Understanding the Supply and Demand Forces behind the Fall and Rise in the U.S. Skill Premium

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Abstract

I develop an assignment model to quantify, in a unified framework, the causal effect of supply and demand forces on the evolution of the college wage premium in the U.S. economy. Specifically, I quantify the relative contribution of four different forces: (1) a within-sector non-neutral technological change, (2) the creation of new high-skill services/sectors, (3) polarizing product demand shifts, and (4) shifts in the relative supply of skilled labor. The model considers endogenous human capital accumulation. I find that, on average, 52% of the change in the U.S. skill premium during the last four decades is explained by demand factors. Supply forces explain the remaining 48% of the skill premium variation. Within the demand-driven change in the skill premium, on average, 39% is explained by the creation of new high-skill sectors, 47% by a polarizing product demand shift within existing sectors and only 14% by a skill-biased technological change. Additionally, I find that the relative contribution of each supply and demand force varies across decades. Supply forces play a major role in the 1970-1980 period when the skill premium falls. On the other hand, the polarization of wages makes a key contribution during the post-1980 period when the skill premium rises.

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1 Introduction

A large literature documents a substantial change in the U.S. wage structure during the past four decades.[1] Changes are observed for different inequality concepts: overall wage inequality, inequality in the upper and lower halves of the wage distribution, between-group wage differentials, and within-group (residual) wage inequality. The literature has paid special attention to the U.S. college wage premium. Figure 1 exhibits the evolution of the college wage premium over the last four decades. We observe that for both men and women the skill premium falls during the pre-1980 period and rises during the post-1980 period.

The pioneering work by Katz and Murphy (1992) proposes a simple supply and demand framework to understand the evolution of the U.S. college wage premium over the past decades. In that framework, the skill premium rises (falls) when the demand for college graduates grows faster (slower) than the supply. Subsequent works have attempted to augment the basic framework by analyzing a richer set of facts (Card and Lemieux 2001), refining the data set used (Lemieux 2006), improving some methodological aspects (Lemieux 2006), and including non-market factors as determinants of the skill premium (Card and DiNardo 2002).

The "canonical" model proposed by Katz and Murphy (1992) and the subsequent works have been extremely useful for proving that a standard supply-demand framework is sufficient to understand the movements of the U.S. skill premium. However, they have been less successful in understanding the underlying factors behind the movement of the supply and demand for college graduates. More concretely, it is difficult to estimate within those frameworks the causal effect of different types of forces on the skill premium. The reason is their lack of structure in the modelling of the supply and demand forces. For instance, in the Katz-Murphy model, the supply of college graduates is assumed to be exogenous and the demand shifts are simply modelled by a linear trend. An exogenous supply of college graduates, a linear trend for demand shifts, and an estimated value for the elasticity of substitution between college and high-school "equivalents" produce changes in supply and demand that fit the data very well, at least in earlier decades. However, the causal effect of different supply and demand forces on the skill premium cannot be quantified within that model. What the canonical model and subsequent works do is to seek consistent findings that at most allow them to speculate about the forces behind the supply and demand shifts.[2]

In this paper I revisit the study of the college wage premium in the U.S. economy with a richer empirical framework. Specifically, I develop an assignment model to quantify, in a unified framework, the causal effect of supply and demand forces on the evolution of the U.S. college wage premium. I quantify the relative contribution of four different forces: (1) a within-sector non-neutral technological change, (2) the creation of new high-skill services/sectors, (3) polarizing product demand shifts, and (4) shifts in the relative supply of skilled labor. The model considers endogenous human capital accumulation.

In the economy model, the production function of the final good is carried out by aggregating the output of a continuum of sectors that produce services of different complexities.

[1]See Bound and Johnson, 1992; Katz and Murphy, 1992; Murphy and Welch, 1992; Juhn, Murphy, and Pierce, 1993, among others.

[2]See Doms, Dunne, and Troske (1997), Dunne, Haltiwanger, and Troske (1997), Autor, Katz, and Krueger (1998), Autor and Katz (1999), Autor, Levy, and Murnane (2003), Levy and Murnane (2004), Bartel, Ichniowski, and Shaw (2007), among others.

Agents are heterogeneous regarding their inherent abilities, which affect their cost of investing in different skills. The model considers heterogeneity in the costs of accumulating human capital. I explicitly model non-pecuniary costs of investing in higher education. These costs depend negatively on the inherent abilities of agents. In that way, the model includes among the supply factors "psychic or effort" costs of accumulating human capital. Cunha and Heckman (2007) and Becker, Hubbard and Murphy (2010) have highlighted the importance of such costs in the investment decisions of agents. The model allows the distribution of abilities of men to differ from that of women, as in Becker, Hubbard and Murphy (2010).

The model developed in this paper has several desirable characteristics not present in the previous empirical frameworks used to study the U.S. college wage premium. First, supply and demand forces are included within a unified model, which allows me to estimate the causal effect of different forces by constructing counterfactuals. Second, I do not impose a priori an inelastic supply curve. That elasticity is estimated by calibrating the degree of heterogeneity of agents based on moments of the empirical distribution of abilities documented in the literature. Third, the model considers different distributions of abilities for men and women (as in Becker, Hubbard and Murphy 2010) and, thus, the elasticity of supply is gender-specific. Fourth, I explicitly model polarizing or non-monotonic product demand shifts, in line with Autor, Levy, and Murnane (2003), Goos and Manning (2007), and Acemoglu and Autor (2010). Fifth, the model allows me to include several facts of the U.S. structure within a single framework. And last, but importantly, the model is tractable and allows for a calibration with few data requirements.

I calibrate the model to match data from the U.S. wage structure. Psychic costs are calibrated by matching the monetary value of psychic costs paid by the agents in the model with those computed in the literature. Data used in the calibration is taken from Acemoglu and Autor (2010) and Cunha and Heckman (2007). I perform counterfactual exercises to estimate the causal effect of each supply and demand force on the skill premium.

The results of this paper show that, on average, 52% of the change in the U.S. skill premium during the last four decades is explained by demand factors. Supply forces explain the remaining 48% of the skill premium variation. Within the demand-driven change in the skill premium, on average, 39% is explained by the creation of new high-skill sectors, 47% by a polarizing product demand shift within existing sectors and only 14% by a skill-biased technological change (SBTC).

Additionally, I find that the relative contribution of each supply and demand force varies across decades. Supply forces play a major role in the 1970-1980 period when the skill premium falls. Positive supply shifts completely explain the fall of the skill premium during the period 1970-1980. On the other hand, the polarization of wages makes a key contribution during the post-1980 period, when the skill premium rises. The results show an increasing polarization of wages over decades. The contribution of this force to the demand-driven change in the college wage premium goes from practically null in the period 1970-1980 to 60% in the last decade included in the analysis (2000-2008), which is equivalent to 75% of the total contribution of the demand forces.

I also disaggregate the analysis by gender. I find that the main asymmetry by gender comes from the supply side. Specifically, the results show a stronger contribution of supply forces in the case of women. On average, supply forces explain around 83% of the changes in the female skill premium, whereas they explain only 35% in the case of the male skill premium. On the demand side, I find in both cases an increasing polarization of wages over the decades.

However, the contribution of polarizing product demand shifts is more pronounced in the case of women.

Additionally, I perform two types of sensitivity analysis. First, I evaluate the sensitivity of the main results to changes in the elasticity of substitution between services. Second, I assess how the results change when the variance of the distribution of abilities falls. I find that as the elasticity of substitution rises, the contribution of demand factors increases and the influence of polarizing product demand shifts becomes more pronounced. This result is consistent with the fact that a higher elasticity of substitution makes the demand curve for more educated workers more elastic and, thus, greater demand shifts are needed to explain the observed changes in quantities and prices. In contrast, as the variance of the distribution of abilities falls, the contribution of supply forces increases and the polarization of wages becomes less important for explaining the skill premium. This is consistent with the fact that a lower variance implies more homogeneous agents and, thus, a more elastic supply curve for more educated workers. With a more elastic supply curve, greater negative supply shifts are needed to explain the rise in the college wage premium in the context of an increasing demand for college graduates during the post-1980 period . Analogously, bigger positive supply shifts are needed to explain the fall in the college wage premium in the context of a stable demand for college graduates during the pre-1980 period.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses the calibration strategy. Section 4 presents and discusses the results of the counterfactual exercises. Finally, section 5 concludes.

2 The Model

In this section, I develop an assignment model to quantify, in a unified framework, the causal effect of supply and demand forces on the evolution of the college wage premium in the U.S. economy. I quantify the relative contribution of four different forces: (1) a within-sector non-neutral technological change, (2) the creation of new high-skill services/sectors, (3) polarizing product demand shifts, and (4) shifts in the relative supply of skilled labor. The model considers endogenous human capital accumulation.

The production function of the final good is carried out by aggregating the output of a continuum of sectors that produce services of different complexities. Agents are heterogeneous regarding their inherent abilities, which affect their cost of investing in different skills. The model considers heterogeneity in the costs of accumulating human capital. I explicitly model non-pecuniary or "psychic or effort" costs of investing in higher education, which are important determinants of human capital investments as highlighted by Cunha and Heckman (2007) and Becker, Hubbard and Murphy (2010). These costs depend negatively on the inherent abilities of agents. The distribution of abilities is gender-specific, as in Becker, Hubbard and Murphy (2010).

The model is static and transforms a life-cycle problem into a one-period problem, which allows for a calibration using very simple data.[3] I model a competitive equilibrium in which heterogeneous agents choose their occupations and years of education to maximize income,

[3]A more complex model (e.g., a dynamic model) would require assumptions about the future path of structural transformations and within-sector SBTCs, but it would not add greatly to the analysis.

taking wage schedules as given. Likewise, a representative firm hires workers, taking the wage schedule as given. Both sectors and workers are measured along a continuous one-dimensional scale. Workers are characterized by a single index variable denoting inherent ability, which affects the cost of investing in education. Sectors are also characterized by a single variable: their level of complexity. Workers of various skill levels are matched to sector types that produce services of different complexities. The market equilibrium is characterized by a mapping of skills (given by the years of education of each worker) on complexities, as in Tinbergen (1956). Because highly skilled workers are assumed to have a comparative advantage in complex services, in equilibrium, they will be allocated to complex services.

I build on Teulings (1995), Kaboski (2009) and Parro (2012). Those authors use variants of an assignment model to study some aspects of the wage distribution (Teulings 1995), the forces behind schooling and wage growth (Kaboski 2009), and the rise and fall in the U.S. gender gap in education (Parro 2012). However, none of them empirically study the fall and rise of the U.S. college wage premium. In this paper I build a model that shares some of the structure of those frameworks. I extend those models by including heterogeneity in the costs of accumulating human capital, by modelling "psychic or effort" costs in the investment decisions of agents (as in Becker, Hubbard and Murphy 2010), and by allowing for the existence of polarizing demand shifts, which could be important for understanding the movement of the skill premium in the most recent decades, as highlighted by Autor, Levy, and Murnane (2003), Goos and Manning (2007) and Acemoglu and Autor (2010), among others. I use the model to quantify the causal effect of different supply and demand forces on the U.S. college wage premium. To the best of my knowledge, no other paper in the literature has studied the causal effect of different factors on the skill premium using this rich structure of supply and demand forces.

2.1 Production Technology

The production of the unique final good Y is performed by aggregating the output S of a continuum of sectors. Sectors are indexed by the "complexity" of the service produced (i). The production function of the final good can be expressed as

$$Y = \left(\int_{\underline{I}}^{\overline{I}} S\left(i\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where σ denotes the elasticity of substitution between services in the production of the final good. $\overline{I}$ and $\underline{I}$ are the least and most complex services produced, respectively.

Before analyzing the production function of each service, I will define some concepts. h is a measure of a worker's years of education, and $A(i, h)$ is the productivity of a worker with h years of education producing a service of complexity i. Additionally, denote by $n(i, h)$ the amount of labor supplied by agents with h years of education in sector i. Total labor supply is normalized to unity and, therefore, $n(i, h)$ is the density function of workers of type h producing a service of type i within the labor supply. Production of service i can be expressed as follows

$$S\left(i\right) = \int_{0}^{\infty} A\left(i,h\right) n\left(i,h\right) dh \tag{2}$$

Function $A(\cdot)$ is assumed to be twice differentiable. Additionally, I make the following two assumptions. First, I assume that more skilled workers have an absolute advantage over less skilled workers $(\partial \log A(i,h)/\partial h > 0)$. That is, workers with higher skills are more productive, irrespective of the job in which they are employed. The direct implication of this assumption is that more educated workers earn higher wages. Second, I assume that more educated workers have a comparative advantage in more complex sectors $(\partial \log A(i,h)/\partial i \partial h > 0)$. That is, the relative productivity gain from an additional unit of skill increases with the complexity of the job.

In order to achieve empirical results, I have to make specific assumptions on the functional form of $A(\cdot)$. I use a convenient parameterization that meets the previous two assumptions regarding $A(\cdot)$ and, in addition, that captures the demand forces that I want to quantify

$$A(i,h) = \exp\left(i^{\delta} h + \lambda (h - 12) + \chi_0 i^2 + \chi_1 i\right) \tag{3}$$

I impose $\chi_0 = -\chi_1/2\bar{i}$ for $\underline{I} < \bar{\imath} < \bar{I}$. Notice that the parameterization for the function $A(\cdot)$ meets the assumptions of absolute and comparative advantages of more skilled workers. Additionally, as I will discuss below, the parameter χ_1 is the source of polarizing product demand shifts.

The representative firm producing the final good hires workers, taking the wage schedule as given. The maximization problem of the representative firm in this economy model is

$$\max_{n(i,h)} \left\{ \left(\int_{\underline{I}}^{\bar{I}} \left[A(i,h)\, n(i,h)\, dh \right]^{\frac{\sigma-|}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} - \int_{\underline{I}}^{\bar{I}} \int_0^{\infty} w(i,h)\, n(i,h)\, dh di \right\} \tag{4}$$

where $w(i,h)$ is the wage earned by a worker with h years of education working in sector i. The first-order condition for labor is

$$w(i,h) = A(i,h) \left(\frac{Y}{S(i)} \right)^{\frac{1}{\sigma}} \tag{5}$$

Equation (5) characterizes the first-order condition of the representative firm.

2.1.1 Demand forces

Three types of demand forces are embodied in the production technology of this economy model. First, the function $A(\cdot)$ allows for the existence of polarizing product demand shifts within the existing sectors of the economy. When $\Delta\chi_1 < 0$, the relative demand for services around complexity $\bar{i}$ falls whereas the relative demand for services produced by low-and high-skill workers rises. When $\Delta\chi_1 > 0$ the opposite polarizing product demand shift is triggered.[4]

[4] Notice that we can alternatively interpret this polarizing effect as a sector-specific technological change that increases the productivity of any worker producing in the sector benefited by the technological improvement.

Second, $A(\cdot)$ captures a within-sector skill biased technological change (SBTC). That is, an increase in λ raises the productivity of workers with more than 12 years of education but decreases the productivity of workers with less than 12 years of education, within each sector. This technological improvement monotonically increases the relative wages of skilled workers by increasing the real wages of workers with 12 or more years of education but decreasing the wages of other types of workers.

A third demand force is a type of structural transformation triggered by a rise in $\overline{I}$. A rise in $\overline{I}$ reflects a structural transformation that creates new sectors that produce more complex services. Those new sectors demand more skilled workers given that more educated workers have comparative advantages in sectors that produce more complex services. Therefore, as the complexity of the services produced by the economy rises, a reallocation of labor toward more complex services should be observed.

2.2 Agents

The economy is populated by a continuum of agents that spend their endowment of time working and accumulating education in formal schooling. Each agent lives for just one period and has an endowment of time T. To get h years of education, agents must spend h years in school, which is an indirect cost of schooling.

In this economy model, agents are heterogeneous and are measured along a continuous one-dimensional scale. They are characterized by a single index variable denoting inherent ability. Agents' inherent abilities are distributed with a positive density across a bounded interval $[\underline{\alpha}, \overline{\alpha}]$ according to a continuously differentiable density function $f(\alpha)$, where α represents inherent ability. Inherent ability affects the cost of investing in education. Specifically, there are "psychic costs" of attending school which are decreasing in the inherent abilities of agents and proportional to the indirect cost of schooling. The proportionality factor is given by a continuous, decreasing, and differentiable function $\Omega(\alpha)$.[5] In the empirical implementation of the model I consider differences between the female and male distributions of abilities, as in Becker, Hubbard and Murphy (2011). These differences in the distribution of abilities allow me to model a gender-specific elasticity of supply.

Agents choose years of education and the sector where they work to maximize lifetime income, taking wage schedules as given. Then, the maximization problem of agents of type α is

$$\max_{i,h} \{[T - h(1 + Z + \Omega(\alpha))] w(i,h)\} \tag{6}$$

$T - h$ is the amount of effective working time (which is decreasing in h), $w(i,h)$ is the indirect cost of each year of schooling, and $\Omega(\alpha) hw(i,h)$ is the monetary value of the psychic costs of acquiring h years of education. In terms of data, $w(i,h)$ is the average annual wage that a full-time, full-year (FTFY) worker with human capital h earns in sector i during his life time.

Z are the supply shiftiers. For instance, Z could be the parameter governing the direct costs of schooling which are assumed to be proportional to the indirect costs of education. In

[5] What I call "psychic costs" are actually residual costs that are not included in tuition costs and that are assumed to depend on the abilities of agents.

general, Z can include tuition costs and the "monetization" of nonmonetary returns to education. A rise in Z reduces the supply of college graduates and increases the skill premium (controlling for compositional effects).[6]

The first-order conditions of the optimization problem of agents of type α is described by the following equations

$$[h] \quad : \quad \frac{1+Z+\Omega(\alpha)}{T-h(1+Z+\Omega(\alpha))} = \frac{\frac{\partial w(i,h)}{\partial h}}{w(i,h)} \tag{7}$$

$$[i] \quad : \quad \frac{\frac{\partial w(i,h)}{\partial i}}{w(i,h)} = 0 \tag{8}$$

where equation (7) is the optimal choice of education for an agent with ability α working in sector i and equation (8) is the optimal choice of sector for an agent with h years of education.

The assumptions regarding the function $A(\cdot)$ ensure that more educated workers earn higher wages in the labor market. Therefore, optimizing workers invest in education until those monetary benefits equalize all costs involved in the accumulation of human capital (direct, indirect and psychic costs of schooling). That is the intuition behind the first-order condition regarding h. Additionally, employers pay workers in accordance with their marginal value product. Workers will choose the job type that offers them the highest wage, since sector characteristics do not enter into any utility function (compensating differentials are ruled out from this model). That optimal decision for a worker of type α is reflected in equation (8).[7]

2.3 Equilibrium

In this section, I first define the competitive equilibrium that I am modelling and, after that, I analyze how the equilibrium is solved.

2.3.1 Competitive Equilibrium

The competitive equilibrium is a set of wages $\{w(i,h)\}$, quantities $\{n(i,h)\}$, and optimal policy functions $\{i(\alpha), h(\alpha)\}$ that solve firms' and agents' maximization problems and the market clearing conditions for labor inputs. The equilibrium allocation of workers to sectors can be

[6] A rise in Z increases the net cost of investing in a higher education, which decreases the relative supply of college graduates and, in general, the average years of schooling of the population. On the other hand, the effect of Z on the college wage premium is, in principle, ambiguous. A rise in Z decreases relative supply of college graduates which increases the average complexity level of the services produced by college graduates. The increase in the average complexity level of the services produced by college graduates pushes the college wage premium up. In addition, there is a composition effect. A rise in Z reduces the average years of education of college graduates and this force pushes the college wage premium down. Therefore, we have that, controlling for compositional effects, the model would predict that as Z increases the college wage premium goes up, which is consistent with the fall in the relative supply of college graduates.

[7] We can verify in the calibrated model that, in equilibrium, $w(i,h)$ is continuous and strictly concave in both i and h. Therefore, the objective is strictly concave in i and the first order condition for the optimal choice of i is satisfied with equality. Second, in the calibrated model all types of agents chose a level of education $h > 0$. Therefore, the first-order condition for the optimal choice of human capital is also satisfied with equality.

described by a one-to-one correspondence between human capital and service complexities, $h(i)$, which therefore has a well-defined inverse function, $i = i(h)$. This implication follows from the assumption of perfect substitutability between types of workers within a single job type. Firms will employ workers only with the lowest cost per efficiency unit of labor. The assumption of comparative advantage guarantees that when two types of workers have an equal cost per efficiency unit of labor in one sector, they cannot have an equal cost in any other sector. Hence, when a specific type of worker is employed in a sector, there is never another type of worker employed in the same sector.[8] Additionally, without proof, I state that $h(.)$ is differentiable in the equilibrium. Furthermore, the assumption of comparative advantage implies that $h'(i) > 0$. Highly skilled workers are allotted to complex jobs.

2.3.2 Solving the Equilibrium

To compute the equilibrium, I solve for the inverse policy mapping of sectors to abilities $\alpha(i)$ and sectors to human capital $h(i)$. Those policy mapping are strictly increasing by the assumptions that more skilled workers have an absolute advantage over less skilled workers and that more educated workers have a comparative advantage in more complex sectors.

The labor market clearing condition requires that the demand for labor of type h working in sector i is equal to the supply. The density of workers in service type i can be derived from a change in variables $f(\alpha(i))\alpha'(i)$, where $\alpha'(i)$ is the Jacobian from transforming the density in terms of α to a density in terms of i. Therefore, the labor market clearing condition is the following

$$n(i,h) = f(\alpha(i))\alpha'(i)(T - h(i)) \tag{9}$$

Then, for sector-education combinations that satisfy $h = h(i)$, the supply is the density of workers of type α that choose sector i. For sector-education combinations that are not optimal, the supply is simply zero.

The output of service i follows from multiplying this density by the effective time that workers spend in the workforce and the productivity of $h(i)$-type workers in service i

$$S(i) = A(i,h(i)) f(\alpha(i))\alpha'(i)(T - h(i)) \tag{10}$$

Taking logs and differentiating equation (10) with respect to i, we have

$$\frac{S'(i)}{S(i)} = \frac{\frac{\partial A(i,h(i))}{\partial i}}{A(i,h(i))} + \frac{\frac{\partial A(i,h(i))}{\partial h}h'(i)}{A(i,h(i))} + \frac{\frac{\partial f(\alpha(i))}{\partial \alpha}\alpha'(i)}{f(\alpha(i))} + \frac{\alpha''(i)}{\alpha'(i)} - \frac{h'(i)}{T - h(i)} \tag{11}$$

Additionally, combining the first-order condition that comes from firm optimization with the agents'optimality condition in the choice of i, we can get an expression of the constant elasticity of substitution

[8] The previous reasoning does not exclude the possibility that some part of the trajectory of $h(i)$ is horizontal or vertical. However, this would require mass points in the distribution of either complexity or abilities. This is ruled out by the assumptions of the model.

$$\frac{S'(i)}{S(i)} = \sigma \left(\frac{\frac{\partial A(i,h(i))}{\partial i}}{A(i,h)} \right) \tag{12}$$

Using equations (11) and (12) produces the following second-order differential equation (SODE) that characterizes the optimal matching

$$\frac{\alpha''(i)}{\alpha'(i)} + \left(\frac{\frac{\partial A(i,h(i))}{\partial h}}{A(i,h(i))} - \frac{1}{T-h(i)} \right) h'(i) + \frac{f'(\alpha(i))\,\alpha'(i)}{f(\alpha(i))} + (1-\sigma)\frac{\frac{\partial A(i,h(i))}{\partial i}}{A(i,h(i))} = 0 \tag{13}$$

Equation (13) is an SODE describing the allocation of workers of type α to sectors in market equilibrium. To solve the previous SODE, I first use the inverse rule for derivatives to express the SODE in terms of abilities. The optimal choice of h given α as its derivative is found using equation (7). After that, I discretize the ability space and use a shooting algorithm to solve for the boundary conditions ($i(\underline{\alpha}) = \underline{I}$; $i(\overline{\alpha}) = \bar{I}$). Appendix A describes in detail the algorithm used to solve the SODE described by equation (13).

3 Calibration

In this section I discuss the calibration strategy. The parameters to be calibrated are the total endowment of time (T), the elasticity of substitution (σ), the complexity of the services produced in the economy $(\bar{I}, \underline{I})$, the location of the supply (Z), the supply shifts (ΔZ), the demand parameters $(\lambda,\ \chi_1,\ \bar{\imath})$, the within-sector technological change $(\Delta\lambda)$, the polarizing product demand shifts $(\Delta\chi_1)$, the rate of creation of new services $(\Delta\bar{I})$, the parameter that determines comparative advantages across sectors (δ), the distribution of inherent abilities for males and females $(f(\alpha))$, and the psychic cost function $\Omega(\alpha)$.

A first group of parameters $(T,\ \sigma)$ are taken from data or previous studies. The supply and demand location parameters $\left(Z, \bar{I}, \underline{I}, \delta, \lambda, \chi_1, \bar{\imath}\right)$ and the supply and demand shifts $\left(\Delta Z, \Delta\lambda, \Delta\chi_1, \Delta\bar{I}\right)$ are calibrated to match data from the U.S. economy. Psychic costs are calibrated to match the monetary value of psychic costs estimated in the literature. Data is taken from Acemoglu and Autor (2010) and Cunha and Heckman (2007). The next sections describe in detail our calibration strategy.

3.1 Parameters Taken from Data or Previous Studies

A first set of parameters $\{T, \sigma\}$ are taken directly from data and previous studies. The total endowment of time T is set equal to 59, the average age of retirement minus 7. In order to calibrate the elasticity of substitution between sectors, σ, I take the parameter estimated by Katz and Murphy (1992), that is, $\sigma = 1.4$. In section 6, I present a sensitivity analysis of the results for different values of σ.

3.2 The Psychic Costs Function

In order to calibrate the psychic costs function $\Omega(\alpha)$, I first impose a linear relationship between inherent abilities and the psychic costs paid by agents:

$$\Omega(\alpha) = E_0 + E_1\alpha \tag{14}$$

The assumed linear functional form implies that the proportionality parameter $\Omega(\alpha)$ will also have a uniform distribution. I assume that gender differences in psychic costs are only explained by gender differences in noncognitive abilities. Therefore, the parameters E_0 and E_1 are not gender-specific.

I calibrate the psychic costs parameters (E_0 and E_1) to make the monetary value of the psychic costs paid by the agents in the model consistent with those computed in the literature. Specifically, I calibrate E_0 and E_1 such that the monetary value of the psychic costs of a college education paid by the average man with a college education and the average man with a high school education in the model are the same as those computed by Cunha and Heckman (2007). By doing so, I can get the boundaries $\Omega(\overline{\alpha})$ and $\Omega(\underline{\alpha})$. Those boundaries are independent of the boundaries of the ability distribution. Therefore, I can normalize men's abilities: $U_m \sim [1, 10]$, where m denotes "male". Then, I impose the condition that the least able agent pays the highest cost and the most able agent pays the lowest cost. Using that information, we get $E_1 = -0.014$ and $E_0 = 0.095$.

In order to calibrate women's abilities, I pick from the literature a proxy for the gender ratio of the mean and variance of abilities. I use the mean and variance of the high school rank (percentiles) reported by Goldin, Katz, and Kuziemko (2006). It is not itself a measure of abilities. However, it is highly correlated with a bundle of abilities. Goldin, Katz, and Kuziemko (2006) present the high school rank deciles by sex from the National Education Longitudinal Survey for the high school graduating class of 1992. The mean high school ranks for men and women are 5.01 and 6.00, respectively. The variances are 8.28 and 7.74 for men and women, respectively. Using this information, we get $U_f \sim [2.24, 10.94]$, where f denotes "female". As a sensitivity analysis, in section 6 I calibrate the model using alternative proxies for the male and female distributions of abilities.

Finally, to calibrate the model for the total sample, I weight the female and male distribution of abilities using the average labor force participation of each group during the whole period. I get $U_t \sim [1.43, 10.33]$, where t denotes "total". Appendix B discusses in further detail the calibration strategy for the psychic cost function.

3.3 Supply and Demand Location and Shifts

The remaining parameters to be calibrated are those determining the supply and demand location $\left(Z, \bar{I}, \underline{I}, \delta, \lambda, \chi_1, \bar{\imath}\right)$ and the supply and demand shifts $\left(\Delta Z, \Delta\lambda, \Delta\chi_1, \Delta\bar{I}\right)$.

I first calibrate the model to match U.S. data for 1970, which is the first year available in the dataset. Notice that $\bar{I}, \underline{I}$, and δ are sufficient to characterize the location and elasticity of the demand. That set of parameters and the set of technological parameters $\{\lambda, \chi_1\}$ are isomorphic at a given moment in time. Therefore a normalization is needed. I normalize $\lambda = \chi_1 = 0$ for the baseline year. The demand $\left(\bar{I}, \underline{I}, \text{and } \delta\right)$ and supply (Z) location parameters determine the equilibrium prices and quantities of skills in the labor market. In order to calibrate those four parameters, I match four facts of the U.S. data: the relative supply of college-educated to the non-college-educated workers, the composition-adjusted ratio of the wages of college graduates to those of high school graduates, the ratio of the $90th$ to the $50th$ percentile of the wage distribution and the ratio of the $50th$ to the $10th$ percentile of the wage distribution. I compute the composition adjusted college wage premium by dividing the college and high school categories into four relevant groups (high school graduate, some college, college graduate, and greater than college) and taking the weighted average wage of the relevant composition adjusted cells using a fixed set of weights equal to the average employment share of each group.[9] The parameter $\bar{\imath}$ is set at the average of the calibrated complexity levels of the previous decade. In that way, I reduce the parameters to be calibrated by imposing the condition that the polarizing product demand shifts occur around the sector that produces the service with the average level of complexity.

Additionally, the supply and demand shifts $\left(\Delta Z, \Delta\lambda, \Delta\chi_1, \Delta\bar{I}\right)$ are calibrated to match the changes in the college wage premium, changes in the relative supply of college graduates, the change in $90th/50th$ ratio of wages, and the change in the $50th/10th$ ratio of wages. The effects of the supply and demand parameters on those facts of the data are not linearly dependent, which allows me to identify the model.[10] Tables 1-3 show how the model fits the data to be matched. We observe that the model is able to closely replicate the chosen data from the U.S. economy. This is the expected result given that I am using n facts of the data to calibrate n parameters. However, I report those results to show that the calibrated model adequately fits the data despite its high degree of non-linearity.

4 Results and Counterfactuals

I present in Table 4 the calibrated parameters for the total sample and in tables 5 and 6 the results for men and women, respectively. On the demand side, we observe that the parameter $\bar{I}$ remains constant during the pre-1980 period but rises sharply during the post-1980 period. The creation of new high-skill sectors seems especially important during the decade 1980-1990, when the skill premium experiences a pronounced increase. Additionally, we observe a continual increase in the parameter λ during the post-1980 period, reflecting an SBTC pushing up the relative wages of more educated workers. Finally, we observe a continual fall of the parameter χ_1 over the post-1980 period, reflecting a polarization of wages in the labor market in favor of low- and high-skill workers.

On the supply side, we observe a positive supply shift during the decade 1970-1980 followed by negative shifts during the post-1980 decades. This movement of the supply is consistent with the fall in the college wage premium during this decade, followed by the rise in the skill premium in the decades that follow. Additionally, we observe in tables 5 and 6 that the results for men and women exhibit the same pattern, although the magnitudes of the effects are different.

[9] This procedure is similar to the one followed by Acemoglu and Autor (2010) to generate the composition adjusted college wage premium, used as one of the target facts in my calibration.

[10] Simulations proving this point are available upon request.

Next, I perform some counterfactual exercises to estimate the causal effect of each supply and demand forces on the U.S. skill premium. I first compute what the skill premium would have been if some forces had not been present in each period—specifically, if only an SBTC had been present. Then, I perform the same exercise considering the SBTC and the polarizing effects. After that I add the effect of a structural transformation and, finally, the supply shifts. When all forces are present, the model predicts the college wage premium observed in tables 1-3.[11] Using that information, I compute the marginal explanatory power of supply and demand forces for the skill premium. Results are reported in tables 7-9.

We observe in Table 7 an increasing contribution of demand forces to the skill premium over time. In the first decade of the analysis, when the skill premium falls, the contribution of the demand forces is practically null. Positive supply shifts completely explain the fall of the skill premium during the period 1970-1980. In the last decade of the analysis (2000-2008) demand forces explain 80% of the rise in the skill premium. Overall, on average, demand forces explain 52% of the variation in the skill premium during the past four decades. The remaining 48% is explained by supply factors.

Among the demand forces, we observe an increasing polarization of wages favoring low- and high-skill workers. That polarization of wages in the labor market had a positive effect on the skill premium that went from zero during the period 1970-1980 to 60% in the last decade analyzed, which is equivalent to 75% of the total contribution of the demand forces. On average, during the whole period 1970-2008, 39% of the demand-driven variation in the skill premium is explained by the creation of new high-skill sectors, 47% by a polarizing product demand shift within existing sectors and only 14% by an SBTC.

Tables 8 and 9 present the analysis disaggregated by gender. We observe that the most remarkable gender asymmetry comes from the supply side. Supply forces make a greater contribution to the female skill premium. This fact is particularly relevant in more recent decades. For instance, in the last decade of the analysis, supply forces completely explain the rise in the female skill premium. In the case of men they only explain 42%. On average, over the entire period, female supply shifts explain 83% of the changes in the skill premium whereas male supply shifts only explain 35%. Particularly important were positive female supply shifts during the decade 1970-1980 and negative female supply shifts during the period 2000-2008. On the demand side, there is an increasing polarization of wages for both men and women, although it seems to be more important in the case of women.

4.1 Sensitivity Analysis

In this section I perform some additional exercises to evaluate the sensitivity of the results to different parameter values. Specifically, I recalibrate the model considering different values for the elasticity of substitution σ and the distribution of abilities $f(\alpha)$.

Some evidence on the elasticity of substitution has been provided by Katz and Murphy (1992), Murphy and Welch (1992), Fernandez Kranz (2000), and Acemoglu and Autor (2010). In general, that literature supports an elasticity of substitution around $1.5 - 2.0$. I choose the

[11] The order in which the forces are introduced does not significantly alter the magnitude of the effects.

middle and the upper bound of that range for my sensitivity analysis. Tables 10-12 present the results considering an elasticity $\sigma = 1.7$ and Tables 13-15 show the results using $\sigma = 2.0$. In order to facilitate the discussion of the results, I summarize them in Table 16 taking the average across decades. While the discussion is focused on Table 16, the decade-by-decade results can be found in Tables 10-15.

First, when comparing the counterfactuals for different values of σ, we observe that the average contribution of the demand forces to the skill premium increases as σ rises. This result is intuitive. Bigger demand shifts are needed to explain a given change in quantities and prices when demand becomes more elastic. We observe that in the case of the total sample, the contribution of demand forces increases from 52% to 74% as σ rises from the baseline value to 2. The increase in the contribution of the demand forces go from 65% to 97% in the case of men and from 17% to 56% in the case of women. Additionally, when looking at the decade by decade results, consistent with the previous results, we observe that in all cases, the polarization of wages becomes more pronounced as σ rises.

Next, I perform a second sensitivity analysis. In the baseline calibration I use the variance of high school rank (percentiles) reported by Goldin, Katz and Kuziemko (2006) to calibrate the ratio between the variance of men's abilities and the variance of women's abilities. However, other proxies for abilities are reported in the literature. Table 17 presents those alternative proxies.

Denote by $\widetilde{\sigma}_\alpha$the ratio between the variance of men's abilities and the variance of women's abilities. We observe in Table 17 that the proxies for the ratio $\widetilde{\sigma}_\alpha$ range from 1.02 to 2.08. In this sense, the proxy used in the baseline calibration constitutes a relatively conservative number ($\widetilde{\sigma}_\alpha = 1.07$ in the baseline scenario). Therefore, as a final sensitivity analysis, I calibrate the model, using as a proxy for $\widetilde{\sigma}_\alpha$ the highest value in Table 17. Considering the highest value for $\widetilde{\sigma}_\alpha$ we get $\bar{\alpha}_w = 9.71$ and $\underline{\alpha}_w = 3.47$, which implies that for the total sample $\alpha \sim U\,[1.86, 9.90]$. In this case the variance of the psychic costs falls by 48.6% in the case of women and by 18.4% for the total sample. Tables 18 and 19 present the results decade by decade. Table 20 summarizes them taking the average across decades.

We observe that as the variance of abilities decreases, the contribution of supply forces rises and the role of wage polarization diminishes. A lower variance of abilities implies that agents are more homogeneous and, thus, the elasticity of the supply of more educated workers is greater. With a more elastic supply curve, greater negative supply shifts are needed to explain the rise in the college wage premium in the context of an increasing demand for college graduates during the post-1980 period . Analogously, bigger positive supply shifts are needed to explain the fall in the college wage premium in the context of a stable demand for college graduates during the pre-1980 period. In the case of the total sample, the average contribution of supply forces increases from 48% to 61%. In the case of women, it increases from 83% to 99%. Consistent with this fact, the polarization of wages in the labor market becomes less pronounced.

5 Conclusions

This paper develops an assignment model to quantify, in a unified framework, the causal effect of supply and demand forces on the evolution of the college wage premium in the U.S. economy. Specifically, it quantifies the relative contribution of four different forces: (1) a within-sector non-neutral technological change, (2) the creation of new high-skill services/sectors, (3) polarizing product demand shifts, and (4) shifts in the relative supply of skilled labor. The model considers endogenous human capital accumulation.

The results show that, on average, 52% of the change in the U.S. skill premium during the last four decades is explained by demand factors. Supply forces explain the remaining 48% of the skill premium variation. Within the demand-driven changes in the skill premium, on average, 39% is explained by creation of new high-skill sectors, 47% by a polarizing product demand shift within existing sectors and only 14% by a skill-biased technological change.

The relative contribution of each supply and demand force is different across decades. Supply forces played a major role in earlier decades. Positive supply shifts explain around 100% of the fall of the skill premium during the period 1970-1980. On the demand side, we observe an increasing polarization of wages favoring low- and high-skill workers. That polarization of wages in the labor market had a positive effect on the skill premium that went from zero during the decade 1970-1980 to 60% in the last decade analyzed (2000-2008), which is equivalent to the 75% of the total contribution of the demand forces.

I also disaggregate the analysis by gender. I find that the most remarkable gender asymmetry comes from the supply side. On average, female supply shifts explain 83% of the changes in the skill premium, whereas male supply shifts only explain 35%. Particularly important were positive female supply shifts during the decade 1970-1980 and negative female supply shifts during the last decade analyzed. On the demand side, the increasing polarization of wages is present for both men and women, although it seems to be more important in the case of women.

I find that as the elasticity of substitution rises, the contribution of demand factors increases and the polarization of wages becomes more pronounced. This result is consistent with the fact that a higher elasticity of substitution makes the demand curve for more educated workers more elastic, and, thus, greater demand shifts are needed to produce a given change in quantities and prices. In contrast, as the variance of the distribution of abilities falls, the contribution of supply forces increases and the polarization of wages becomes less important for explaining the skill premium. This is consistent with the fact that a lower variance implies more homogeneous agents and, thus, a more elastic supply curve for more educated workers.

This model presents several desirable characteristics not present in the previous empirical framework studying the U.S. college wage premium. First, the model allows us to estimate the causal effect of different supply and demand forces on the college wage premium. Unlike in previous frameworks, all of those forces are included within a unified model. Second, the model considers an endogenous response of the supply for higher education without restricting the elasticity of the supply curve (which is inelastic in the standard model). Third, I explicitly model non-pecuniary costs of investing in higher education. Fourth, I allow the distribution of abilities of men to differ from that of women. Fifth, the model allows for the existence of polarizing or non-monotonic product demand shifts. Sixth, the model allows me to analyze several moments of the U.S. structure within a single framework. Finally, the model is tractable and allows for a calibration with few data requirements. As far as I know, no such framework has previously been used in the literature to estimate the causal effect of different supply and demand forces on the U.S. skill premium.

This paper contributes to disentangling the causal effects of different demand and supply forces on the U.S. skill premium. An interesting and important area for future research would be to include international trade in this closed economy model. This aspect could be more relevant when using this framework to understand the movement in the skill premium in developing countries. In those countries, specific forces such as the Stolper-Samuelson effect, imports of capital goods, and capital skill complementarities are relevant. In that way, this model would become relevant for understanding the skill premium not only in big economies such as the U.S. economy but also in small open economies. The framework developed in this paper constitutes a stepping stone towards a more complete open-economy model.

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Appendix A: Solution of the SODE

The actual SODE solved is not (13), but its equivalent in terms of the inverse $i(\alpha)$. Using the chain rule, I first substitute $h'(i) = h'(\alpha)\alpha'(i)$ in (13). Additionally, using the inverse rule for derivatives, we have that $\alpha'(i) = \frac{1}{i'(\alpha)}$ and $\frac{\alpha''(i)}{\alpha'(i)} = -\frac{i''(\alpha)}{[i'(\alpha)]^2}$. Given the uniform distribution for α, we have that $\frac{f'(\alpha(i))}{f(\alpha(i))} = 0$. Therefore, I can express the SODE in terms of abilities

$$i''(\alpha) = \left(\frac{\frac{\partial A(i(\alpha),h(\alpha))}{\partial h}}{A(i(\alpha),h(\alpha))} - \frac{1}{T-h(\alpha)} \right) h'(\alpha)\, i'(\alpha) + (1-\sigma) \frac{\frac{\partial A(i(\alpha),h(\alpha))}{\partial i}}{A(i(\alpha),h(\alpha))} \left[i'(\alpha)\right]^2 \quad \text{(A.1)}$$

where

$$\frac{\frac{\partial A(i,h(i))}{\partial h(i)}}{A(i,h(i))} = i(\alpha)^{\delta} + \lambda \quad \text{(A.2)}$$

$$\frac{\frac{\partial A(i,h(i))}{\partial i}}{A(i,h(i))} = \delta i(\alpha)^{\delta-1} h(\alpha) + 2\chi_0 i(\alpha) + \chi_1 \quad \text{(A.3)}$$

The remaining step is to find an expression for $h(\alpha)$ and $h'(\alpha)$. From the optimality condition for the representative firm, I get

$$\frac{\frac{\partial w(i,h)}{\partial h}}{w(i,h)} = \frac{\frac{\partial A(i,h(i))}{\partial h(i)}}{A(i,h(i))} = i(\alpha)^{\delta} + \lambda \quad \text{(A.4)}$$

Then, using the optimality condition for h we have

$$h(\alpha) = \frac{T}{1+Z+\Omega(\alpha)} - \frac{1}{i(\alpha)^{\delta} + \lambda} \quad \text{(A.5)}$$

$$h'(\alpha) = \frac{\delta i(\alpha)^{\delta-1} i'(\alpha)}{\left(i(\alpha)^{\delta} + \lambda\right)^2} - \frac{T\Omega'(\alpha)}{(1+Z+\Omega(\alpha))^2} \quad \text{(A.6)}$$

To solve the SODE, I discretize the ability space and use a shooting algorithm to solve for the boundary conditions ($i(\underline{\alpha}) = \underline{S}$; $i(\overline{\alpha}) = \overline{S}$).

The Matlab command is ODE45. The inputs of the algorithm are the following two initial conditions: $i(\underline{\alpha}) = \underline{S}$ and $i'(\underline{\alpha}) = S_0$. To solve for the boundary conditions $i(\overline{\alpha}) = \overline{S}$ and $i(\underline{\alpha}) = \underline{S}$, I implement the following algorithm. First, I guess some $S_{\max}(0)$ and $S_{\min}(0)$ such that for $S_0 = S_{\max}$ the model produces $i(\overline{\alpha}) > \overline{S}$ and for $S_0 = S_{\min}$ the model produces $i(\overline{\alpha}) < \overline{S}$. Then, I define $S_{avg}(0) = \frac{S\max(0)+S_{\min}(0)}{2}$ and run the model using $S_0 = S_{avg}(0)$. If $i(\overline{\alpha}) > \overline{S}$, then I update $S_{max}(1) = S_{avg}(0)$ and run the model using $S_0 = S_{avg}(1) = \frac{S\max(1)+S_{\min}(0)}{2}$. If $i(\overline{\alpha}) < \overline{S}$, then I update $S_{\min}(1) = S_{avg}(0)$ and run the model using

$S_0 = S_{avg}(1) = \frac{S\max(0)+S_{\min}(1)}{2}$. I repeat this sequence until $i(\overline{\alpha}) = \overline{S}$. This algorithm requires some monotonicity in the problem. Specifically, I require that $i(\overline{\alpha})$ be increasing in S_0. Without a formal proof, I state that the model satisfies this monotonicity.

Appendix B: Calibration of the Psychic Costs Function

Denote by $PV_c(h_c)$ the mean monetary value of the ability cost (in year 2000 dollars) of attending college for college graduates, by $PV_{hs}(h_c)$ the mean monetary value of the ability cost (in year 2000 dollars) of attending college for high school graduates, by $w_c(h_c)$ the average annual wage that a college graduate earns during his lifetime, by $w_{hs}(h_c)$ the average annual wage that a high school graduate would earn during his lifetime if he had chosen to be a college graduate, h_c the average years of schooling of a college graduate in 2000, by α_c the mean inherent ability of agents with $h \geq 16$ (college graduates), and by α_{hs} the mean inherent ability of agents with $12 \leq h < 16$ (high school graduates). Following this notation, we have that the indirect costs of going to college for the typical college and high school graduate are $h_c w_c(h_c)$ and $h_c w_{hs}(h_c)$, respectively. Therefore, given that I have assumed that the monetary value of the psychic cost of going to college is proportional to the indirect costs, with data on the $PV_c(h_c), PV_{hs}(h_c), h_c, w_c(h_c)$, and $w_{hs}(h_c)$, I compute

$$\Omega(\alpha_c) = \frac{PV_c(h_c)}{h_c w_c(h_c)} \tag{B.1}$$

$$\Omega(\alpha_{hs}) = \frac{PV_{hs}(h_c)}{h_c w_{hs}(h_c)} \tag{B.2}$$

Equations (B.1) and (B.2) show the proportionality factor $\Omega(\cdot)$ for the typical college and high school graduate, respectively.

To compute the upper and lower limits of that distribution of the psychic costs ($\Omega(\underline{\alpha})$ and $\Omega(\bar{\alpha})$, respectively), I use the properties of a normal distribution and data on the fraction of the population with a college education. Denote by p_c the fraction of the population with a college education. $\Omega(\underline{\alpha})$ is the psychic cost parameter of the least able agent (who has the highest cost) and $\Omega(\bar{\alpha})$ is the psychic cost parameter of the most able agent (who has the lowest cost). Therefore, if the fraction of agents with college education is p_c and the distribution of Ω is uniform, it must be true that the psychic cost parameter for the least able college graduate is $(\Omega(\underline{\alpha}) - \Omega(\bar{\alpha}))p_c + \Omega(\bar{\alpha})$. The psychic cost parameter for the most able college graduate is $\Omega(\bar{\alpha})$.Therefore, the psychic cost parameter for the typical college graduate (the one with the mean abilities among college graduates) is given by

$$\Omega(\alpha_c) = \frac{(\Omega(\underline{\alpha}) - \Omega(\bar{\alpha}))p_c + 2\Omega(\bar{\alpha})}{2} \tag{B.3}$$

Additionally, denote by p_{hs} the fraction of the population with a completed high school education (but who have not earned a college degree). Then the psychic cost parameter for the least able high school graduate is $(\Omega(\underline{\alpha}) - \Omega(\bar{\alpha}))(p_c + p_{hs}) + \Omega(\bar{\alpha})$. The psychic cost parameter for the most able high school graduate is $(\Omega(\underline{\alpha}) - \Omega(\bar{\alpha}))p_c + \Omega(\bar{\alpha})$. Therefore, the psychic cost parameter for the typical high school graduate is given by

$$\Omega(\alpha_{hs}) = \frac{(\Omega(\underline{\alpha}) - \Omega(\bar{\alpha}))(2p_c + p_{hs}) + 2\Omega(\bar{\alpha})}{2} \tag{B.4}$$

Then, equations (B.3) and (B.4) constitute a system of two equations and two unknown variables ($\Omega(\underline{\alpha})$ and $\Omega(\bar{\alpha})$). Therefore, using (B.3) and (B.4) I get the limits of the uniform distribution for the psychic costs function. Notice that those boundaries are independent of the boundaries of the ability distribution. Therefore, we can normalize men's abilities: $U \sim [1; 10]$.

Finally, by imposing the condition that the least able agent in the distribution pays the highest cost and the most able agent pays the lowest cost, I get E_0 and E$_1$

$$\Omega(\underline{\alpha}) = E_0 + E_1\underline{\alpha} \tag{B.5}$$

$$\Omega(\bar{\alpha}) = E_0 + E_1\bar{\alpha} \tag{B.6}$$

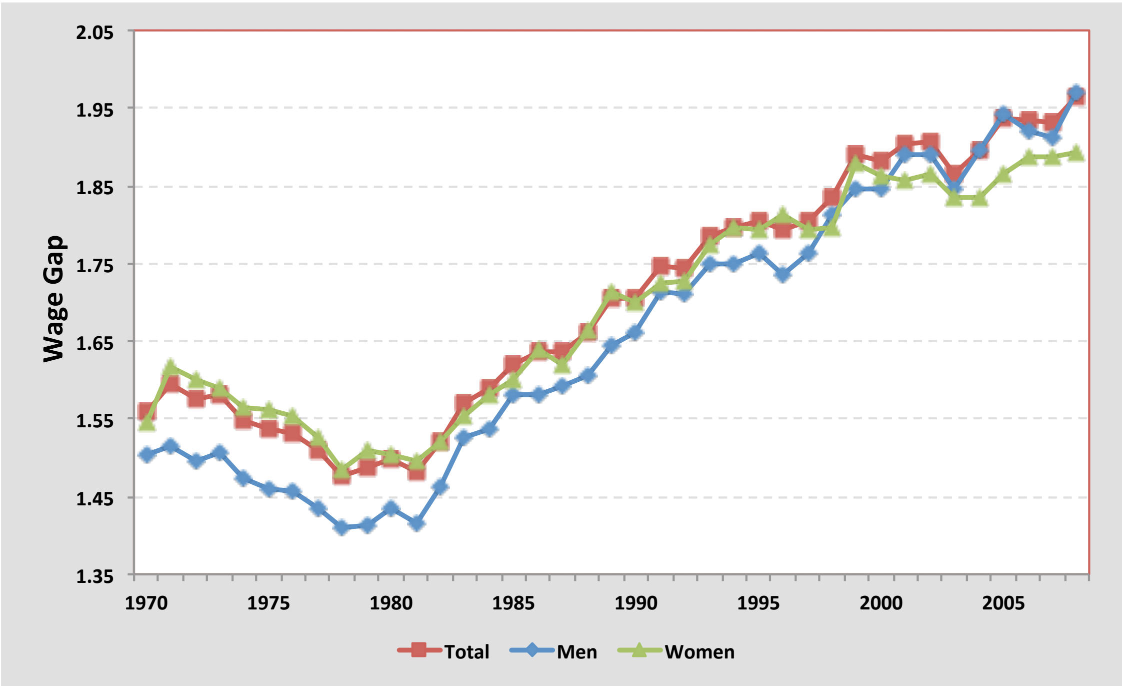
Notice that (B.5) and (B.6) constitute a system of two equations and two unknowns.

5.0.1 Parameter Values

Using a sample of white males from the National Longitudinal Survey of Youth 1979 (NLSY79), Cunha and Heckman (2007) estimate that the mean monetary value of the ability cost (in year 2000 dollars) of attending college is $-\$14,892$ for college graduates $PV_c(h_c)$ and $\$12,715$ for high school graduates $PV_{hs}(h_c)$. Additionally, Cunha and Heckman (2007) estimate that the present the value of earnings of a typical college graduate is $\$1,390,321$ (in year 2000 dollars). The typical high school graduate would earn $\$1,125,785$ if he had chosen to be a college graduate. The average years of schooling of a college graduate is 16.9 in 2000. Therefore, I get $w_c(h_c) = 1,390,321/(59-16.9) = 33,024$ and $w_{hs}(h_c) = 1,125,785/(59-16.9) = 26,741$. Additionally, from census data we get $p_c = 0.25$ and $p_{hs} = 0.64$. Using those inputs, and equations (B.3) and (B.4), I get $\Omega(\underline{\alpha}) = 0.081$ and $\Omega(\bar{\alpha}) = -0.043$. Using equations (B.5) and (B.6) I get $E_1 = -0.014$ and $E_0 = 0.095$.

Figures and Tables

Figure 1: College/High-School Weekly Wage Ratio



Source: Acemoglu and Autor (2010).

Table 1: Model Fit, Total (Baseline)

	Data				
	1970	1980	1990	2000	2008
Skill Premium	1.5580	1.4979	1.7055	1.8813	1.9655
Supply	0.3256	0.4295	0.4949	0.5488	0.5858
90th/50th	1.8580	1.8920	2.0306	2.2188	2.3
50th/10th	1.9829	2.0328	2.1489	2.1333	2.1622
	Model				
	1970	1980	1990	2000	2008
Skill Premium	1.5567	1.4896	1.6910	1.8809	1.9685
Supply	0.3252	0.4301	0.4795	0.5461	0.5853
90th/50th	1.8586	1.8947	2.0340	2.2190	2.2986
50th/10th	1.9830	2.0666	2.1512	2.1334	2.1608

Table 2: Model Fit, Men (Baseline)

	Data				
	1970	1980	1990	2000	2008
Skill Premium	1.5031	1.4357	1.6617	1.8473	1.9698
Supply	0.3387	0.4482	0.5018	0.5476	0.5722
90th/50th	1.7126	1.7333	1.9588	2.1622	2.2444
50th/10th	1.8383	2	2.2548	2.2672	2.2500
	Model				
	1970	1980	1990	2000	2008
Skill Premium	1.5146	1.4385	1.6601	1.8315	1.9700
Supply	0.3401	0.49	0.5202	0.5473	0.5721
90th/50th	1.7266	1.7285	1.9385	2.1582	2.2443
50th/10th	1.8240	1.9121	2.2287	2.2653	2.2499

Table 3: Model Fit, Women (Baseline)

	Data				
	1970	1980	1990	2000	2008
Skill Premium	1.5457	1.5039	1.6994	1.8611	1.8923
Supply	0.2831	0.3813	0.4799	0.5512	0.6121
90th/50th	1.7412	1.7491	1.8725	2.0679	2.1429
50th/10th	1.7230	1.7348	1.9760	2.1049	2.0588
	Model				
	1970	1980	1990	2000	2008
Skill Premium	1.5353	1.4845	1.7002	1.8592	1.8941
Supply	0.2828	0.3598	0.48	0.5511	0.6119
90th/50th	1.7254	1.7510	1.8721	2.0687	2.1425
50th/10th	1.7351	1.7479	1.9757	2.1056	2.0577

Table 4: Calibrated Parameters, Total (Baseline)

	1970	1980	1990	2000	2008
$\overline{I}$	0.1888	0.189	0.2252	0.2324	0.2335
$\underline{I}$	0.0215	0.0215	0.0215	0.0215	0.0215
χ_1	0	0.0112	-4.1571	-13.0355	-16.8479
$\bar{\imath}$	0.1052	0.1052	0.1052	0.1052	0.1052
λ	0.0001	0.0002	0.0123	0.0316	0.0399
δ	0.8992	0.8992	0.8992	0.8992	0.8992
Z	1.6632	1.5351	1.6929	1.7583	1.7653

Table 5: Calibrated Parameters, Men (Baseline)

	1970	1980	1990	2000	2008
$\overline{I}$	0.1677	0.1562	0.2182	0.2270	0.2255
$\underline{I}$	0.0256	0.0256	0.0256	0.0256	0.0256
χ_1	0	3.0428	-0.0117	-8.2219	-13.5923
$\bar{\imath}$	0.0967	0.0967	0.0967	0.0967	0.0967
λ	0.0001	-0.003	0.0002	0.007	0.0175
δ	0.874	0.874	0.874	0.874	0.874
Z	1.6124	1.3725	1.5897	1.6635	1.6969

Table 6: Calibrated Parameters, Women (Baseline)

	1970	1980	1990	2000	2008
$\overline{I}$	0.2131	0.2128	0.2644	0.28	0.2699
$\underline{I}$	0.0452	0.0452	0.0452	0.0452	0.0452
χ_1	0	-1.2	-1.596	-8.2923	-15.6134
$\bar{\imath}$	0.1292	0.1292	0.1292	0.1292	0.1292
λ	0	0	0.0237	0.0376	0.0586
δ	0.9854	0.9854	0.9854	0.9854	0.9854
Z	1.7139	1.6536	1.7768	1.8253	1.8314

Table 7: Explanatory Power (%), Total (Baseline)

	1970 – 1980	1980 – 1990	1990 – 2000	2000 – 2008	Avg.
SBTC	0.01	11.6	11.8	5.2	7.1
Polarization	0.1	4.0	32.9	60.2	24.3
Creation of New Sectors	-0.8	45.7	21.8	14.1	20.2
Supply	100.7	38.8	33.5	20.5	48.4

Table 8: Explanatory Power (%), Men (Baseline)

	1970 – 1980	1980 – 1990	1990 – 2000	2000 – 2008	Avg.
SBTC	1.8	1.7	10.3	26.1	10
Polarization	12.2	0.8	36.5	43.6	23.2
Creation of New Sectors	21.3	90.5	28.4	-11.2	32.3
Supply	64.7	7.1	24.8	41.6	34.6

Table 9: Explanatory Power (%), Women (Baseline)

	1970 – 1980	1980 – 1990	1990 – 2000	2000 – 2008	Avg.
SBTC	0	11.2	2.8	1.9	4
Polarization	-1.4	0.1	22.2	128.0	37.2
Creation of New Sectors	5.6	30.1	13.7	-147.2	-24.5
Supply	95.8	58.6	61.3	117.3	83.3

Table 10: Explanatory Power (%), Total (Elasticity $\sigma = 1.7$)

	1970 − 1980	1980 − 1990	1990 − 2000	2000 − 2008	Avg.
SBTC	1.2	14.9	16.7	16.2	12.2
Polarization	−2.7	1.2	52.5	77.4	32.1
Creation of New Sectors	11.9	53.2	4.2	−8.6	15.2
Supply	89.6	30.6	26.6	15	40.5

Note: σ is the elasticity of substitution between services in the production of the final good.

Table 11: Explanatory Power (%), Men (Elasticity $\sigma = 1.7$)

	1970 − 1980	1980 − 1990	1990 − 2000	2000 − 2008	Avg.
SBTC	23.1	14.5	18.9	18.5	18.7
Polarization	38.2	6.5	83	115.6	60.8
Creation of New Sectors	14.3	77.4	−7.3	−44.6	10
Supply	24.4	1.7	5.4	10.5	10.5

Note: σ is the elasticity of substitution between services in the production of the final good.

Table 12: Explanatory Power (%), Women (Elasticity $\sigma = 1.7$)

	1970 − 1980	1980 − 1990	1990 − 2000	2000 − 2008	Avg.
SBTC	2.6	15.2	6.2	4.3	7.1
Polarization	0	1.9	45.5	137.2	46.2
Creation of New Sectors	7.3	33.5	12.5	−111.2	−14.5
Supply	90.1	49.4	35.8	69.7	61.3

Note: σ is the elasticity of substitution between services in the production of the final good.

Table 13: Explanatory Power, Total (Elasticity $\sigma = 2$)

	$1970 - 1980$	$1980 - 1990$	$1990 - 2000$	$2000 - 2008$	Avg.
SBTC	32.4	13	3.1	2.1	12.6
Polarization	-24.6	8.6	88.2	97.4	42.4
Creation of New Sectors	34.0	52.3	-12.1	-0.1	18.5
Supply	58.3	26.1	20.8	0.6	26.5

Note: σ is the elasticity of substitution between services in the production of the final good.

Table 14: Explanatory Power, Men (Elasticity $\sigma = 2$)

	$1970 - 1980$	$1980 - 1990$	$1990 - 2000$	$2000 - 2008$	Avg.
SBTC	14.1	6.2	12	20.3	13.1
Polarization	64.9	13.3	102.4	141.8	80.6
Creation of New Sectors	18.6	80.4	-18.4	-66	3.7
Supply	2.5	0.1	3.9	3.9	2.6

Note: σ is the elasticity of substitution between services in the production of the final good.

Table 15: Explanatory Power, Women (Elasticity $\sigma = 2$)

	$1970 - 1980$	$1980 - 1990$	$1990 - 2000$	$2000 - 2008$	Avg.
SBTC	0	12.8	1.2	5.2	4.7
Polarization	-0.2	1.3	80.7	152.3	58.5
Creation of New Sectors	18.8	49.8	-10.2	-87.6	-7.3
Supply	81.4	36.2	28.3	30.1	44

Note: σ is the elasticity of substitution between services in the production of the final good.

Table 16: Average Explanatory of Power Supply and Demand Forces (%) and σ

Total			
	$\sigma = 1.4$	$\sigma = 1.7$	$\sigma = 2$
Demand	51.6	59.5	73.5
Supply	48.4	40.5	26.5
Men			
	$\sigma = 1.4$	$\sigma = 1.7$	$\sigma = 2$
Demand	65.4	89.5	97.4
Supply	34.6	10.5	2.6
Women			
	$\sigma = 1.4$	$\sigma = 1.7$	$\sigma = 2$
Demand	16.8	38.7	56
Supply	83.2	61.3	44

Note: σ is the elasticity of substitution between services in the production of the final good.

Table 17: Alternative Proxies for $\widetilde{\sigma}_\alpha$

8th grade composite ability	1.02
Hours homework/wk in 8th grade	1.03
High school grades	1.06
12th grade composite ability	1.07
Class rank (percentile)	1.08
Middle school grades	1.11
Behavior problem	1.54
Hours of homework/wk in 12th grade	1.72
Behavior composite	2.08

Source: Jacob (2002). Note: $\widetilde{\sigma}_\alpha$ is the ratio between the variance of men's abilities and the variance of women's abilities.

Table 18: Explanatory Power, Total ($\widetilde{\sigma}_\alpha = 2.08$)

	$1970-1980$	$1980-1990$	$1990-2000$	$2000-2008$	Avg.
SBTC	-21.8	10.2	16.6	1.0	1.5
Polarization	-8.3	2.9	21.0	42.1	14.4
Creation of New Sectors	22.5	44.8	21.4	2.2	22.7
Supply	107.6	42.1	41.0	54.7	61.4

Note: $\widetilde{\sigma}_\alpha$ is the ratio between the variance of men's abilities and the variance of women's abilities.

Table 19: Explanatory Power, Women ($\widetilde{\sigma}_\alpha = 2.08$)

	$1970-1980$	$1980-1990$	$1990-2000$	$2000-2008$	Avg.
SBTC	-14.5	6.5	2.2	1.2	-1.2
Polarization	-11.8	0	11.1	67	16.6
Creation of New Sectors	20.3	25.8	18.9	-125.3	-15.1
Supply	106.1	67.7	67.8	157.1	99.7

Note: $\widetilde{\sigma}_\alpha$ is the ratio between the variance of men's abilities and the variance of women's abilities.

Table 20: Average Explanatory of Power Supply and Demand Forces (%) and $\widetilde{\sigma}_\alpha$

	Total	
	$\widetilde{\sigma}_\alpha = 1.07$	$\widetilde{\sigma}_\alpha = 2.08$
Demand	52.6	38.6
Supply	48.4	61.4
	Women	
	$\widetilde{\sigma}_\alpha = 1.07$	$\widetilde{\sigma}_\alpha = 2.08$
Demand	26.7	0.03
Supply	83.3	99.7

Note: $\widetilde{\sigma}_\alpha$ is the ratio between the variance of men's abilities and the variance of women's abilities.