$. Thus, changes in the median wage, $\tilde{m}\left(w^{*}\right)$, could be driven by either or both of changes in $p_{k t}$ or $\gamma_{k t}$. In other words, the two forces are not separately identified.

One potential response to this problem is to impose an assumption that restricts $\theta_{t}=\theta$. However, we can separately identify movements in $p_{k t}$ and time-varying components of $\theta_{t}$ under

[^8]the assumption that $f_{k}(\cdot)$ is additively separable as follows:
\[

$$
\begin{equation*}
w_{k j t}^{*}=f_{k 1}\left(x_{j t}, p_{k t} ; \theta_{1}\right)+f_{k 2}\left(x_{j t}, \eta_{k j}, e_{k j t} ; \theta_{2 t}\right) \tag{27}
\end{equation*}
$$

\]

where $\theta_{1}$ contains only time invariant elements of $\theta$ and any time-varying elements are contained in $\theta_{2 t}$. The key features of this formulation are that $f_{k 1}(\cdot)$ contains only $p_{k t}$ and observable covariates while $f_{k 2}(\cdot)$ is a non-additively separable function of $\eta_{k j}$ and $\theta_{2 t}$. Our earlier log wage specification follows this form, with $\theta_{2 t}$ including $\gamma_{k t}$. With a wage function of this form, we can identify movements in $\theta_{2 t}$ through movements in higher moments. In particular, in this formulation a change in $p_{k t}$ will change all quantiles of the wage distribution in the same way (i.e., it only affects the location of the distribution conditional on $x$ ). In contrast, since $\theta_{2 t}$ interacts with $\eta_{k j}$ in the second sub-function, changes in $\theta_{2 t}$ will alter the higher moments of the log wage distribution. To take advantage of this, note that the bounding procedure we have described can be applied to any quantile (not just the median). Working with our earlier expression for log wages, we can write the inter-quartile difference for the $\log$ wage in sector $C$ in period $t$ as:

$$
\begin{gather*}
\ln p_{C t}+\gamma_{C t} q 3\left[\ln \eta+e_{C t} \mid \ln \eta>A_{C R t}\right]-\ln p_{C t}+\gamma_{C t} q 1\left[\ln \eta+e_{C t} \mid \ln \eta>A_{C R t}\right]  \tag{28}\\
=\gamma_{c t}\left(q 3\left[\ln \eta+e_{C t} \mid \ln \eta>A_{C R t}\right]-q 1\left[\ln \eta+e_{C t} \mid \ln \eta>A_{C R t}\right]\right)
\end{gather*}
$$

where $q 1[\cdot]$ and $q 3[\cdot]$ refer to the first and third quartiles of a distribution, respectively.
Then, using trimming as before, we can generate a difference for period $t+1$ as:

$$
\begin{equation*}
\gamma_{c t+1}\left(q 3\left[\ln \eta+e_{C t+1} \mid \ln \eta>A_{C R t}\right]-q 1\left[\ln \eta+e_{C t+1} \mid \ln \eta>A_{C R t}\right]\right) \tag{29}
\end{equation*}
$$

Subtracting (28) from (29) provides a bound on the change in $\gamma_{C t}$, identified up to a factor of proportionality. The bound on the change in $\gamma_{k t}$, in turn, can be used to adjust our interpretation of the bounds on the movements in the composition adjusted median wage. If the trend in $\gamma_{k t}$ is flat (i.e., if there are no significant changes in the inter-quartile range of log wages within a sector) then $\Delta \gamma=0$ and changes in the median wage identify changes in $p_{k t}$. If, instead, $\gamma_{k t}$ follows a discernible trend that is the opposite to the trend in the median wage then we an conclude that the trend in $p_{k t}$ is in the same direction as the trend in the median wage and at least as large as the median wage trend. However, if trends in both $\gamma_{k t}$ and the median wage are in the same direction then we will be unsure of the extent or direction of any trends in $p_{k t}$.

One component of the wage function that is unavoidably time varying is $e_{k j t}$. However, if we assume that its distribution does not vary with $t$ then it does not raise any further problems. In that case, for new jobs starters, it behaves much like $\eta$ in the sense that each cohort has a new set of draws on it. In our simple log wage specification, this disturbance enters in an additively separable way, but we could allow for interactions with time varying components of $\theta$. In that case, it would simply be a contributor to the movements in higher moments of the wage distribution just discussed.

The second identification issue is with the function of $p_{k t}, f_{k 1}(\cdot)$. If $p_{k t}$ enters the wage determination function through a monotonically increasing transformation then we can bound the
direction of the price change but not the magnitude. If it enters through more general functions then even the direction may not be identifiable. To get the magnitude of the price change as well as its direction, we need to assume that $p_{k t}$ enters in a simple linear (or log-linear) fashion with a coefficient of 1 .

In summary, in our initial log linear wage specification, with the factor loadings on ability being time invariant, changes in the composition constant sectoral median wages identify the direction and magnitude of changes in $p_{k t}$. Allowing for more general functions of $p_{k t}$ and for other timevarying elements of the wage determination function can still permit identification of bounds on, at least, the direction of the trend in $p_{k t}$ under specific assumptions about additive separability.

Finally, it is important to notice that we do not require exogeneity of the price changes in the sense of their not being jointly determined with the supplies of labour to sectors. We are not trying to provide bounds on either supply or demand elasticities but rather to provide bounds on selection-free reduced form price changes that emerge from the operation of the economy. For example, if there were a decline in the cost of entering the $C$ sector then we would observe an increase in the number of $C$ sector workers and the equilibrium price, $p_{C t}$ would decline. Our goal is to bound that movement in $p_{C t}$, addressing the change in the composition of $C$ sector workers that accompanies the change in the size of the sector. Bounding those equilibrium price changes narrows the relevant set of models of the determination of the price changes that can be said to be consistent with the data.

### 3.1.3. Tightening the Bounds using Stochastic Dominance

We now move to tightening the bounds while working within the Hierarchical model. As Blundell et al. (2007) show, knowing that one of the unobserved distributions stochastically dominates the other can help in tightening bounds. In our case, the logic of the model does imply such a condition. Consider, in particular, sector $C$ wages when the relative price of tasks in $C$ is rising. In that case, it is simple to show that the wage distribution of the stayers first order stochastically dominates that of the people who move into sector $C$ in period $t+1$. That is:

$$
\begin{equation*}
\operatorname{Pr}\left(\ln W_{C t+1}<x \mid A_{C R t} \geq \ln \eta>A_{C R t+1}\right)>\operatorname{Pr}\left(\ln W_{C t+1}<x \mid \ln \eta>A_{C R t}\right) . \tag{30}
\end{equation*}
$$

The intuition for this is straightforward and, as we will see, carries over to versions of the Roy model with other ability patterns. Those who are drawn into a sector by an increase in its relative price must be of lower ability than those already in the sector since the latter had such high sector specific abilities that they were willing to choose the sector even when its price was lower. Thus, the stochastic dominance assumption ultimately rests on the assumption that people are voluntarily choosing their sectors in response to price or cost changes.

We can use (30), replacing $x$ with $\tilde{m}_{C t+1}$, instead use the bound defined by $\operatorname{Pr}\left(\ln W_{C t+1}<\right.$ $\left.\tilde{m}_{C t+1} \mid A_{C R t} \geq \ln \eta>A_{C R t+1}\right)>0$. Using this new bound with equation (21), implies that the
new lower bound is $\operatorname{Pr}\left(\ln W_{C t+1}<\tilde{m}_{C t+1} \mid \ln \eta>A_{C R t+1}\right)$, i.e., the actual observed median wage in sector $C$ in period $t+1$. Thus, the lower bound on price changes is just the observed change in median wages in sector $C$. Bounds for the change in $\ln p_{C t}$ when people are leaving the sector are formed analogously. Sector $M$ can also be solved analogously, with the key exception that now the people moving into the sector have wages that stochastically dominate the incumbents. The operations for dealing with $R$ occupation are, again, more complicated. A detailed description of how to do the bounding in each case is provided in Appendix ??.

### 3.1.4. Introducing Nonemployment

The results so far can easily be extended to a case with nonemployment. In particular, assume that there is a fourth sector, $H$, or home. We do not observe any measure related to the price in this sector and so are only interested in it to the extent it affects wages in the other sectors. In the Hierarchical model, we assume that it fits in below the $M$ sector, i.e., there will be a cut-off, $A_{H S t}$ such that people with $\eta$ values below this cut-off will be in the home sector.

There are two adjustments we need to make in the model set out above. First, the $M$ sector is no longer an extreme end sector. Instead, it will be treated as the $R$ sector was previously. Second, we need to get values for the flows between $H$ and $M$, and use them to get values for the flows between $M$ and $R$ (the latter were just the changes in the size of $M$ previously). Under the structure of the model, any change in the number in $H$ (i.e., the number nonemployed) will be the negative of the flows from $H$ to $M$,i.e., $\Delta N_{H t}=-N_{H M t}$, where $N_{H M t}$ is the number flowing from $H$ to $M$. Using this, we then back out the number flowing from $M$ to $R$ as, $N_{M S R}=\Delta N_{M t}-N_{H M t}$. Once we have these numbers, the bounds for both $M$ and $R$ are constructed in the same way as the bounds for $R$ alone were constructed in the three sector case. Nothing changes for the $C$ sector. Again, implementation details are in Appendix ??.

### 3.1.5. Introducing Covariates

Next, consider the possibility that the characteristics affecting productivity in each sector can be divided into ones that are observed and ones that are unobserved. In particular, we assume that ability in a sector $\eta_{k j}$ can be produced using a combination of unobserved characteristics, $\eta_{k j}^{*}$, and observable characteristics such as age and education that are represented in a vector, $\mathbf{x}$. Based on this, we can write:

$$
\begin{equation*}
\eta_{k j}=\eta_{k j}^{*} e^{\mathbf{x}_{j} \beta_{k}} \tag{31}
\end{equation*}
$$

Given this, the median wage in sector $C$ would now be defined by:

$$
\begin{equation*}
\operatorname{Pr}\left(\ln W_{C t}<m_{C t} \mid\left(\gamma_{C}-\gamma_{R}\right) \ln \eta^{*}+\mathbf{x} \cdot\left(\beta_{C}-\beta_{R}\right)>\left(\ln p_{R t}-\ln p_{C t}\right)\right)=0.5 . \tag{32}
\end{equation*}
$$

A key question in seeking to identify changes in the $p_{k t}$ 's is whether the $\beta$ 's are time invariant. Even if they are, as the relative prices change, and assuming that $\beta_{C} \neq \beta_{R}$, the combination of values of $\eta^{*}$ and $\mathbf{x}$ that satisfy the conditioning statement in (32) will vary. That is, the composition of workers in the sector in terms of both observable and unobservable characteristics will change, and sectoral wage changes will reflect both that change and changes in the sectoral task price. In this situation, holding the composition constant in terms of observable characteristics will go part way toward allowing us to identify movements in task prices alone. With time invariant $\beta$ 's a simple way to do this is to generate estimates for workers with one specific value of the $\mathbf{x}$ vector since all types of workers face the same price trend. Holding the value of the $\mathbf{x}$ vector constant in this way, the expression in (32) is the same as what we worked with in earlier sessions apart from the inclusion of the fixed component $\mathbf{x} \cdot\left(\beta_{C}-\beta_{R}\right)$. Thus, all of the same analysis goes through. ${ }^{11}$ Alternatively, if the shape of the $\eta$ distribution does not vary with $\mathbf{x}$ then one could run a median regression on the $x$ vector, obtain residuals and work with those residuals as the equivalent of the wages in the expressions in the earlier sections. However, if the distribution of $\eta$ does vary with $\mathbf{x}$ then this approach would not work since one would want to trim more from some covariate groups than others. The approach we describe below allows for the possibility of the $\eta$ distribution varying with $\mathbf{x}$.

To the extent that the $\beta$ 's are not time invariant, we will see different median wage trends for different worker types defined by covariate values. If we assume that the common trend across observable worker types as being the task price trend then a weighted average of type trends with fixed weights will give us the task price trend. We will make this assumption in interpreting our estimates. ${ }^{12}$

The approach we pursue to obtaining a common trend while holding the $\mathbf{x}$ vector composition constant is to trim within groups defined by each possible value of the $\mathbf{x}$ vector and then re-combine the resulting, trimmed samples into an overall sample (using time constant weights) and obtaining the median of that adjusted sample. This, in principle, generates a trimmed sample with the $\eta$ composition within each covariate group being constant. Since these sub-samples are re-combined with fixed weights, the end result is an overall sample with a time invariant $\eta$ distribution and the

[^9]covariate composition held constant at the base year value. ${ }^{13}$
The specific steps in our procedure are as follows (all carried out separately for different task groups):

1. Divide data into cells based on observable skill groups. Because of our arguments from the implicit contracting model, we focus on individuals who are under age 30 and have 5 years or less of predicted experience. This means that we do not need to further control for age or experience. We define skill groups based on education groups. We work with males and females separately and will present results for both.
2. Re-weight data in each cell to keep the number of observations constant at the base year (1990) value.
3. Pool the data across cells and then get the median. This gives composition constant median wage series
4. For the Cognitive occupation group, compare the number of observations in the cell in a given year to the number in the base year. ${ }^{14}$ If the current year has $N$ more observations than the base year then trim $N$ observations from the bottom of the current year. If the base year is larger, trim $N$ observations from the bottom of the base year.
5. For each of the base year and current year, pool the cell samples, weighting to maintain the true base year composition for each. Note that the base year trimming could be different for each current year.
6. Calculate the median wage for the base year sample ( $m_{t}^{B}$ ) and current year sample ( $m_{t}^{C}$ ), and take the difference.

The result is composition constant and a bound on the selection-free median. Under our second set of assumptions, it is also a bound on the change in the task price.

### 3.1.6. Incorporating Selection into Education

So far, we have ignored the possibility that $\eta^{*}$ and the $\mathbf{x}$ 's are correlated. During our time period, there was a substantial rise in the education level of the workforce. That means that the size

[^10]of the high education cells in our procedure rise over time, with offsetting declines in the size of low education cells. In our procedure, we re-weighted the data in each cell so that it has the same size as in the base period. This is the equivalent of trimming the cell samples by removing (or adding) workers randomly. But if the newly added high educated workers do not have the same ability distribution as those who previously had that education level then the median wage for a given occupation in the cell could change because the ability composition of the cell has changed even if the task price does not change. This is the point made earlier about the need for an assumption of time-invariant ability distributions.

We address this problem through the use of a trimming exercise that is analogous to the one we use to address occupational selection. For tractability reasons, we work with two education groups: low (high school or less education) and high (any post-secondary education). We provide some robustness checks on the effects of this grouping relative to using multiple education categories. We continue to assume that there is one underlying, time invariant ability, $\eta$, and that selection into high education is potentially related to that ability. In our analysis, we first consider an extreme case in which all of the increased number of high educated workers between the base year and a year, $c$, have lower ability than the high educated workers in the base year. This case can be justified in an hierarchical ability model in which there is a falling cost of obtaining a high education. In such a model, higher ability individuals would already have obtained a high education in the base year and all those who are added would be of lower ability. We implement this case in a two step approach: 1) obtain the increase in the number of high educated workers between the base year and year $c$, and trim that number from the bottom of the period c wage distribution for high educated workers and from the top of the base period wage distribution for low educated workers; 2) continue with the steps outlined above, starting at step 3.

At the other extreme, we could assume that the new additions to the high education cell were higher ability than those who obtained high education in the base year. But this seems to us to be unnecessarily extreme and unrealistic. Instead, as in the occupational trimming, we assume that the other extreme is formed based on the assumption that the ability distribution of those with high education in the base year first order stochastically dominates that of the new additions to the cell. As in the occupational trimming, this is equivalent to working with the actual samples in both base and c years. Thus, it is what we would implement in the procedure outlined earlier.

Once we implement these trimming steps and, as in the procedure, re-combine the cell samples using base period weights, we obtain four sets of bounds for the cognitive task price: upper and lower bounds when we assume that any expansion of the high educated cells occurs at the bottom of the high educated ability distribution (with offsetting exits from the top of the low-educated distribution); and upper and lower bounds when we work with the observed ability distributions (i.e., use the stochastic dominance result).

### 3.2. Alternative Ability Models

We now re-examine the conclusions we have reached so far in the context of other assumptions about the errors in the wage and selection equations. These other assumptions correspond to other models of sector specific abilities and moving costs.

### 3.2.1. Pure Single Ability Model

An extreme version of the hierarchical ability model arises if there are no $e$ 's in the wage equation: both wages and selection depend only on a single ability factor. In that case, we would know exactly where movers between sectors fit in the sectoral wage distributions. For example, if we observe an increase in the number of workers in the $C$ sector, $\Delta N_{C t}>0$, then we know that the lowest $\Delta N_{C t}$ workers in the period $t+1 C$ sector wage distribution are those movers. We could then trim those $\Delta_{C t}$ observations from the period $t+1$ distribution, find the median and difference from it the median wage in sector $C$ in period $t$. In the more general hierarchical model, where movers could end up anywhere in the period $t+1$ distribution, this generated the upper bound on the change in the sector $C$ price. Under this more restrictive model, no bound is needed: the number calculated from the trimming exercise is the estimate of the price change. Essentially, this is the most extreme version of the monotonicity assumption.

### 3.2.2. Independent Productivity Shocks Model

The other main category in Willis (1986)'s taxonomy of selection models can be represented in our case by a model in which the $\eta$ 's and $u$ 's don't exist and the $e$ 's enter the sector decision equation. Thus, the log wage would be written as:

$$
\begin{equation*}
\ln w_{k j t}=\ln p_{k t}+e_{k j t} \text { for } k \in\{C, R, M\} . \tag{33}
\end{equation*}
$$

Again, we will assume that the $e$ 's are mean zero, white noise variates - independent of the prices and of other values for $e$ in other sectors and across time, even for the same person. Importantly, in this model, there is no natural ordering of occupations. People can be good lawyers but bad carpenters, with the draws on how good a person is at each being independent.

The $e$ 's now enter the selection process:

$$
\begin{equation*}
z_{C R j t}=\left(e_{C j t}-e_{R j t}\right)-\left(\ln p_{R t}-\ln p_{C t}\right) . \tag{34}
\end{equation*}
$$

A key first point arises if we think about this model while following individuals through time. With people getting different draws on the $e$ 's in each period, we will see people moving both from $C$ to $M$ and from $M$ to $C$. This appears to imply a loss of the monotonicity assumption. But consider a situation where the prices don't change. In that case, the median wage in $C$ does not change even
though the specific people in the sector will have changed. One way to look at this is that we care only about the set of $e$ 's in the sector, not which people actually possess those $e$ 's. The conditional distribution of the $e$ 's in a sector only changes when the price changes and it is this change in the distribution that we need to characterize. This is a particularly useful characterization in our case since we are examining different entry cohorts which, by definition, consist of different people.

The logic of the Roy model implies that addressing selection with these ability assumptions is easier than with the Hierarchical Ability model. One can show that with Independent Shocks model, the wage distribution of people who would be in this sector under both period $t$ and period $t+1$ prices will first order stochastically dominate that of people who would be induced to join the sector because of an increase in its price between period $t$ and $t+1$. The logic underlying the latter result again comes from the monotonicity implicit in the selection model: the people induced to join the sector by a price increase will be of lower ability in that sector than those who were in at the lower price, otherwise they, too, would have already been in the sector. With sector specific abilities being independent, workers may be moving between any pair of sectors in a given period depending on the set of relative price changes (e.g., unlike in the Hierarchical Model, we could see workers moving from $C$ to $M$ and from $R$ to $C$ in the same period). But, again because of the independence, in considering changes in a specific sector, there is no relevant information about that sector's abilities in where new workers come from or departing workers exit to. That means that only the net change in the size of the sector matters for the ability composition in the sector. Thus, we can form two bounds: 1) trim the net change in number of workers from the bottom of the period $t+1$ wage distribution if the sector grew and from the bottom of the sector $t$ distribution if it shrank; and 2) the actual changes in the median wages (under the Stochastic Dominance assumption). Notice that this is the same as the bounds for the $C$ sector in the Hierarchical Model since all workers enter and leave that sector at the bottom of the distribution in that model. But it is different from the $R$ and $M$ sectors in the Hierarchical Model. For those sectors, workers entered and left from either the top of the distribution or from both the top and bottom, depending on the number of sectors.

### 3.2.3. Combination Model

Now consider a case that is a combination of the Hierarchical and Independent Shocks model. In this case, the log wage would again be written as:

$$
\begin{equation*}
\ln w_{k j t}=\ln p_{k t}+\gamma_{k} \ln \eta_{j}+e_{k j t} \text { for } k \in\{C, R, M\} . \tag{35}
\end{equation*}
$$

The $e$ 's enter the selection process:

$$
\begin{equation*}
z_{C R j t}=\left(\gamma_{C}-\gamma_{R}\right) \eta_{j}+\left(e_{C j t}-e_{R j t}\right)-\left(\ln p_{R t}-\ln p_{C t}\right) . \tag{36}
\end{equation*}
$$

First, consider the $C$ sector. Since this continues to be the top occupation in terms of selection based on the $\eta$ 's, entering or departing workers from the sector will tend to come from the bottom
of the $\eta$ distribution and will also come from the bottom of the $e_{C}$ distribution for the reasons discussed in the independent shocks model. Thus, the bounds for this sector are the same as in both the previous models.

For the $R$ sector (and also the $M$ sector in the four sector model), the situation is different. In particular, suppose that $p_{R t}$ increases relative to all other prices. Inspection of (36) indicates that those entering from the $C$ sector will tend to be at the top of the $\eta$ distribution for the $R$ sector, while those entering mainly because of comparisons of $e$ values will tend to enter at the bottom. This means that for entrants from the $C$ sector, no stochastic dominance argument is possible. On the other hand, entrants from both the $M$ and $H$ sectors will enter at the bottom for both reasons and so a stochastic dominance argument remains a possibility. In the appendix we show that for these sectors the key distinction is whether inflows from sectors that rank (in terms of the hierarchical model ranking) above versus below the $M$ and $R$ sectors have the same sign. For example, if there are inflows from $C$ to $R$ but outflows from $R$ to $M$ then any potential trimming related to the $C \rightarrow R$ flow would be done in period $t+1$ while potential trimming related to the $R \rightarrow M$ flow would be done in period $t$. For the reasons just stated, we cannot use stochastic dominance to tighten the parts of the bounds related to the $C \rightarrow R$ flow but we can do some trimming in period $t+1$ related to the $R \rightarrow M$ flow. If, on the other hand, both sets of flows were, for example, outflows then one could not be sure where the $C$ part of the outflow came from in the $R$ distribution and only the extreme bounds would be possible. Thus, in the most complete model, there will tend to be limited opportunities for tightening the bounds based on economic arguments in the $M$ and $R$ sectors and we will be forced to resort to the Extreme Bounds.

Finally, one could introduce shocks to the costs of being in a sector that do not directly affect productivity (the $u$ 's in the initial model). If these are independent of all other shocks then the resulting model is the same as the Combination Model in terms of its implication for bounding. This is true because the implications of the $u$ 's for bounding are the same as for the $e$ 's. That is, decreases in these costs for a particular sector will lead people to enter the sector and the entrants will tend to be at the lower end of the wage distribution since those with higher productivity in the sector would tend to be already working there. If, however, the cost shocks and the productivity shocks for a sector are positively correlated then one can no longer make stochastic dominance arguments for anyone and the extreme bounds are the only ones that are clearly proven.

## 4. Summarizing the Possibilities

At this point, there are three possible set of bounds:

1. Extreme bounds: This involves trimming changes in the size of the sector from, alternately, the top and the bottom of the sectoral wage distribution. This is the widest set of bounds that we calculate.
2. Tight bounds using stochastic dominance under the Hierarchical Model: Under this approach, the hierarchical model tells us whether to trim from the bottom or top of a given sector's wage distribution as one bound and the other bound is the observed change in medians due to the stochastic dominance assumption. One could also justify this approach as being reasonable in a version of the combination model in which the variance of the $\eta$ disturbances is much larger than those for the $\epsilon$ 's and so sector choice is predominantly based on the $\eta$ 's. These are the tightest of the available bounds.
3. Tight bounds using stochastic dominance under the Independent Shocks Model: In this case, one bound is formed by trimming from the bottom of the wage distribution in all sectors and the other is based on the observed changes in medians due to the stochastic dominance assumption. This approach be justified as a possible but not proven version of the combination model.
4. Tight bounds using stochastic dominance under the combination model:. In the $C$ sector, these bounds are the same as under the Independent Shocks Model but for the $M$ and $R$ sectors they tend to be only slightly better than the extreme bounds.

## 5. Data

The data we use for our empirical work comes from the the Current Population Survey's Outgoing Rotation Group, from the years 1984 to 2013. Our initial extraction includes all individuals aged 18-64 years with positive potential experience, and excludes full- or part-time students. ${ }^{15}$ Following Lemieux (2006), we use the hourly wage as our wage measure and do not use observations with allocated wages when calculating wage statistics. ${ }^{16}$ Wages and employment status refer to the week prior to the survey week, and we only use wage and occupation data on individuals who are currently employed in the reference week. Our main empirical work focuses on a subset of these data, with the the goal of capturing workers at the beginning of their careers. To do this, we select young workers, between the ages of $18-30$, with no more than 5 years of potential experience.

Following Acemoglu and Autor (2011) and the extensive literature on polarization they survey, we aggregate occupations into three broad groups: Cognitive (occupations with a high intensity of abstract thinking tasks that are often viewed as complementary to capital and organizational forms embedding information technology (IT)); Routine Production and Clerical (blue-collar occupations intensive in routine tasks that can be easily substituted for by IT, white-collar occupations intensive in routine tasks); and Manual (service and manual occupations that tend to be low skilled but not easily substituted for with IT). The occupation codes are based on 1980/90 Census categories and

[^11]are consistent from 1983-2002. For post-2002 data, we use Bureau of Labor Statistics cross-walks to convert the data into 1980/90 categories before aggregating into the three broad occupation categories we use below. ${ }^{17}$

## 6. Results

### 6.1. Observed Wage Trends

In Figure 1, we plot median hourly log wages for workers of all ages for the three occupation groups, separately for men and women. For ease of comparison, median wages for each occupation we plot the difference between the log median wage in each year and the log median wage in the base year(1984). When calculating the median wage, we delete observations with allocated wages but we do not trim the remaining sample for outliers.

The sectoral wage movements for men fit with the argument in papers such as (Autor et al. 2003, 2007; Acemoglu and Autor 2011) that since the 1990s, the U.S. has undergone a polarization in wages, with wages for both low skilled $M$ sector workers and high skilled $C$ sector workers growing relative to wages in the $R$ sector. For women, the period before the mid- 90 s contains only weak evidence of polarization. In the last part of the 90 s , it is the $M$ sector wages for women that outstrip both the other sectors. For both genders, the patterns fit with the claim in Beaudry et al. (2012) that something changed in the U.S. economy around 2000. Post-2000 sharply increasing 1990s trends for the S and C sectors for men and for all sector for women give way to flat trends.

As discussed previously, shifts in the wage series can arise because of changes in task prices or because of composition shifts within each occupation. Composition shifts might occur because of shifts in the age structure due to the ageing of the baby boom cohort or because of increases in education during the sample period. To account for these shifts, we perform a simple reweighting exercise. In particular, we follow Lemieux (2010) and divide the data into demographic cells based on five education groups and eight age groups. For each occupation, we create a set of fixed weights based on the average proportion of each of the demographic groups during the entire sample period. Using these weights, we recalculate occupational median wage series holding the demographic mix of each occupation constant at the average fraction of all years combined. In Figure 2, we report the results of this exercise separately by gender.

This figure is our first indication that composition does matter in this period. For men, real wage declines for the $R$ sector double once we control for age and education composition. Perhaps most strikingly, for the $M$ sector, a pattern of modest overall gains, with wage increases in the 1990s and a flat wage trend after 2000, is converted to one of general decline, with wages falling

[^12]

Fig. 1.- Indexed Median Wage by Occupation Group: All Workers
particularly strongly after 2000. For both men and women, once we take account of education increases and the rise in the average age of the workforce, wages in all sectors fall after 2000.

Our main empirical work will focus on younger workers at the beginning of their careers in order to abstract from the fact that longer tenured workers' wages may be determined by contracting issues and not directly related contemporaneously to movements in productivity. In Figure 3 we plot indexed wages by occupation group for our sample of young workers, where, again, we have held the education composition of each occupation constant at the average for the entire sample. For this age group, the wage movements are actually less dramatic than for the overall age sample but continue to show the pattern of increases in the 1990s followed by declines after 2000. For males, there is not a strong indication of polarization since all three task prices follow broadly similar trends. For females, the main sectoral difference is the strong rise in wages in the $M$ sector relative to the other two in the late 1990s.


Fig. 2.- Indexed Median Wage by Occupation Group: All Workers, Composition Adjusted

### 6.2. Bounds

We turn, now, to our main empirical results: those showing our constructed bounds for movements in the median sectoral wages when both the composition of observables and unobservables is held constant. All of these results are for our young age/experience sample.

### 6.2.1. Cognitive Task Sector

In Figure 4, we present plots of the two bounds for the cognitive sector task price along with a bootstrapped confidence interval for the line where we trim from the bottom of the wage distribution. It is worth recalling that the same procedure is used for forming bounds for the Cognitive task price under all three of our models. To form the confidence interval, we draw 100 samples of the data of size $90 \%$ of the actual sample size with replacement. We obtain the standard deviation of the value of the bound in each year across the 100 samples. The plotted confidence


Fig. 3.- Indexed Median Wage by Occupation Group: Young Workers, Composition Adjusted
interval is plus or minus 1.96 times that standard deviation. It is worth pointing out that these lines correspond to changes relative to the base period, 1984. As the sector grows and contracts, it is possible for the lines to cross since trimming is done either in the current year or the base year depending on growth versus contraction. Thus, unlike standard bounding, in any given year, one line shows the upper and the other the lower bound on the change relative to the base year but which is upper and which is lower can vary. ${ }^{18}$

The dashed line in figure 4 is simply the covariate constant median wage. For males, in the left panel, it increases by approximately $15 \%$ between 1990 and 2000 before declining by approximately $10 \%$ by the end of our sample period. In comparison, the bound formed by trimming from the bottom of the distribution shows a greater run-up in the 90s (experiencing nearly a $20 \%$ increase) but also a larger decrease, ending up at a lower level than the covariate constant median. It is

[^13]also the case that the lower end of the confidence interval in 2001 is above the upper end of the confidence interval in 2012, indicating that we would not be able to reject that cognitive task prices fell over this decade. For females, the pattern is similar but more muted.

In Figure 5, present the same plot but restricted to those with a college education or higher or higher education. Thus, in this figure, we are controlling for both experience and education. The patterns for both males and females are similar to the all-education plot but, in both cases, show larger movements. For example, for males, the bound obtained by trimming at the bottom end increases by over $20 \%$ in the 1990 s and then declines by more than that in the 2000 s. The similarity of the two figures is reassuring in that it suggests a common trend for the task price whether we control for covariates through an averaging approach or by examining one covariate group.


Year

$$
\begin{aligned}
& \text { ———Composition Contsant Median } \quad \text { Bound with Trimming } \\
& \text {--.... Confidence Interval }
\end{aligned}
$$

Fig. 4.- Cognitive Task Price Changes, Young, All Education
Recall that our conclusions about task price movements from figures like 4 and 5 can be altered if the factor loading on ability is changing at the same time. To gauge this, in Figure 6, we plot the difference between the 75 th and 25 th percentiles of the trimmed samples. The general trend for the difference is quite flat for males up until the early 2000's after which it appears to increase to


Fig. 5.- Cognitive Task Price Changes, Young, High Education
some extent. However, given that the lower edge of the confidence interval never exceeds the upper edge from earlier years, we cannot reject the hypothesis that the difference does not change over time. This would imply that the median wage changes in the earlier figures correspond to changes in the task price. To the extent that the inter-quartile range is rising after 2000 , since the median wage is declining over this period, the implication would be that the task price is falling even more than the bounds shown for the composition constant median wage.

Finally, in Figure 7 we plot the bounds on the median wage for all education groups combined after we first trim in relation to education changes. The substantial increase in the proportion of workers with a high education implies a corresponding substantial amount of trimming from samples after the base year. This results in very large increases in the trimmed bounds relative to what was observed in figure 4. Once we also trim from the bottom of the cognitive wage distribution, we observe increases of over $40 \%$ between 1984 and 2000 followed by about a $20 \%$ decline over the 2000s. Thus, the post- 2000 decline is similar in size to what we obtained without trimming for education. In contrast to the earlier figures, trimming for education implies that the general


Fig. 6.- Interquartile Range for Cognitive Task Price Changes, Young, High Education
increase in the cognitive task price swamps cyclical movements. Overall, our conclusion is that the cognitive task price increased substantially (i.e., by between $15 \%$ and $35 \%$ between 1990 and 2000 before declining by approximately $20 \%$ after 2000 .

### 6.2.2. Routine Task Sector

In Figure 8, we present plots of the bounds formed for the Routine Sector task price under the Hierarchical Model with no advance trimming for education shifts. For brevity, we just show this figure for the combined education groups. The patterns for individual education groups are very similar. For the Routine Sector, the specific model of ability we assume alters the way the trimming is done and results in different bounds. In this figure, the strong impression is of a long term ratcheting down of wages, with significant drops during recessions that are only partially offset by increases in subsequent booms. For both men and women, for the bound formed using the actual medians, the decline in the task price over the whole period is on the order of $12 \%$. For the


Fig. 7.- Cognitive Task Price Changes with Education Trimming, Young, High Education
bounds formed by trimming the bottom tail of the distribution, the decline is on the order of $20 \%$ between 1984 and 2007 but this is followed by a much larger drop during the 2008 recession. This drop is actually an artifact of the Hierarchical Model trimming protocol. The large increase in the share of workers in the $H$ sector (i.e., non-employed sector) is necessarily seen as being generated as a flow out of the bottom of the $M$ sector. Since, in reality, not all of the increased number of non-employed workers would have otherwise been in the $M$ sector, the net change in the $M$ sector will be much smaller than the increase in the $H$ sector as the U.S. entered the recession. In the Hierarchical Model this implies that there must have been an offsetting entry of workers to the $M$ from the $R$ sector. This then requires us to trim a considerable number of workers from the bottom of the base period $R$ wage distribution, which yields large drops in the trimmed sample median wage. In Figure 9, we plot the bounds on the interquartile differences for Routine sector wages for all education groups combined. For the untrimmed samples, the range shows a decline in the mid-90s but it otherwise follows a flat trend. Thus, the 25 th, 50 th and 75 th percentiles of the wage distribution is moving down together. However, once we trim according to the Hierarchical


Fig. 8.- Routine Task Price Changes, Young, All Education, HM Trimming

Model, the trend for the inter-quartile difference is flat until about 2000 for both men and women but then rises strongly (and statistically significantly). This rise is driven by a stronger decline in the 25 th than the 75 th percentile of the trimmed samples after 2000 . The implication is that there is something of a polarization within the Routine sector after 2000: the 25th percentile of the trimmed sample (which would be more like the 40th percentile of the untrimmed sample) declines more than the 25 th percentile of the untrimmed sample. In terms of implications for bounds on task prices, the rise in the inter-quartile range after 2000 implies an increase in the ability factor loading after that year and that the decline in the task price is even larger than the large declines we observe in the bounds on the composition constant median wage.

The restrictions related to the allowable support that bounding approaches impose become important once we move to the education trimmed samples for the Routine sector. For the low educated sample, once we trim for the large decline in the number of low educated workers from the base period low education wage sample, the number of observations that we would need to trim from the wage distribution to account for the decline in the size of the Routine sector is more


Fig. 9.- Interquartile Range for Routine Task Price Changes, Young, All Education, HM Trimming
than the number of remaining base period workers for that education group. Thus, we are simply unable to form bounds for the low educated groups after the early 1990s. This means that we also cannot examine bounds for the combined education sample since trends in that case are partly driven by the sudden removal of low educated workers from the sample after the early 1990s. We can, however, still form bounds throughout our period for the high educated group because the $R$ sector movements are sufficiently small for this group that we don't run into the same type of violations. We plot this set of bounds in figure 10. In contrast to the earlier figures, here the bounds do not show a long term decline in the $R$ task price before 2008, though they still show increases in the 1990s, declines after 2000, and particularly strong drops after 2008. In essence, the increase in education is being assumed to add workers at the low end of the high education ability distribution. Once we trim those additions from year $t$, and taking account that it is in this part of the distribution where most of the $R$ sector workers are located, we end up with smaller declines in the median wage of the trimmed sample between the base year and year t. In essence, this says that given the size of the education and $R$ sector quantity changes, the observed decline in $R$ wages
before 2008 could arise with a relatively flat trend in $R$ task prices combined with a downgrading of the ability composition of $R$ workers. Taking shifts in supply into account can matter.


Fig. 10.- Routine Task Price Changes with Education Trimming, Young, High Education, HM Trimming

Figure 11 contains the bounds for the case where we trim according to the Independent Shocks model. Recall that this means we either trim observations equal to the net change in the number of workers in the sector from the bottom of the wage distribution, or (using Stochastic Dominance) we don't trim at all. While this figure is similar in general patterns to the equivalent figure based on Hierarchical Model trimming, the scales in the figures are currently different and the declines in the task price bound with trimming are on the order of 20 percentage points smaller in the IS model. We do not yet have results combining education trimming with IS trimming.


Fig. 11.- Routine Task Price Changes, Young, All Education, IS Trimming

### 6.2.3. Manual Task Sector

We next turn to bounds on the price for the $M$ sector. With four sectors, the analysis is similar to that in the $R$ sector in the sense that the type of ability distribution model can matter for our bounds. This is seen most clearly in figure 12, where we plot the bounds for all education groups with HM trimming. As with the $R$ sector, movements in non-employment imply substantial trimming of the wage distribution after 2008, leading to declines in the trimmed sample bound of approximately $100 \%$ in five years. Prior to 2008, the untrimmed sample bound (the one that relies on the Stochastic Dominance arguments) shows no long term decline for men and only a small decline for women. In both cases, this is a result of declines through the 1980s, increases in the 1990s and a renewal of declines after 2000. For the bound formed with HM model trimming, the trend is reminiscent of what we observed for the R sector, with a ratcheting down in the price across cycles and an ultimate drop between 1984 and 2008 that is on the order of $40 \%$. We do not present the interquartile range for this sector for brevity, but it is essentially flat over the entire period for men and women, implying that the changes plotted in figure 12 correspond to task price changes.


Fig. 12.- Manual Task Price Changes, Young, All Education, HM Trimming

In figure 13, we plot bounds after trimming for education movements. As with the $R$ sector, we have to restrict our attention to high educated workers to be able to form bounds for the entire sample period. Those bounds show essentially flat M sector task prices when we do not also trim for occupation changes. When we do trim for occupation, both men and women show increases in the M sector task price in the early 1980s followed by a relatively continual decline thereafter. The result is about a $20 \%$ decline between 1984 and 2008 for women and a slightly larger decline for men. Finally, in figure 14, we plot bounds associated with the IS model of ability. For men, both bounds show increases in the 1990s and declines in the 2000s but no long term change between 1984 and 2008. Post 2008, they show about $10 \%$ declines. For women, there are strong increases in the 1990s associated with strong growth in employment in this sector (recalling that in the IS model, these additions are assumed to enter at the bottom of the ability distribution) and then declines of just under $10 \%$ after 2000.


Fig. 13.- Manual Task Price Changes with Education Trimming, Young, High Education, HM Trimming

## 7. Discussion and Conclusion

The bounding exercises we have implemented indicate that taking account of changes in both observable and unobservable characteristic changes can substantially alter our picture of relevant factor price changes for the U.S. in the last thirty years. Simple median wages in both the $C$ and $M$ sectors rise strongly through the 1990s and then rise more slowly after 2000 , while $R$ sector wages fall for men over the same period but rise in much the same way as $M$ sector wages for women. Taken together, the male wages, in particular, fit with arguments that the U.S. has experienced wage polarization in recent decades.

However, our bounds taking account of composition shifts tell quite a different story. For cognitive sector prices, our conclusion is that they increased substantially (i.e., by between $15 \%$ and $35 \%$ between 1990 and 2000 before declining by approximately $20 \%$ after 2000 . Given movements in the interquartile range in the $C$ sector, the latter decline may be an understatement. For both the


Fig. 14.- Manual Task Price Changes, Young, All Education, IS Trimming
$R$ and $M$ sectors, HM model bounds are strongly affected by the increase in non-employment after 2008, generating potentially unrealistic sized bounds on drops in the task price in that sub-period. If we focus on the period before 2008, long term trends in $R$ sector task prices between 1984 and 2008 are bounded between no change and a decline of approximately $40 \%$. For the 2000 's, our $R$ series bounds vary between a high of just under a $10 \%$ decline and just over a $20 \%$ decline. Finally, for the $M$ sector, our bounds for the change from 1984 to 2008 range between no change and an approximately $40 \%$ decline. Between 2000 and 2008, the bounds are as small as an approximately $10 \%$ decline and an approximately $40 \%$ decline.

Combined, these patterns may fit with wage polarization in the 1990s, though this is far from clear. The bounds in the $C$ sector are increasing in this period but in many cases so are the $R$ sector bounds while some $M$ sector bounds show declines. For the 2000-2008 period, however, the dominant pattern is one of decline in all sectors to extents that are relatively similar in magnitude. This implies that the models that should be considered are ones the incorporate a change in the labour market in approximately 2000 and one that does not include an increase in demand for

Table 1: Implications for Men

|  | $2000-1990$ |  | $2008-2000$ |  | $2013-2000$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta_{O}$ | $\Delta_{B}$ | $\Delta_{O}$ | $\Delta_{B}$ | $\Delta_{O}$ | $\Delta_{B}$ |
| Cognitive | 0.01 | 0.06 | 0.00 | -0.08 | -0.04 | -0.11 |
| Routine | 0.01 | -0.03 | -0.04 | -0.14 | -0.12 | -0.36 |
| Manual | 0.09 | -0.12 | -0.05 | -0.34 | -0.12 | -0.92 |
| Cognitive | 0.01 | 0.06 | 0.00 | -0.08 | -0.04 | -0.11 |
| Routine | 0.01 | -0.06 | -0.04 | -0.12 | -0.12 | -0.26 |
| Manual | 0.09 | 0.06 | -0.05 | -0.13 | -0.12 | -0.16 |

workers in any sector as a dominant force.

## REFERENCES

Acemoglu, D. and D. H. Autor (2011). Skills, tasks and technologies: Implications for employment and earnings. Handbook of Labor Economics 4, 1043-1171.

Autor, D., F. Levy, and R. Murnane (2003). The Skill Content of Recent Technological Change: An empirical exploration. Quarterly journal of economics 118(4), 1279-1333.

Autor, D. H., L. F. Katz, and M. S. Kearney (2007). Trends in U.S. Wage Inequality: Revising the Revisionists.

Beaudry, P. and J. DiNardo (1995). Is the Behavior of Hours Worked Consistent with Implicit Contract Theory? The Quarterly Journal of Economics 110(3), 743-768.

Beaudry, P., D. A. Green, and B. M. Sand (2012). Does Industrial Composition Matter for Wages? A test of search and bargaining theory. Econometrica 80(3), 1063-1104.

Beaudry, P., D. A. Green, and B. M. Sand (2013). The Great Reversal in the Demand for Skill and Cognitive Tasks. Technical Report NBER Working Papers 18901, National Bureau of Economic Research, Inc.

Blundell, R., A. Gosling, H. Ichimura, and C. Meghir (2007). changes in the distribution of male and female wages accounting for employment composition using bounds. Econometrica 75(2), 323-363.

Bowlus, A. J. and C. Robinson (2012, December). Human Capital Prices, Productivity, and Growth. American Economic Review 102(7), 3483-3515.

Cortes, G. M. (2012). Where Have the Routine Workers Gone? A Using Panel Data Study of Polarization. Technical Report October, Manchester Discussion Paper Series: 1224.

Ham, J. and K. Reilly (2002). Testing Intertemporal Substitution, Implicit Contracts, and Hours Restriction Models of the Labor Market Using Micro Data. The American Economic Review 90(4), 905-927.

Heckman, J. (1979). Sample selection bias as a specification error. Econometrica 47(1), 153-161.
Heckman, J., L. Lochner, and C. Taber (1998). Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents. Review of economic dynamics 1(1), 1-58.

Horowitz, J. L. and C. F. Manski (2014). Nonparametric Analysis of Randomized Experiments With Missing Covariate and Outcome Data. 95(449), 77-84.

Katz, L. F. and K. M. Murphy (1992). Changes in relative wages, 1963-1987: supply and demand factors. The Quarterly Journal of Economics 107(1), 35-78.

Lee, D. (2009). Training, wages, and sample selection: Estimating sharp bounds on treatment effects. The Review of Economic Studies $76(3), 1071-1102$.

Lemieux, T. (2006). Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill? The American Economic Review 96(3), pp. 461-498.

Lemieux, T. (2010). What Do We Really Know about Changes in Wage Inequality? In K. G. Abraahm, J. Spletzer, and M. Harper (Eds.), Labor in the New Economy, Number October, pp. 17-59. University of Chicago Press.

Manski, C. (1990). Nonparametric Bounds on Treatment Effects. The American Economic Review $80(2), 319-323$.

Roy, A. (1951). Some thoughts on the distribution of earnings. Oxford economic papers 3(2), 135-146.

Vytlacil, E. (2002). Independence, Monotonicity, and Latent Index Models: An Equivalence Result. Econometrica $70(1), 331-341$.

Willis, R. (1986). Wage determinants: A survey and reinterpretation of human capital earnings functions. In O. Ashenfelter and R. Layard (Eds.), Handbook of labor economics, Volume I, Chapter 10, pp. 525-602. Elsevier Science Publishers BV.

Willis, R. and S. Rosen (1979). Education and Self-Selection. Journal of Political Economy 87(5), S7-S36.


[^0]:    ${ }^{1}$ We thank seminar participants at UBC and the University of Michigan for helpful comments.

[^1]:    ${ }^{2}$ Which set of bounds is appropriate can potentially be determined using observations on worker movements in panel data even though, as just argued, panel data cannot be used directly to get task price movements. We have not yet pursued this possibility.

[^2]:    ${ }^{3}$ Note that if we allow for $L$ task types and allow for workers of any task type to be employed in sector $k$ then the average wage at a point in time would be,

    $$
    \begin{equation*}
    \ln ^{-} w_{k t_{1}}=(1-\delta) \sum_{\tau=0}^{\infty} \delta^{\tau} \sum_{l=1}^{L} \ln w_{l t_{1}, t_{1}-\tau} \cdot \pi_{k l}\left(t_{1}, t_{1}-\tau\right) \tag{8}
    \end{equation*}
    $$

    where $\pi_{k l}\left(t_{1}, t_{1}-\tau\right)$ is the proportion of the employed workers in sector k who started working in period $t_{1}-\tau$ and are still working in period $t_{1}$ who are of type $l$.

[^3]:    ${ }^{4}$ Our approach is similar in spirit to Heckman et al. (1998) and Bowlus and Robinson (2012) both of whom generate skill price series using older workers (who are on a flat portion of the wage-experience profile). While they work across cohorts at an older age, we do the same for new entrants. In the context of their life-cycle human capital model, we would need to add to the identifying assumptions listed below, the assumption that the proportion of time a worker devotes to human capital at the start of his or her working life is invariant to the current task prices.

[^4]:    ${ }^{5}$ Median wages in the other sectors have analogous expressions.

[^5]:    ${ }^{6}$ This statement is true if $\operatorname{Med}\left[\ln \eta+e_{C t}\right]=\operatorname{Med}\left[\ln \eta+e_{C t+1}\right]$ which holds if, as we have assumed, $e_{C \tau}$ is drawn from a distribution that is stationary over time.

[^6]:    ${ }^{7}$ Note that we could allow for gross flows in both directions between sectors in the model by including exogenous shocks reflecting, for example, whether a close friend had joined a specific sector. If these shocks are independent of price movements then the net flows will reflect the responses to the price changes that are of interest to us.

[^7]:    ${ }^{8}$ As described earlier, there can be random movements of workers both in and out of the sector. We just require the movements induced by the price or cost changes to satisfy Monotonicity in order to identify task price changes since it is movements that alter the ability composition in the sector that are of concern to us.
    ${ }^{9}$ Increases in the number of university educated workers over time has the potential to generate a violation of this assumption, even if we focus just on one education group. The assumption would require that any new additions to the education group are random draws from the same ability distribution as those already in the education group. We address this explicitly in our discussion of covariates and observable skill groups in section 3.1.5.

[^8]:    ${ }^{10}$ Our identifying assumptions are re-statements of those in Lee (2009).

[^9]:    ${ }^{11}$ Note that, even with time invariant $\beta$ 's the model could fit well known patterns such as increases in the BA/HS wage differential in recent decades. A change in the BA/HS differential arises in the model to the extent that BA and HS educated workers are differentially distributed across sectors. Then the wage differential will change if the sectoral compositions of the two groups change differently and/or the sector prices change. Whether these mechanisms are sufficient to explain observed movements in the BA/HS wage differential is potentially testable.
    ${ }^{12}$ A potential alternative to the approach we define below would be to re-weight the data so that the distribution of the $\mathbf{x}$ 's in the weighted data does not change over time then implement the trimming approach discussed earlier on the overall, weighted sample. The trouble with this is approach is that it would always trim either the lowest or highest earners and those will correspond closely to low and high educated workers. But, even in an hierarchical model, it is not only the workers with the least remunerated values of observed characteristics who will leave the $C$ sector in the event that $p_{C t}$ declines. The most educated workers who have relatively low values of $\eta$ will also leave the sector, and those workers would leave from the middle of the overall wage distribution, not the lowest tail.

[^10]:    ${ }^{13}$ Note that our final sample could contain a set of relatively high ability workers within one covariate cell and a low ability set in a different cell, depending on how the trimming proceeds. This is not a problem because we are working in differences.
    ${ }^{14}$ For the middle skill occupations, such as $R$, deciding how many to trim and from where is more complex, as described earlier.

[^11]:    ${ }^{15}$ We begin our series in 1984 because this is the first year in which we can identify students.
    ${ }^{16}$ The Data appendix provides additional information on our data processing.

[^12]:    ${ }^{17}$ Details on adjustments and cross-walks used in generating consistent occupation categories are given in Beaudry et al. (2013).

[^13]:    ${ }^{18}$ We are able to identify changes in the log task price rather than levels. At this point, we are adding those changes to start of period median log wages for the sector but in the future we will simply plot the differences.

