Wage Dispersion with Heterogeneous Wage Contracts*

Cynthia L. Doniger†
Federal Reserve Board

October 30, 2015

Abstract

I study a labor market in which identical workers search on- and off-the-job and heterogeneous firms employ using either an ex-ante posted wage or flexible wage contracts contingent on outside options. Firm level costs for contingent contracts generate a separating equilibrium in which less productive firms post wages. Using German employee-level administrative data, I estimate roughly 70 percent of firms post wages and employ nearly 50 percent of workers under such contracts. The model with heterogeneous contracts can achieve wage dispersion, labor share, employment transitions, and flow value of unemployment that are simultaneously consistent with empirical observations while capturing information frictions and search externalities modeled by ex-ante wage posting.

---

*The views expressed in this paper solely reflect those of the author and not necessarily those of the Federal Reserve Board, the Federal Reserve System as a whole, or anyone else associated with the Federal Reserve System.

†Email: Cynthia.L.Doniger@frb.gov. I would like to thank Dmitriy Stolyarov, Jeff Smith, Matthew Shapiro, and Charlie Brown for advising and support. I also thank the insight and comments of the participants of the Second Joint Paris SaM Workshop and the Sixth Annual Joint Data Workshop, BI Oslo Group and LMDG (CAP) Group. Finally, I thank Aditya Aladangady, Isaac Sorkin, and the participants of the labor search reading group and informal macro and labor seminars at the University of Michigan. This study uses the weakly anonymous Sample of Integrated Labour Market Biographies (Year 2006). Data access was provided via on-site use at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) in Ann Arbor, Michigan, USA.

1 Introduction

Equilibrium search theories yield a fruitful theoretical paradigm for assessing the efficiency and design of labor market policy and how these interact with wage dispersion and inequality. A key component of search models is the mechanism used to divide the output of employment between an employer and employee and set the wage. However, different wage setting mechanisms yield different – sometimes radically different – theoretical implications for market efficiency and the distribution of output between and across workers and firms, with some of these in contradiction with empirical evidence. Discrepancies between model and data pose challenges for application to policy analysis. This paper demonstrates that a model featuring a mixture of wage setting mechanisms can simultaneously match a long list of policy-relevant observables. This result suggests that explicit modeling of wage contract heterogeneity is an important component of a policy-relevant, search-theoretic model of the relationship between wage dispersion and unemployment.

Survey evidence indicates that a significant fraction of employers hire under contracts that are nonnegotiable while others are willing to renegotiate. Barron et al. (2006) find 49 percent of U.S. firms surveyed in the 2001 Small Business Administration Survey reported willingness to *re*negotiate the wage of the most recently hired employee. Hall and Krueger (2012) find 31 percent of U.S. workers surveyed as part of the Princeton Data Improvement Initiative in 2008 reported negotiating over pay at the time of hire. Importantly for the present empirical results, Brenzel et al. (2013) find 38 percent of German firms surveyed in the 2011 Germany Job Vacancy Survey negotiated the wage of the most recently hired employee.These data suggest that modeling contract heterogeneity increases the realism of the model in addition to the aforementioned gains in goodness of fit.

This paper characterizes the interactions; resulting wages; and, *most importantly*, impact on the aggregate wage distribution and search incentives of the unemployed when firms compete for workers using either a non-negotiable contract or a renegotiable contract. Specifically I model firms choosing between setting wages using either wage posting (WP) or sequential auction

(SA). Wages set under WP are characterized by a single and inflexible wage for all employees through the duration of their contracts as in Burdett and Mortensen (1998) and Bontemps et al. (2000). In contrast, wages formed under the SA contract are contingent on a worker's best-to-date outside option and may be updated during a workers tenure as in Postel-Vinay and Robin (2002b).

The modeling approach I adopt is simple and designed to expose the impact on the aggregate wage distribution and value of search for the unemployed most starkly. A per-firm cost for employing under the SA contract induces firms to sort between WP and SA wage contracts such that the most productive firms select SA while all less productive firms choose WP. The cost may be thought of as reflecting the legal or human resources costs of offering a job in which wages are set under a contingent wage contract. Consider that maintaining the contingent pay schedule requires knowledge of each workers best-to-date alternative employment offer. Such knowledge may be costly to obtain or contract upon and may require firms to maintain human resources or legal department, or both.

This simple cost structure enables me to isolate and exploit a single mechanism that drives contract choice: differentials in rent allocation between contract types. As a result, I am able to consider a continuous distribution of firm types and optimal wage-setting strategies under *both* contract types.[1] These features, particularly optimal wage setting in the WP sector, facilitates a link between search incentives of the unemployed and wage dispersion. For extreme values of the cost the pure-contract direct antecedents of the model are recovered: under null costs the model is identical to Bontemps et al. (2000) and as costs approach infinity the model becomes identical to Postel-Vinay and Robin (2002b).

[1]The environment is more general than that considered by my nearest neighbors in the literature: Postel-Vinay and Robin (2004) and Holzner (2011). Generality is achieved by abstracting from the micro-foundations of the cost of the SA contract. Each neighbor provides micro-foundations that derive from differential search effort under contract types. The general labor market considered here is at odds with such micro-foundation since a continuous distribution of productivity types and optimal wage setting strategies in the WP sector imply that workers should have search incentive in *both* sectors, and search incentives are not always strongest in a single sector.

An advantage to explicitly modeling wage contract heterogeneity is the ability to capture analytical and policy-relevant insight regarding firms that face information or negotiating frictions, as in the pure-WP model, at the same time as an improved overall fit to both micro- and macro-data. In particular, the mixed-contract model is able to produce substantial wage dispersion without contradicting data on unemployment and employment duration, the ratio of value added to compensation of employees (labor share), or plausible ranges for the value of non-employment. Further, good fit is achieved even when a large portion of firms hire under WP. This stands in contrast to the well known result that the WP model is at odds with data on wage dispersion (Hornstein et al., 2011).

I estimate the parameters of the model using wages and labor market histories of West German workers in 2006 from the Sample of Integrated Labour Market Biographies (SIAB) provided by the Research Data Center of the German Federal Employment Agency at the Institute for Employment Research and the Simulated Generalized Method of Moments.[2] I estimate that 73 percent of firms employ using the SA contract. Due to a job ladder present in the model this results in an estimated 47 percent of workers employed in a renegotiable contract.

With the estimated parameters, the model implies a labor share of 54 percent and a ratio of the flow value of non-employment to the average wage of 71 percent. This is achieved when the model is required to explain *raw* wage dispersion. Larger labor share would be achievable if some dispersion were accounted for by observables: e.g. industry and occupation. For comparison, the EU-KLEMS database reports German labor share as 63.7 in 2006. My own estimates suggest a value of non-employment well in excess of 30 percent of average wages.[3]

[2]2006 represents a relatively stable period postdating the Hartz reforms and predating the financial crisis.

[3]I focus on West Germany in order to abstract from differences in East and West German wage scales that are not addressed by the model. All statistics other than labor share reflect the focus on West Germany. Survey evidence analyzed in Doniger (2014) suggests that labor share is not substantially different in the two regions. The 30 percent ratio of value of non-employment to average wages reflects the ratio of the average unemployment benefits to average wages paid in the first week of January 2006. Due to selection on who

Moderate ratio of the value of non-employment to average wages facilitates analysis of policies, such as unemployment insurance, that impact search incentive while allowing search to be more effective when unemployed than employed.[4] In contrast, when constrained such that all firms SA or all WP, the model suggests ratio of the value of non-employment to average wages near unity or large and negative, respectively. The first begs the question "why do workers search at all?" while the second begs "since leisure is (plausibly) not actually so hateful, why do they ever stop?" Meanwhile, divergence from labor share as recorded in National Income and Product Accounts – when constrained to only WP the model suggest labor share of only 23 – begs the question "who owns the remaining labor rents?" and renders calculations of changes in total output due to policy spurious.

The remainder of the paper proceeds as follows. Section 2 lays out the labor market setting under consideration, including details of contracts that are available; describes worker and firm behavior; and demonstrates that, for every fixed per-firm cost for the right to SA, there is a separating equilibrium in which low productivity firms employ under WP while higher productivity employ under SA. Section 3 provides comparative statics for a fixed distribution of firm types when the cost of the SA contract increases. As more firms join the WP sector, labor share increases but the distribution of wages becomes more compressed, setting up a trade-off whereby one can achieve substantial labor share even when many firms WP, since realistic degree of wage dispersion can be supplied by the remaining firms in the SA sector. Section 4 structurally estimates the model using social security register data for German workers. Section 5 concludes. Proofs, derivations, and details of the estimation strategy are found in the Appendix.

becomes unemployed and nonpecuniary value of non-employment, this 30 percent forms a very cautious lower bound.

[4]If the ratio is too near unity feedback is eliminated. If the ratio is too small feedback overwhelms the model and produces spurious results.

2 Model

2.1 Setting

I consider the steady state equilibrium of a search market in which firms and workers are brought together by a sequential process of random matching. Measure N of firms operate technologies that produce flow output p per worker. Technology is distributed continuously according to an exogenous distribution $\Gamma(p)$ on support $[\underline{p}, \bar{p}]$ with $0 < \underline{p}$ and $\bar{p}$ potentially infinite. Measure M of workers search both off- and on-the-job using uniform sampling, meaning the probability of sampling a firm of productivity p or less is $\Gamma(p)$. Job offers arrive at exogenous Poisson arrival rates λ_0 when unemployed and λ_1 when employed. Workers are exogenously separated from employment contracts at Poisson arrival rate δ and die and are replaced an equal mass of unemployed newborns at Poisson arrival rate μ. Workers receive flow b when unemployed. Each worker has linear utility and seeks to maximize the present discounted value of wages and unemployment benefits. Firms seek to maximize steady state current operating surplus: output less wages and costs paid for the right to SA, if any.

Firms hire workers under one of two wage-setting mechanisms: WP or SA. If WP, the firm offers a nonnegotiable wage for as long as the worker wishes or until exogenous separation. In equilibrium, this wage offer is uniform for all workers within the same firm. If SA, the firm offers a wage chosen to match the value of each worker's best-to-date outside offer whenever profitable. Wages are updated as outside offers evolve and the SA firm bids up the worker's wage even in the event of a job-to-job transition. SA wages are thus described by the wage-setting mechanism and the productivity of both the incumbent firm and best-to-date outside option.

If the firm chooses to set wages under SA it must pay a flow cost of c. The cost is independent of firm size. Modeling of costs is reduced form for the sake of tractability; however, one possible story is that there are legal and/or administrative fees associated with posting a vacancy in which wages will be set by SA. These fees must be paid whether or not the vacancy is filled. Note that although firms differ in size in equilibrium, each offers an identical number of vacancies. Firm size is pinned down by the rate of vacancy filling and the duration for which contracts persist.[5]

The remainder of this section proves the following proposition:

Proposition 1. *For each cost, c, there is a threshold, $\check{p}$, such that a Nash equilibrium exists in which firms with productivity less than the threshold ($p < \check{p}$) all strictly prefer WP while more productive firms ($p > \check{p}$) all strictly prefer SA. Threshold productivity firms ($p = \check{p}$) are indifferent.*

I begin by characterizing labor supply and wage schedules. I prove that for every threshold a cost exists such that no firm wishes to unilaterally deviate from the separating equilibrium. I then observe that the mapping $\check{p} \mapsto c$ is continuous, that null costs are consistent with all firms selecting SA, and all firms select WP for sufficiently large costs. The claim then follows from the intermediate value theorem. Conditions for uniqueness are provided in the Appendix.

2.2 Labor supply

As stated, workers seek to maximize the value of their current employment contract. This yields the following lemma governing the flow of workers between firms:

Lemma 1. *In the mixed contract model with proposed separating equilibrium, labor flows are constrained efficient.*

In other words, no worker ever rejects a job offer from a more productive employer than their current incumbent. Note that 1) flows within sectors

[5] The cost structure used in this paper is highly stylized. Weaker stylization is possible with restrictions on the distribution of productivity or hazard parameters. For example, modeling costs as cost per worker may be desirable and is possible when firm size does not increase too rapidly in productivity. Micro-foundations of costs based on search incentive of employees are extremely interesting and have been explored in the literature; see Postel-Vinay and Robin (2004) and Holzner (2011). Such micro-foundation poses a major drawback for the present project, however: with a continuum of firm types and optimally chosen WP wages, search incentive cannot be ranked between the WP and SA sectors.

remain efficient in the mixed equilibrium and 2) (more importantly) the flow between WP and SA sectors is efficient. The key ingredient is that in the proposed separating equilibrium, all SA firms are more productive than, and can outbid, all WP firms. Proof is provided in the Appendix.

Since workers accept any wage offer originating from a more productive firm, labor supply to a p-type firm can be pinned down by the method of mass balance. In steady state, the mass of workers flowing into firms of p-type or less must be equal to the mass flowing out:

$$\underbrace{U\lambda_0\Gamma(p)}_{\text{in}} = \underbrace{[\delta + \mu + \lambda_1\bar{\Gamma}(p)](M - U)L(p)}_{\text{out}},$$

where $\bar{\Gamma}(p) = 1 - \Gamma(p)$ is the fraction of firms with productivity greater than p, U is the mass of unemployed workers, and $L(p)$ is the mass of workers employed in a firm of productivity no greater than p.

Evaluating the mass balance equation at the supremum of productivity types yields steady state unemployment rate: $u = U/M = 1/(1 + k_0)$, where $k_0 = \lambda_0/(\delta + \mu)$. Also, the fraction of workers working for a firm with technology p or less is $L(p) = \Gamma(p)/(1 + k_1\bar{\Gamma}(p))$, where $k_1 = \lambda_1/(\delta + \mu)$ is the expected number of job offers per employment spell.

The supply of labor to a firm of type p can then be expressed as:

$$\ell(p) = \overbrace{\frac{\lambda_0 U + \lambda_1(M - U)L(p)}{N}}^{\text{expected hiring}} \overbrace{\frac{1}{\delta + \mu + \lambda_1\bar{\Gamma}(p)}}^{\text{expected duration}} = \frac{1 + k_1}{[1 + k_1\bar{\Gamma}(p)]^2}\frac{M - U}{N}.$$

When selecting a wage schedule, firms consider the impact on both size, $\ell(p)$, and expected flow profit per worker, $p - \mathbb{E}[wages]$. I now turn to the optimal wage schedule.

2.3 Wage choice

Characterizing an equilibrium that features both kinds of wage contracts requires characterizing wages that arise when firms operating different wage-setting mechanisms compete for the same worker: I will call this the "transitional wage" and denote it $w_{PA}(q,p)$ where p is the productivity of the incumbent and q is the productivity of the best-to-date outside option. Recalling that the SA firm's optimal wage choice equates the value of employment in the SA firm at the optimal wage with the value of employment at the best-to-date outside option at that competitor's optimal wage choice. The transitional wage can be characterized for an arbitrary SA firm of type p employing a worker with best-to-date outside option from an arbitrary WP firm of type q. Note that, since I am considering a separating equilibrium, if q is WP and p is SA then $q < \check{p} \leq p$.[6] Denote the values of employment in a WP and an SA contract are denoted as V^P and V^A.

The value of employment at an SA firm at some transitional wage, $w_{PA}(q,p)$, consistent with best-to-date outside offer being a q-productivity WP competitor is:

$$\begin{aligned}
\mu V^A(w_{PA}(q,p),p) &= w_{PA}(q,p) \\
&+ \underbrace{\lambda_1[\Gamma(\check{p}) - \Gamma(q))][\mathbb{E}[V^P(w_{PP}(x),x)|q < x < \check{p}] - V^A(w_{PA}(q,p),p)]}_{\text{on-the-job wage gain due to a credible threat from a WP competitor}} \\
&+ \underbrace{\lambda_1[\Gamma(p) - \Gamma(\check{p})][\mathbb{E}[V^A(x,x)|\check{p} < x < p] - V^A(w_{PA}(q,p),p)]}_{\text{on-the-job wage gain due to a credible threat from a SA competitor}} \\
&+ \underbrace{\lambda_1[\bar{\Gamma}(p)][V^A(p,p) - V^A(w_{PA}(q,p),p)]}_{\text{job-to-job transition to a SA competitor}} \\
&+ \underbrace{\delta[V^U - V^A(w_{PA}(q,p),p)]}_{\text{unemployment shock}}, \qquad (2.1)
\end{aligned}$$

where $w_{PP}(x)$ and x are the optimal competing wage offers of x-type WP and SA firms with productivity less than p. The difference in the wage setting strategies of a WP competitor and a SA competitor is reflected in potential wages changes. If the incumbent SA firm meets a new WP competitor wages will rise only enough to just best the value of the competitor WP firm's posted

[6] The functional form of the full wage schedules under all combinations of contract types (WP, $w_{PP}(p)$; transitional, $w_{PA}(q,p)$; SA, $w_{AA}(q,p)$; and reservation entry wages from unemployment into each contract type $w_{UP}(p)$ and $w_{UA}(\cdot,p)$) are derived in the Appendix. These are helpful for analysis of equilibrium properties, but are not required to establish the existence of equilibrium.

wage offer. On the other hand, if the incumbent SA firm meets a new SA competitor and is able to retain the worker, it must be the case that wages rise to just best the *maximum* value that the competitor SA firm is able to offer: the value of a wage equal to the productivity of the competitor SA firm at the competitor firm. This is reflected also in the option value of moving to a more productive firm. The transition would yield value just larger than the maximum the current incumbent is able to offer.[7]

Meanwhile, the value of employment in the employees' best-to-date outside option, a q-productivity WP firm, is:

$$\begin{aligned}
\mu V^P(w_{PP}(q), q) &= w_{PP}(q) \\
&+ \underbrace{\lambda_1[\Gamma(\check{p}) - \Gamma(q)][\mathbb{E}[V^P(w_{PP}(x), x)|q < x < \check{p}] - V^P(w_{PP}(q), q)]}_{\text{job-to-job transition to a WP competitor}} \\
&+ \underbrace{\lambda_1[\bar{\Gamma}(\check{p})][\mathbb{E}[V^A(w_{PA}(q, x), x)|\check{p} < x] - V^P(w_{PP}(q), q)]}_{\text{job-to-job transition to a SA competitor}} \\
&+ \underbrace{\delta[V^U - V^P(w_{PP}(q), q)]}_{\text{unemployment shock}}. \qquad (2.2)
\end{aligned}$$

The optimal transitional wage choice equates the value of the two contracts: $V^P(w_{PA}(q, p), p) = V^A(w_{PP}(q), q)$ for all $q < \check{p} \leq p$ and can thus be expressed as the posted wage at the best-to-date outside offer and the difference in the option values of the two employment contracts:

$$\begin{aligned}
&w_{PA}(q, p) = w_{PP}(q) + \\
&\underbrace{\lambda_1\left\{\bar{\Gamma}(\check{p})V^P(w_{PP}(q), q) - [\Gamma(p) - \Gamma(\check{p})]\mathbb{E}[V^A(x, x)|\check{p} < x < p] - \bar{\Gamma}(p)V^A(p, p)\right\}}_{\text{difference in option values in the SA contract and best-to-date WP outside option}}.
\end{aligned}$$

This expression is surprisingly simple. Notably, both the WP outside option and SA incumbent promise exactly the same schedule of option values when

[7] Note that the model is written as if the firm commits to a new flow wage yielding said value. It would be equivalent to write the firm as paying the flow value of leisure almost always and making lump sump payments equal to the innovations in the value of workers' labor market history when outside offers arrive. This second contract is a poor fit for the world we observe, but illustrates why the firm cannot only temporarily raise the wage and then revert to a lower wage when the competition is "over."

competing against passive WP competitors and in the event of an unemployment shock. Thus, the difference in option values depends *exclusively* on the difference between how the SA employer and WP best-to-date outside option would compete with SA competitors.

Inspection of the option values reveals that the WP firm offers less option value than the SA firm due to its refusal or inability to renegotiate and bid up wages in Bertrand competition. Indeed, the option value of an encounter with a SA firm when employed at a WP firm is null, since the SA firm offers exactly the reservation wage for the transition. The result is that the SA firm is always able to employ workers at wages *lower* than the wages offered by the best-to-date outside option WP firms:

$$w_{PA}(q,p) < w_{PP}(q) \text{ for all } q < \check{p} \leq p. \tag{2.3}$$

Further, a more productive SA firm offers even greater option value than a less productive one: it is able to promise a greater schedule of on-the-job raises and greater wages following job-to-job transition since it is willing to bid up wages to a larger value commensurate with its productivity. The result is that the more productive the SA firm, the larger the wage cut for any best-to-date outside option:

$$\frac{dw_{PA}(q,p)}{dp} < 0 \text{ for all } q < \check{p} \leq p. \tag{2.4}$$

Conditions 2.3 and 2.4, which govern the transitional wage schedule, w_{PA}, are all that is required to prove that a separating equilibrium is a Nash equilibrium under an appropriate cost of SA.

2.4 Cost and contract choice

If the prescribed contract choice and wage schedule are consistent with Nash equilibrium, then there must not be any deviation that increases current operating surplus for any firm. Current operating surplus for WP firms is pinned down simply as rent per worker times labor supply: $\pi^P(p) = \mathbb{E}[w|p,P]\ell(p) = [p - w_{PP}(p)]\ell(p)$. Deriving current operating surplus for SA firms requires deriving the mass of their employees earning each wage. Following the usual solution strategy, the mass flowing in and out of such wages must balance. Note that workers willing to accept wage $w_{PA}(q,p)$ (or $w_{AA}(q,p)$ if q is a SA firm) or less must have best-to-date outside option q or less. The mass flowing into such contracts will be $U\lambda_0\Gamma(q)$ and the mass flowing out must be $[\delta+\lambda_1\bar{\Gamma}(q)](M-U)L(q)$, which yields $\ell(w(q,p)|p)=\ell(q)$.[8] The current operating surplus (exclusive of the cost of SA) for a firm of type p offering the SA contract is: $\pi^A(p)=\mathbb{E}[w|p,A]\ell(p)=(p-w_{PA}(\underline{p},p))\ell(\underline{p})+\int_{\underline{p}}^{\check{p}}(p-w_{PA}(q,p))d\ell(q)+\int_{\check{p}}^{p}(p-w_{AA}(q,p))d\ell(q)$.

If $\check{p}$ is the threshold, it must be the case that the $\check{p}$-type firms are indifferent between contract types and that all less productive firms prefer WP while all more productive firms prefer SA. For threshold productivity in the interior of the support of the productivity distribution, the threshold productivity firm's willingness to pay for the right to SA is:

$$c=\pi^A(\check{p})-\pi^P(\check{p})=\{w_{PP}(\check{p})-\mathbb{E}[w|\check{p},A]\}\,\ell(\check{p}).$$

The strategies described constitute a Nash equilibrium if, when costs are equal to the threshold firm's willingness to pay, firms maximize profit by offering the prescribed contract choice and wage schedule. The proof follows from noting that 1) the profit-maximizing WP wage of any firm is an increasing function of productivity, 2) for a given labor market history, SA firms pay a lower wage than WP firms, and 3) for any given labor market history, the SA wage is decreasing in the SA firm's productivity.

That a separating equilibrium exists for every choice of the cost of SA follows from the intermediate value theorem. For a fixed set of active firms, the mapping between cost, c, and threshold, $\check{p}$, is unique when marginal wage schedule under WP, $w_{PP}(\check{p})$, increases more rapidly with respect to the change in threshold than the marginal wage schedule under SA, $w_{PA}(q,\check{p})$. This requires placing restrictions on how rapidly the value of search rises with threshold productivity. Formal conditions for this criteria are presented in the Appendix. Under these conditions the productivity type of the threshold firm is an increasing function of the cost of SA. Formal proofs are in the Appendix.

[8]Note, these results can also be found in Postel-Vinay and Robin (2002a, pg. 999-1001).

3 Changes in the composition of contracts

I compare models with mixed contracts of the type just developed when the cost of SA, c, varies. Throughout, I consider a fixed set of transition hazards, $\{\lambda_0, \lambda_1, \delta, \mu\}$, and distribution of productivity, $\Gamma(p)$.[9] The main takeaways are 1) search is more valuable to workers with poor employment histories when more firms WP, which will have implications for the implied flow value of leisure; 2) the distribution of output is more favorable to labor when more firms employ under WP; and 3) wage distributions are more disperse when more firms SA.

The second takeaway initially seems at odds with the stated need to generate a model that features WP and a large labor share. I reiterate, the problem is to develop a model that generates large labor share *and* substantial wage dispersion *at the same time*. With 2) and 3) established, I pursue the goal of matching observed wage dispersion in the section 4. The intuition that will be established in this section and utilized in that section is that the larger the share of firms employing under SA in equilibrium, the lighter the tail of the productivity distribution required to match any given level of wage dispersion.

3.1 Value of search

Refer to the value of employment in a WP firm, equation 2.2. The option value of receiving an employment offer from a more productive WP firm is positive since posted wages are not contingent on workers' labor market histories. The option value of receiving an employment offer from a SA firm behaves quite differently. Pitting the fully flexible and informed wage setting policy of SA firms against the passive wage setting strategy of WP firm generates *no increase in the value of employment* upon transition! Thus, the value of unemployment or employment in a WP contract depends only on the distribution of WP firms.

Assuming that $w_{PP}(x)$ is a differentiable function (this is verified in the

[9] As in both antecedents, both labor share and the value of leisure are larger when the right tail of the productivity distribution is lighter. Also, as in both antecedents, wage dispersion is smaller for productivity distributions with lighter right tails.

Appendix), the value of employment in a p-productivity WP firm is:

$$V^P(w_{PP}(p),p) = \frac{w_{PP}(p)}{\mu+\delta} + \frac{\lambda_1}{\mu+\delta}\int_p^{\check{p}} \frac{\Gamma(\check{p})-\Gamma(x)}{\mu+\delta+\lambda_1[\Gamma(\check{p})-\Gamma(x)]}\frac{dw_{PP}(x)}{dx} + \frac{\delta V^U}{\mu+\delta}.$$

Similarly, the value of unemployment is:

$$V^U = \frac{b}{\mu} + \frac{\lambda_0}{\mu}\int_{\underline{p}}^{\check{p}} \frac{\Gamma(\check{p})-\Gamma(x)}{\mu+\delta+\lambda_1[\Gamma(\check{p})-\Gamma(x)]}\frac{dw_{PP}(x)}{dx}$$

The value in both states is larger when more firms operate the WP contract – $V^P(w_{PP}(p))/d\check{p} > 0$ and $V^U/d\check{p} > 0$ – since, as the threshold rises, a greater share of firms yield rents and the new rent-yielding firms yield large rents. Thus, we have:

Claim 1. *For a given distribution of firms, as the cost of SA increases, the option value of unemployment rises and the flow value of unemployment consistent with equilibrium falls.*

Pure-SA has the least option value of unemployment with $\mu V^U = b$, mixed contract value of unemployment increases in the threshold, and the value of unemployment is maximized under pure-WP. Conversely, one can rank equilibria in order of the flow value of unemployment consistent with a given set of active firms. The ranking falls in the reverse order. In other words, Claim 1 states that if two economies are each described by the same distribution of firms $\Gamma(p)$, the same set of transition hazards, and *different* composition of wage contracts, it must be the case that workers in the economy with fewer SA contracts have a lower flow value of unemployment. As we will see, this is at odds with modeling the economy as containing too large a share of WP contracts: with many WP contracts and realistic wage dispersion, the flow value of unemployment must be *extremely* low.

Similar logic and calculations yield:

Claim 2. *For a given distribution of active firms, as the cost of SA increases, the value of employment with best-to-date outside option a WP firm rises as* *does the transitional wage from such firms to the SA sector.*

As we will see, an increasing share of firms operating WP contracts *significantly* impacts the support and shape of the wage distribution, shifting the support upward and increasing the skew of the distribution.

3.2 Distribution of output among factors

The model divides output into three shares: current operating surplus, payment for the right to SA, and wages. These can be interpreted in comparison to statistics from national income and product accounts: capital input, intermediate service input, and compensation of employees. Interpretation of wages as compensation of employees is direct. Interpretation of aggregate payments for the right to SA as intermediate service input is consistent with conceiving of these as payment to a human resources or legal department or a head hunter that is responsible for negotiating employment contracts at least cost. Human resources does not produce final goods; rather it insures that final goods are produced by line workers who are employed with least cost. Under the structure of cost for the right to SA modeled in this paper, the human resources department or head hunter is equally effective regardless of the number of employees it is required to manage.

Current operating surplus may be interpreted as capital input using the following logic. It is possible to micro-found the distribution of productivity on market-clearing conditions for capital input. Let there be some function $f(K) = p$, which transforms capital input K into labor productivity p with the usual conditions $f'(\cdot) > 0$ and $f"(\cdot) < 0$. The capital market is in equilibrium when all firms are indifferent between all active productive technologies: $\Pi = \pi^i(p) - rf^{-1}(p)$ for all choices of productivity $p \in [\underline{p}, \bar{p}]$ and contract $i \in \{P, A\}$, where r is the rental rate of capital. Since $\pi^i(p)$ is continuous and increasing on the intervals $[\underline{p}, \check{p})$ and $[\check{p}, \bar{p}]$ and $\pi^P(\check{p}) = \pi^A(\check{p})$, a function $f(\cdot)$ exists that is consistent with distribution of productivity $\Gamma(p)$ (or visa versa). With free entry, $\Pi = 0$ and $\pi^i(p) = rf^{-1}(p)$ for all productivities and both contract types. Here I do not explicitly micro-found the capital market. However, noting that such a market can be written, I interpret current operating surplus,

$\pi^i(p)$, as rents paid to capital, $rf^{-1}(p)$, in the un-modeled capital market.

I consider the distribution of output between capital, intermediate service, and labor for a fixed distribution of active productivity types, $\Gamma(p)$, under different costs of SA. Throughout, I assume that the distribution of productivity, $\Gamma(p)$, meets the condition for uniqueness of equilibria, and thus that threshold productivity, $\check{p}$, is increasing in the cost of SA, c.

Current operating surplus (capital input)

Claim 3. *As the cost of SA increases, the share of output paid to capital falls.*

This is straightforward to show. Consider a SA firm under an initial, low cost of SA. When costs rise the firm has two options: 1) continue to employ under SA but pay a larger share of output to intermediate service or 2) switch to WP, saving the cost of SA but paying a larger fraction of output to labor. Regardless of the firm's decision, each firm that employed under SA under the initial low cost earns strictly less current operating surplus under high costs. Meanwhile, the current operating surplus of firms that employed under WP, even under low costs, are unaffected. In aggregate, firms earn less current operating surplus under high costs of SA. Interpreting current operating surplus as payments to a capital input, as described above, gives the result.

Aggregate spending on cost of SA (intermediate service input)

Claim 4. *Aggregate spending on cost of SA (intermediate service input) is zero when all firms WP ($\check{p} = \bar{p}$), zero when all SA ($\check{p} = \underline{p}$), and maximized in some mixed-contract equilibrium.*

As the cost for the right to SA increases, some firms change wage-contracting strategies, so fewer firms pay the cost. The result arises immediately. The first follows since when all firms WP ($\check{p} = \bar{p}$), none pays any cost and aggregate payments are thus zero. The second follows since $\check{p} = \underline{p}$ when $c = 0$, all firms pay a zero cost and the aggregate payment is zero.

Compensation of employees (labor input)

Claim 5. *As the cost of SA, c, increases, the share of output paid to labor rises.*

As the cost for the right to SA increases, more firms offer WP contracts that immediately yield rent to workers. These firms, obviously, operate larger wage bills when the cost of SA increases. On the other hand, firms that employ under SA come into Bertrand competition with other SA firms less often, leading to a drop in their wage bill, since wages are lower for workers whose best-to-date outside option no longer competes via SA. However, this drop is counteracted by a simultaneous increase in the wage schedule for hiring from WP firms. This increase arises since a larger share of WP firms implies a larger value of search in unemployment and in WP firms. The second effect dominates the first. The proof, which also shows that the wage bill rises within every firm, is provided in the Appendix.

3.3 Distribution of wages

Increasing the cost associated with SA, and therefore increasing the fraction of firms that select WP, decreases the spread and increases the skew of the distribution of wages. The impact is most starkly evident in the support of the distribution. Measuring wage dispersion as the spread between $\underline{w}$ and $\bar{w}$ models can be ranked by increasing threshold: pure-SA has the most dispersion, mixed contract dispersion decreases in threshold, and pure-WP with the least dispersion.

Every SA firm offers some wages smaller than the wage offered by the least productive WP firm: $w_{PA}(\underline{p}, p) < w_{PP}(\underline{p})$ for all $p \geq \check{p}$. Every SA firm also offers some wages larger than the most productive WP firm: $w_{AA}(p, p) < w_{PP}(\check{p})$ for all p. Additionally, in all equilibria where some firms SA, both the largest *and* the smallest wages are paid by the most productive firm under the SA contract operated by this firm.

Upper bounds, $\bar{w}$, of implied wage distributions under equilibria can be ranked as follows:

- For $\check{p} < \bar{p}$ the upper bound of the implied wage distribution is equal to the upper bound of the productivity distribution: $\bar{w} = \bar{p}$.

When two $\bar{p}$ productivity SA firms meet the resulting wage is $\bar{p}$, and if such firms have positive probability, the interaction must also occur with some probability; thus the resulting wage must also have positive mass in the wage distribution.[10]

Lower bounds, $\bar{w}$, of implied wage distributions under equilibria can also be ranked as follows:

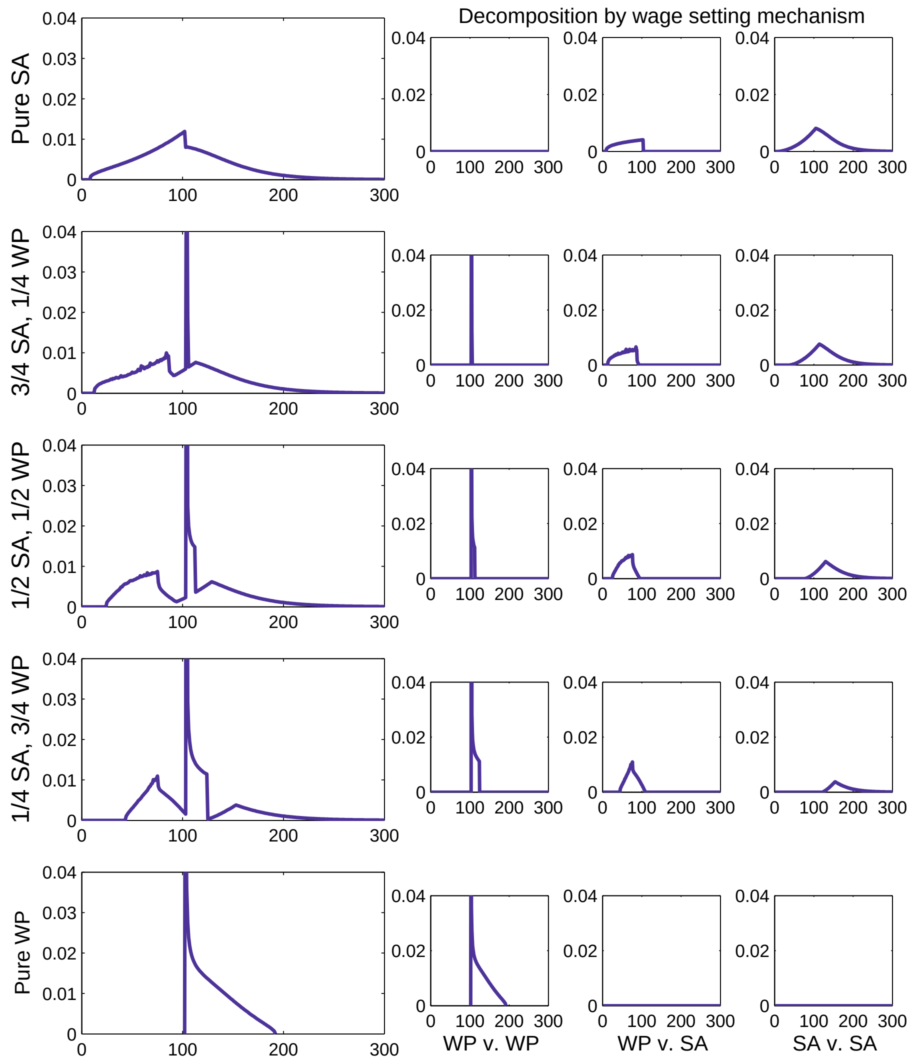
- As cost of SA, c, increases the lower bound of the wage distribution, $\underline{w} = w_{UA}(\underline{p}, \bar{p})$, increases.

Remember that the value of unemployment is increasing in the fraction of firms that hire under WP. Since entry wages in SA firms are chosen to make workers indifferent between unemployment and accepting the wage offer, entry wages in each SA firm rise: $dw_{UA}(p)/d\check{p} > 0$. This is true, in particular, for the most productive SA firm, and this firm's entry wage is the smallest wage in the economy: $\underline{w} = w_{UA}(\bar{p})$.

Expressions for the density of wages give a set of hazards $\{\lambda_0, \lambda_1, \delta, \mu\}$ and productivity distribution $\Gamma(p)$ are provided in the Appendix. Figure 1 plots wage distributions implied by a pure-SA (top), pure-WP (bottom), and mixed-contract models (middle three). The mixed contract models feature 25, 50, and 75 percent WP firms, respectively. These are decomposed into wages set under WP, wages set under SA with best-to-date outside option WP (or unemployment), and wages set after Bertrand competition between

[10]If $\check{p} = \bar{p}$ the upper bound of the implied wage distribution is strictly less than the upper bound of the productivity distribution: $w_{PP}(\bar{p}) = \bar{w} < \bar{p}$. This follows from observing that if all firms WP then the largest posted wage is strictly less than the largest productivity. This follows from the concavity of the posted wage schedule. See the appendix for the functional form of the posted wage schedule, w_{PP}, and Bontemps et al. (2000) and Mortensen (2003) for further details.

Figure 1: Wage distributions



two SA firms.

As the fraction of WP firms climbs from zero to one, the distribution becomes more compressed. Compression is localized in the left tail, where increasing value of unemployment and employment in WP firms, due to increasing size of the WP sector, puts upward pressure on entry and transitional wages in SA firms. The result is increased skew of the distribution.

4 Empirical performance

An econometrician or policy maker seeks a model that fits an observed distribution of wages and unemployment durations. The econometrician may also have auxiliary data on the relation between aggregate output and compensation of employees from national income and product accounts, and a notion of the generosity of social insurance with which the flow value of unemployment ought to be bounded. Therefore, from the econometrician's standpoint, models ought to be compared on the plausibility of the implied distribution of productivity and implied flow value of unemployment when the observed empirical distributions of wages and (un)employment durations are matched. As demonstrated in the previous section, the mixed contract model presents a tradeoff between wage dispersion on the one hand and value of leisure and labor share on the other hand. The econometrician or policy maker can harness this tradeoff and use it to select a distribution of output that is both consistent with microeconomic data on workers (wage dispersion and transition rates) and aggregate and/or policy relevant moments (labor share and leisure value).

Simulations target empirical observations for Germany in 2006. Before providing details of the data and method, one important choice should be highlighted. I target *raw* wage dispersion rather than residual wage dispersion. I acknowledged that individual characteristics account for some wage dispersion and that the ex ante identical workers who populate the model abstract from this. However, explicitly modeling worker types and, more importantly, transition between these is beyond the scope of the present work.[11]

Raw wage dispersion presents an upper bound on the inequality a model need achieve. That simulations presented here attain a good match while

[11] Consider that the usual decomposition by industry or occupation masks significant transition between these that is unmodeled. For example: does a worker who changes occupation change types and populate a different segment of the labor market or is occupational change a feature described by the job ladder? An extreme interpretation of the results presented here would be to take the view that all sectoral or occupational changes are captured as "rungs" on the job ladder. I prefer to interpret my results as an illustration of the (large) amount of dispersion that can be feasibly generated from a random search model with contract heterogeneity. Decomposition of dispersion into frictional and neoclassical components is the subject of a related active literature.

targeting raw inequality is indicative of capacity to exceed the level of residual inequality. The conclusion, contrary to some standing results in the literature, is that reasonable levels of inequality are well within the reach of a random search model even when respecting national accounting data and leisure value consistent with social insurance. Further, my results indicate that these are attainable even when a large majority of firms employ under the WP contract. Finally, as the amount of wage dispersion the model need supply decreases, the share of firms employing under the WP contract that can be consistent with the data increases.

4.1 Data

Data come from the *Sample of Integrated Labour Market Biographies (SIAB)* from the Research Data Center (FDZ) of the German Federal Employment agency at the Institute for Employment Research (IAB). The SIAB is a 2 percent sample drawn from the population of individuals in Germany who are employed and subject to social security, marginal part-time employed, a benefit recipient to the German Social Code II, official registered as a job-seeker at the German Federal Employment Agency, or participate in programs of active labor market policies.[12] I select 2006 as a relatively stable period that postdates the Hartz labor market reforms and predates the financial crisis. I calibrate the model to match labor market histories of West Germans. I select persons age 20 to 60 who were employed full time in the first 7 days of 2006.[13]

From the cross section of workers employed in the first 7 days of 2006, I compute moments of the wage distribution. Wages contained in the SIAB

data are daily wages computed as total pay during a registered employment spell divided by total days worked. Restricting the data to full-time workers

[12] See Dorner et al. (2010) for a full description of the data.

[13] I restrict the sample to West Germany to facilitate simulation. As will be noted, data are limited by censoring at the value of the maximum mandated social security contribution. This differs by region with the states that make up the former German Democratic Republic subject to lower maximum contributions. While national accounting data are available only for unified Germany, micro data on value added and wage bills indicate small differences in labor share across regions, with workers in West Germany capturing only minutely more of the surplus than in East Germany. See Doniger (2014) for further information.

Table 1: Empirical Moments

Wage distribution (2006 Euros)		Fraction of workers with ...	
Mean	98.04	Job staying	0.842
Variance	39.7	Job-to-job change	0.058
		Job change w/ unemp.	0.053
Duration (in years) of ...		Job losing	0.047
Employment	0.924	On-the-job raise	0.035
Nonemployment[a]	0.035		
Initial pay	0.906	Observations	268,721

[a] Duration of unemployment is coded to nil if no unemployment spell.

minimizes the impact of differential hours on this measure of wages. Wages in the SIAB data are censored at the maximum mandated contribution to social security, 172 Euros in 2006. Mean and variance of this distribution are presented in Table 1. 8.56 percent of wages are censored. To account for this, I match moments from censored data to moments from analogously censored simulated data.

From longitudinal data on the same sample, I compute the average duration of the initial employment spell and initial pay level, the fraction experiencing no job change, job-to-job transition, temporary job separation or persisting employment, and the fraction who experience nominal pay gain on-the-job. Job-to-job transitions are defined as transitions between employers with no greater than 7 days of nonemployment intervening and no registration of unemployment with the Employment Agency.[14] Empirical moments are presented in Table 1. Censoring biases the duration of initial pay level and incidence of nominal pay gain. Again, I address this by matching moments of censored simulated data to censored empirical moments.

In evaluating model performance, I exploit auxiliary estimates of labor share, share of firms that WP, and income replaced by social insurance. The first, labor share of 63.7% in 2006, comes from the EU KLEMS Growth and Productivity Accounts (O'Mahony and Timmer, 2009). The second, share of

[14] This definition is somewhat more restrictive than typically applied in the literature. See Doniger (2014) for a discussion of alternate definitions of pay gain and job transition and checks of robustness to these.

WP, firms is estimated as 62 percent by Brenzel et al. (2013) from a 2011 survey of German employers. To my knowledge, this is the only estimate for Germany outside of the present paper. The final, a value of non-employment of 30% – calculated as the ratio of the average unemployment benefit received in the first week of January 2006 to the average wage in the same time interval – represents a lower bound, and is likely a severe underestimate. Shimer (2005) puts the figure at 41 percent. Subsequent work in the macro-search literature suggests substantially higher values; for example, Hall and Milgrom (2008).

4.2 Estimation method

In estimation, I model productivity as distributing according to a Pareto distribution, $\Gamma(p) = 1 - (\underline{p}/p)^{\alpha}$, with unknown shape, α, and scale, $\underline{p}$. Parameters of the Pareto distribution also have convenient interpretation. The shape parameter, α, is inversely related to the density in the tail of the distribution. Meanwhile, with free entry the scale parameter pins down the reservation wage of the unemployed for employment in the WP sector: $w_{UP}(p) = \underline{p}$. The functional form assumption is not innocuous, but selection of the Pareto distribution is supported on both theoretical and empricial grounds.[15] Further, parameters $\{\lambda_0, \lambda_1, \delta, \mu, s_f\}$ can be identified without any knowledge of the distribution of firm types – up to a minor adjustment to account for censoring. Estimates of these five parameters obtained without modeling productivity do not differ meaningfully from estimates obtained when all seven parameters are estimated jointly.

Parameters to be estimated are the hazards of job offer arrival, separation, and exit; the share of firms employing under SA; and the shape and scale of the productivity distribution: $\{\lambda_0, \lambda_1, \delta, \mu, s_f, \underline{p}, \alpha\}$. I estimate these via the Simulated Generalized Method of Moments, minimizing the sum of squared percent differences between empirical moments regarding job mobility, pay

[15]On theoretical grounds, the Pareto distribution is consistent with a balanced growth path for growth driven by technology adoption, for example: Perla and Tonetti (2014); Kortum (1997); Jones (2005) and Eaton and Kortum (1999). Empirically, Gabaix (2009) give evidence that firm size distributes Pareto, and Aw et al. (2001) give evidence productivity across industries and time distributes Pareto.

changes and mean and variance of wages (summarized in table 1), and analogues in simulated labor market histories for 10,000 workers. Specifics and further data description are in the Appendix. An advantage of the simulation methods employed is the ability to be agnostic about the relation between modeled productivity and measured firm-level characteristics. All that is required for identification is that there exists some common ranking of firms such that when presented with the opportunity to move from a firm of type q to a firm of type p, all workers do so whenever $q < p$.

4.3 Results

Table 2 reports parameter estimates. I estimate that 73 percent of firms employ under the WP contract. Together with estimated hazard of on-the-job offer arrival, λ_1, of 0.21, and separation and exit hazards, δ and μ, of 0.10 and 0.01, this implies that 47 percent of workers are employed with a nonnegotiable wage contract. This estimate is broadly in line with the estimate of Brenzel et al. (2013) using survey data from 2011.[16] With these estimated parameter values the model produces labor share equal to 54 percent and a ratio of the flow value of nonemployment to the average wage of 71 percent, significantly outperforming the nested pure contract equilibria.

Recelebrating the model such that hazard parameters are unchanged but all firms are constrained to SA produces labor share in this range but a value of leisure that is nearly as large as the average wage. Meanwhile, repeating

the same exercise but constraining all firms to WP produces a labor share of only 23 and a large and negative value of leisure. Recall, that the EU-KLEMS database reports German labor share as 63.7 in 2006. My own estimates suggest a value of leisure well in excess of 30 percent of average wages, while

[16]Note that the model implies a strict ranking: the share of firms with WP is always less than the share of workers with WP (except in the pure-WP equilibrium). Survey data summarized in the introduction appears to contradict. Note, however: 1) surveys are not conducted contemporaneously and the two U..S. surveys are not designed for comparison. 2) As Hall and Krueger (2012) note, even if a worker is employed in a negotiable contract it is not necessary that actual negotiations have occurred and thus no negotiation may be reported. Unfortunately, no survey asks a prospective question about negotiating to employees.

Table 2: Estimated Parameters

Share of WP firms :	s_f	0.727	(0.067)
Unemployed offer arrival :	λ_0	1.783	(0.472)
Employed offer arrival :	λ_1	0.213	(0.002)
Job losing :	δ	0.099	(0.006)
Quit to nonparticipation :	μ	0.010	(0.005)
Reservation productivity :	R	95.1	(0.702)
Shape of prod. dist. :	α	2.610	(0.143)
	$k_0 = \lambda_0/(\delta + \mu)$	16.34	
	$k_1 = \lambda_1/(\delta + \mu)$	1.95	
Share of WP workers :	$s_w = s_f/[1 + k_1(1 - s_f)]$	0.474	

Note: Standard errors in parentheses.

Shimer (2005) suggests a value closer to 40 percent and Hall and Milgrom (2008) suggest a value in excess of 70 percent (both for the United States).[17] For robustness, I repeat this exercise an additional time constraining the share of WP firms to 62 percent as estimated by Brenzel et al. (2013). Constraining the model to this share of WP firms requires a productivity distribution with a lighter tail and results in a slightly higher labor share, 58 percent, and a larger ratio of the flow value of leisure to average wages, 84 percent.

These relations persist broadly over the range of shares of WP firms. Table 3 records the scale and shape parameters required to achieve wage dispersion comparable to that observed for a range of possible shares of WP

contracts while constraining hazards to the estimated values. The table also presents the implied cost of the SA contract, labor share, and ratio of the flow value of leisure to average wages. Costs are expressed as percent of aggregate and as percent of output in the threshold firm. That costs are increasing and take an increasingly large share of the threshold firm's output when the

[17] I focus on West Germany in order to abstract from differences in East and West German wage scales that are not addressed by the model. All statistics other than labor share reflect the focus on West Germany. Survey evidence analyzed in Doniger (2014) suggests that labor share is not substantially different in the two regions. 30 percent replacement rate reflects the ratio of the average unemployment benefits to average wages paid in the first week of January 2006. Due to selection on who becomes unemployed and nonpecuniary value of leisure, this 30 percent forms a very cautious lower bound.

Table 3: Labor share and ratio of flow value of leisure to mean wages under differing compositions of contracts when targeting moments of the wage distribution and labor flows.

% firms Utilizing WP	Scale	Shape	Cost of SA: *% of output in $\check{p}$ firm*	Cost of SA: *% of aggregate output*	Labor Share	Ratio: flow value nonemp. to mean wage
0	100.96	3.24	0	0	0.557	0.996
10	100.95	3.24	5.71	1.17	0.563	0.995
20	100.79	3.24	10.61	2.32	0.572	0.992
30	100.47	3.24	14.95	3.48	0.580	0.981
40	99.93	3.20	18.64	4.61	0.585	0.961
50	99.09	3.12	21.79	5.65	0.587	0.923
60	97.79	2.95	24.5	6.55	0.578	0.861
70	95.85	2.67	27.01	7.2	0.553	0.752
80	92.90	2.17	29.9	7.42	0.479	0.539
90	79.84	1.64	29.15	6.16	0.363	0.096
100	49.87	1.29	100	0	0.295	-0.755

threshold rises is straightforward.[18]

As the portion employing under wage posting approaches 100%, the required distribution of productivity has an increasingly heavy tail (the shape parameter falls substantially). Implausibility of such a heavy tail is captured by implausibly low labor share, also documented in Table 3. In contrast, the tail remains quite light for compositions of contracts up to seventy percent WP. The difficulty is not the presence of WP, rather it is the presence of WP in the tail if the distribution.

Table 3 also records the ratio of the value of leisure to the average wage. When nearly all firms select WP, the best-fit simulation is unable to produce substantial value of leisure. The difficulty arises from the supposition that

[18]The magnitude of costs is sensitive to the choice of the size of the mass of workers, M, as compared to the size of the mass of firms, N. In the simulation the ratio is calibrated as 1. A larger ratio would produce larger firm sizes and thus larger aggregate costs. However, a larger ratio also entails larger output. In simulations the fraction of output that must be paid to service the cost of SA never exceeds eight percent.

starting wages may be drawn from the full empirical distribution, resulting in an implausibly large option value of search for the unemployed. Inclusion of a SA sector mitigates this problem: large wages arise only after on-the-job search and competition for a worker by two firms employing under SA. The result is that the simulated distribution of posted wages is more compressed, resulting in a more reasonably sized option value of search while unemployed.

The ratio of the flow value of leisure to average wages is problematic for both extremes, largely WP and largely SA. In the largely WP case, the problem is again the presence of WP in the tail: the possibility of receiving such wage offers out of unemployment coupled with short unemployment spells contradict a large flow value of unemployment. In the other extreme, when many firms employ under SA the problem stems from the full rent extraction assumed. With flow value of leisure very near to or higher than the average wage, the question arises: why are the unemployed searching for work at all? Inclusion of the WP sector supplies a search incentive, and for intermediate mixtures the value of leisure is a (large) proportion of the average wage.[19]

In sum, the calibration demonstrates that the mixed contract model can attain labor share near the observed value and reasonable value of the flow value of unemployment when matching data on employment transitions, incidence of wage increases, and on *raw* wage dispersion. Further, this is possible for homogeneous workers with null bargaining power and linear preferences, suggesting that this model, as well as more nuanced versions, are capable of matching any level of residual wage dispersion, in particular whatever portion for which search frictions are accountable.

5 Conclusion

This paper offers a model of on-the-job search in which firms select between hiring under a WP and a SA contract. The mechanism described, a fixed per-firm cost for the right to hire under SA, provides sharp predictions. This simple cost structure enables me to characterize the equilibrium under any

[19] An alternate and complementary solution would be to model workers with preferences that reduce the option value of search: increased discounting and/or risk aversion.

continuous distribution of productivity and for optimal WP strategies. The equilibrium described is separating, with some intermediate firms being indifferent between contract types, all less productive firms selecting WP, and all more productive firms selecting SA. The model featuring both contract types is able to match key policy relevant features of the data – wage dispersion, labor share, employment transitions, and flow value of unemployment – at the same time as most firms are bound by information frictions that may yield policy relevant search externalities.

Optimal contracting in the context of on-the-job search is an active literature important in its own right, independent of questions of goodness of fit.[20] A key insight, is that different contracts provide differential search and transition incentives to employees. Matching outside offers rewards and incents search on-the-job. On the other hand, failure to match leads to potentially preventable turnover. The literature on competing pricing mechanisms is also active.[21] The literature regarding labor search with heterogeneous wage setting, however, is small. This author is aware of four examples: Ellingsen and Rosén (2003); Michelacci and Suarez (2006); Postel-Vinay and Robin (2004); and Holzner (2011)

The model presented here is most closely related to Postel-Vinay and Robin (2004) and Holzner (2011). Each pursues the question of optimal contract type in a similar environment and with similar contracting alternatives as considered here. An important distinction, however, is that each explicitly models workers' search incentive and the implication of firms' contract choices for search intensity and turnover. The intuition pursued in these papers is that SA rewards workers for on-the-job search and thus might provide greater incentive for search effort.

This argument is clearly true at the level of an individual firm – the complication is to find a setting in which differential incentive for search effort is global, with search always being more attractive in the SA sector than the WP sector. This ranking is not easy to obtain. To gain a handle on the prob-

[20]Some prominent and some newer contributions: Stevens (2004), Postel-Vinay and Robin (2004), Burdett and Coles (2010), Holzner (2011), and Lentz (2014)

[21]A non-exhaustive list of examples: Bester (1993), Arnold and Lippman (1998), and Camera and Delacroix (2004). See Peters (2013) for a summary.

lem, Postel-Vinay and Robin (2004) and Holzner (2011) each simplify some elements of the setting. Postel-Vinay and Robin (2004) require all WP firms to post identical wages, thus eliminating search incentive entirely in the WP sector. Holzner (2011) restricts the set of firm types to simply a high type and a low type. These restrictions have undesirable consequences for the implied wage distribution and associated search incentives of the unemployed.

Compared with its nearest neighbors, Postel-Vinay and Robin (2004) and Holzner (2011), the model presented here accepts general productivity distributions and allows for a WP sector in which wages are optimally set by firms and, more importantly, in which WP wages are disperse. Further, my model nests the continuous productivity case of the pure-WP with on-the-job search (Bontemps et al., 2000) and the pure-SA model (Postel-Vinay and Robin, 2002b) as limiting cases. These results are achieved by abstracting from the micro-foundations of the cost structure and instead directly exploiting a single, separation-driving mechanism: the differential in wage bills for like-firms employing under different wage contracts.

The cost structure and the mechanism that it isolates could be extended to include more complex contract types. Particularly interesting candidates are tenure-contingent contracts. In principle, as long as flows between sectors of the labor market are constrained efficient, it should be possible to find a similar separating equilibrium even with tenure-contingent contracts. The equilibrium plausibly features three sectors, with the least productive firms populating the WP sector, moderately productive firms offering tenure-contingent contracts, and the most productive firms offering fully contingent SA contracts. I leave full exposition of such an equilibrium to future work.

The model presented here suggests additional insights. First, flexible contracts allocate a greater share of surplus to firms and induce greater dispersion in outcomes for workers. These results suggest a line of inquiry into trends in both labor share and inequality experienced in developed countries in the last several decades. Second, the equilibrium described here interacts with workers' on-the-job search in an interesting way: as workers gain employment experience, on-the-job pay gain becomes an increasingly important source of wage growth as compared to job-to-job transition. This result is consistent with empirical evidence – for example Topel and Ward (1992) – but suggests a new interpretation of patterns in job mobility and wages over the life cycle.

References

Arnold, M. A. and S. A. Lippman (1998): "Posted Prices versus Bargaining in Markets with Asymmetric Information," *Economic Inquiry, Western Economic Association International*, 36, 450–57.

Aw, B. Y., X. Chen, and M. J. Roberts (2001): "Firm-Level Evidence on Productivity Differentials and Turnover in Taiwanese Manufacturing," *Journal of Development Economics*, 66, 51 –86.

Barron, J. M., M. C. Berger, and D. A. Black (2006): "Selective Counteroffers," *Journal of Labor Economics*, 24, 385–410.

Bester, H. (1993): "Bargaining versus Price Competition in Markets with Quality Uncertainty," *The American Economic Review*, 83, 278–88.

Bontemps, C., J.-M. Robin, and G. J. van den Berg (2000): "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation," *International Economic Review*, 41, 305–58.

Brenzel, H., H. Gartner, and C. Schnabel (2013): "Wage Posting or Wage Bargaining? Evidence from the Employers' Side," IZA Discussion Papers 7624, Institute for the Study of Labor (IZA).

Burdett, K. and M. Coles (2010): "Wage/Tenure Contracts with Heterogeneous Firms," *Journal of Economic Theory*, 145, 1408–35.

Burdett, K. and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39, 257–73.

Camera, G. and A. Delacroix (2004): "Trade mechanism selection in markets with frictions," *Review of Economic Dynamics*, 7, 851–68.

Diamond, P. A. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3, 156–68.

Doniger, C. L. (2014): "Labor Search, Inequality, and Public Policy: Chapter 3: Empirical identification of the composition of contracts in an on-the-job search equilibrium with heterogeneous wage contracts using administrative," Ph.D. thesis, University of Michigan.

Dorner, M., J. Heining, P. Jacobebbinghaus, and S. Seth (2010): "Sample of Integrated Labour Market Biographies (SIAB) 1975-2008," *FDZ - Datenreport*, 01/2010 (en), Nürnberg.

Eaton, J. and S. Kortum (1999): "International Technology Diffusion: Theory and Measurement," *International Economic Review*, 40, 537–70.

Ellingsen, T. and Å. Rosén (2003): "Fixed or Flexible? Wage-Setting in Search Equilibrium," *Economica*, 70, 233–50.

Gabaix, X. (2009): "Power Laws In Economics and Finance," *Annual Review of Economics*, 1, 255–94.

Hall, R. E. and A. B. Krueger (2012): "Evidence on the Incidence of Wage Posting, Wage Bargaining, and On-the-Job Search," *American Economic Journal: Macroeconomics*, 4, 56–67.

Hall, R. E. and P. R. Milgrom (2008): "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 98, 1653–74.

Hochfellner, D., J. Montes, M. Schmalz, and D. Sosyura (????): "Winners and Losers of Financial Crises: Evidence from Individuals and Firms," .

Holzner, C. (2011): "Optimal Wage Policy with Endogenous Search Intensity," Macro.economics.uni-mainz.de.

Hornstein, A., P. Krusell, and G. L. Violante (2011): "Frictional Wage Dispersion in Search Models: A Quantitative Assessment," *American Economic Review*, 101, 2873–98.

Jones, C. I. (2005): "The Shape of Production Functions and the Direction of Technical Change," *The Quarterly Journal of Economics*, 120, 517–49.

Kortum, S. S. (1997): "Research, Patenting, and Technological Change," *Econometrica*, 65, 1389–1420.

Lentz, R. (2014): "Optimal Employment Contracts with Hidden Search," NBER Working Papers 19988, National Bureau of Economic Research, Inc.

Michelacci, C. and J. Suarez (2006): "Incomplete Wage Posting," *Journal of Political Economy*, 114, 1098–123.

Mortensen, D. T. (2003): *Wage Dispersion: Why are Similar Workers Paid Differently?*, Cambridge, MA: The MIT Press.

O'Mahony, M. and M. P. Timmer (2009): "Output, Input and Productivity Measures at the Industry Level: Rhe EU KLEMS Database," *Economic Journal*, 119, F374–F403.

Perla, J. and C. Tonetti (2014): "Equilibrium Imitation and Growth," *Journal of Political Economy*, 122, pp. 52–76.

Peters, M. (2013): "Competing Mechanisms," in *The Handbook of Market Design*, ed. by N. Vulkan, A. E. Roth, and Z. Neeman, Oxford University Press, chap. 24.

Postel-Vinay, F. and J.-M. Robin (2002a): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, 2295–350.

——— (2002b): "The Distribution of Earnings in an Equilibrium Search Model with State-Dependent Offers and Counteroffers," *International Economic Review*, 43, 989–1016.

——— (2004): "To Match or Not to Match? Optimal Wage Policy With Endogenous Worker Search Intensity," *Review of Economic Dynamics*, 7, 297–330.

Rothschild, M. (1973): "Models of Market Organization with Imperfect Information: A Survey," *Journal of Political Economy*, 81, 1283–1308.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *The American Economic Review*, 95, 25–49.

Stevens, M. (2004): "Wage-Tenure Contracts in a Frictional Labour Market: Firms' Strategies for Recruitment and Retention," *Review of Economic Studies*, 71, 535–51.

Topel, R. H. and M. P. Ward (1992): "Job Mobility and the Careers of Young Men," *The Quarterly Journal of Economics*, 107, 439–79.

A Proofs and Derivations

A.1 Further details of the proof of Proposition 1

A.1.1 Constrained efficient labor flows

The largest possible WP wage is, at most, equal to the productivity of the threshold firm. In the proposed separating equilibrium, the least productive SA firm has larger productivity than the most productive WP firm. So, each SA firm can profitably pay a wage at least as large as the largest possible WP wage and the SA contracting mechanism permits the firm to offer such a wage. Therefore, every SA firm must hire employees of every WP firm, if it meets them, in equilibrium, and the flow between sectors is efficient.[22]

Within the WP sector, firms continue to behave as if the whole economy posted wages, and so flows are efficient. The presence of SA firms does not alter the solution to the WP firms' maximization problem since 1) profitable wage choices are only competitive against other WP firms and 2) there is no mass of WP firms at the threshold productivity, since $\Gamma(p)$ is continuous. The proof due to Burdett and Mortensen (1998, pg.268) applies: more productive firms can employ workers of less productive firms at trivially greater wages and at greater profits. Finally, within the SA sector, more productive firms are still able to outbid less productive firms and so the flows are efficient. In total, flows of employed workers in the separating equilibrium are efficient.

A.1.2 Separating Nash equilibrium for $\{c, \check{p}\}$

The proposed separating equilibrium is a Nash equilibrium of the labor market if each firm prefers the prescribed wage contract and wage schedule conditional on all other firms playing the assigned contract and wage schedule and labor flowing toward more productive firms. To prove this, I must show that current operating surplus from the proposed strategies exceed current operating surplus from each firm's best deviation.

Suppose WP is prescribed: A firm for which WP is prescribed must have $p < \check{p}$.

[22] We will see that, in equilibrium, the SA firm is actually able to hire these workers for less than their best-to-date posted wage. The reason is that the SA firm compensates its workers partially through the option value of contingent pay.

For the p-productivity firm, current operating surplus from playing optimal wage under the prescribed wage contract, WP, and the best deviation to SA can be written as

$$
\begin{aligned}
\pi^P(p) =& [p - w_{PP}(p)]\ell(p) \\
&\text{and} \\
\pi^{BD}(p) =& [p - w_{PA}(\underline{p}, p)]\ell(\underline{p}) + \int_{\underline{p}}^{\grave{p}} [p - w_{PA}(q,p)] d\ell(q) - c(\check{p})
\end{aligned}
$$

where $\grave{p}$ is the productivity of the most productive firm that offers a posted wage less than p (e.g., the most productive firm that the p-type firm can outbid by switching to SA).

Simplifying,

$$
\begin{aligned}
\pi^{BD}(p) =& [p - w_{PP}(\check{p}) + \underbrace{w_{PA}(\underline{p}, \check{p}) - w_{PA}(\underline{p}, p)}_{<0, \text{ since } \frac{dw_{PA}(\underline{p},p)}{dp} < 0}]\ell(\underline{p}) \\
& + \int_{\underline{p}}^{\grave{p}} [p - w_{PP}(\check{p}) + \underbrace{w_{PA}(q, \check{p}) - w_{PA}(q, p)}_{<0, \text{ since } \frac{dw_{PA}(q,p)}{dp} < 0}] d\ell(q) \\
& - \int_{p}^{\check{p}} [\underbrace{w_{PP}(\check{p}) - w_{PA}(q, \check{p})}_{\geq 0}] d\ell(q) \\
<& [p - w_{PP}(\check{p})]\ell(\grave{p}) \leq \pi^P(p).
\end{aligned}
$$

The last line follows from noting $w_{PP}(p)$ was the unique profit-maximizing posted wage choice for the p-type firm.

In other words, the WP firm could increase its labor supply by deviating to SA. However, the firm could also increase its labor supply by the same amount by deviating to a larger posted wage. Willingness to pay for the right to SA is then strictly less than the difference between the wage bill under the deviation to SA and the deviation to a higher posted wage, which in turn is strictly less than the cost of SA.

Suppose SA is prescribed: A firm for which SA is prescribed must have $\check{p} \leq p$. For the p-productivity firm, current operating surplus from playing the prescribed

SA wage schedule and deviating to the best posted wage are

$$
\begin{aligned}
\pi^A(p) =& [p - w_{PA}(\underline{p}, p)]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - w_{PA}(q, p)] d\ell(q) \\
&+ \int_{\check{p}}^{p} [p - w_{AA}(q, p)] d\ell(q) - c(\check{p}) \\
&\text{and} \\
\pi^{BD}(p) =& [p - \grave{w}]\ell(\grave{w}).
\end{aligned}
$$

Note that $\grave{w} \geq w_{PP}(\check{p})$ since $p \geq \check{p}$. Simplifying,

$$
\begin{aligned}
\pi^{BD}(p) =& [p - \grave{w}]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - \grave{w}] d\ell(q) + \int_{\check{p}}^{\grave{w}} [p - \grave{w}] d\ell(q) \\
<& [p - \underbrace{w_{PP}(\check{p})}_{\leq \grave{w}}]\ell(\underline{p}) + \int_{\underline{p}}^{\check{p}} [p - \underbrace{w_{PP}(\check{p})}_{\leq \grave{w}}] d\ell(q) + \int_{\check{p}}^{\grave{w}} [p - \underbrace{w_{AA}(q, p)}_{< \grave{w}}] d\ell(q) \\
<& [p - w_{PA}(\underline{p}, p) - \underbrace{w_{PP}(\check{p}) + w_{PA}(\underline{p}, \check{p})}_{>0, \text{ since } \frac{dw_{PA}(\underline{p},p)}{dp} < 0}]\ell(\underline{p}) \\
&+ \int_{\underline{p}}^{\check{p}} [p - w_{PA}(q, p) - \underbrace{w_{PP}(\check{p}) + w_{PA}(q, \check{p})}_{>0, \text{ since } \frac{dw_{PA}(q,p)}{dp} < 0}] d\ell(q) \\
&+ \int_{\check{p}}^{p} [p - w_{AA}(q, p)] d\ell(q) < \pi^A(p).
\end{aligned}
$$

The best deviation to WP involves a reduction in the SA firm's labor supply. I can find a bound on the minimum willingness to pay for the right to SA by considering only the labor supply that would arise under the *smallest possible* best deviation the SA firm might select: $w_{PP}(\check{p})$. Willingness to pay for the right to SA is then larger than the difference between the wage bill under the deviation to WP and the wage bill for these employees under the prescribed SA contract, which in turn is strictly greater than the cost of SA.

Since no firm wishes to unilaterally deviate, the pair $\{c, \check{p}\}$ form a Nash equilibrium.

A.1.3 Existence of $\check{p}$ for any c.

Since $\check{p}$ was chosen arbitrarily, c is defined for any possible threshold in the support of Γ.

First, consider $\Gamma(p)$ with finite support $[\underline{p}, \bar{p}]$. To show that for every cost, c, there exists a threshold, $\check{p}$, I must first extend the definition of c to include the boundaries of the support of $\Gamma(p)$.

- $c(\underline{p}) = (-\infty, 0]$ (when SA is subsidized or free, all firms select SA).
- $c(\bar{p}) \supset [\ell(\bar{p})\bar{p}, \infty)$ (if the cost of SA exceeds the output of the most productive firm, $\ell(\bar{p})\bar{p}$, then no firm selects SA).

So, c is upper hemicontinuous on support $[\underline{p}, \bar{p}]$ and continuous on support $(\underline{p}, \bar{p})$. The intermediate value theorem implies that there exists at least one threshold for every cost.

The result can be generalized to Γ with infinite upper support by considering the limit as $\bar{p} \to \infty$: for every $\bar{p}$ there exists a $c = \ell(\bar{p})\bar{p}$ such that all firms SA.

A.1.4 Uniqueness of equilibria

Equilibrium is unique if $[(w_{PP}(\check{p}) - \underline{p})k_1]^{-1} \geq d\Gamma(\check{p})$ for all $\check{p}$. This condition requires that the distribution of productivity be "thin enough" everywhere in the tail that the shift $\frac{dw_{PA}(q,\check{p})}{d\check{p}}$ due to indirect upward pressure on schedules $w_{PA}(q, p)$ from the now larger WP sector is dominated by the direct downward pressure on the schedule in the marginal firm due to the now larger productivity of the marginal firm. Proof, which stems from differentiating the marginal wage schedule, is available upon request.

This guarantees that $\frac{dc}{d\check{p}}$ is increasing for all $\check{p}$ in the interior of the support of $\Gamma(p)$ and the mapping from $\check{p}$ to c is one-to-one.

A.2 Wage schedules

SA vs. SA

If the best-to-date outside offer originates from a SA firm the problem is analogous to that considered by Postel-Vinay and Robin (2002b), since all firms with productivity greater or equal to the best-to-date outside option also SA. The least productive firm that is able make a credible threat to hire the worker is a SA firm, and it is the firm that is able to offer a value equal to $V^A(w_{AA}(q,p),p)$ with a wage offer of q.

Postel-Vinay and Robin (2002b) observe that $V^A(q,q) = (q + \delta V^U)/(\delta + \mu)$. Equating $V^A(q,q)$ and $V^A(w_{AA}(q,p),p)$ identifies the reservation wage for accepting a job at the p-productivity SA employer when the best-to-date outside option is a q-productivity SA employer:

$$w_{AA}(q,p) = q - k_1 \int_q^p \bar{\Gamma}(x)dx \qquad \text{for } \check{p} < q \leq p, \tag{A.1}$$

where $k_1 = \lambda_1/(\mu + \delta)$ is the expected *discounted* number of job offers per employment spell.[23] This pins down the portion of the optimal wage schedule for a p-productivity SA firm and the best-to-date outside option a q-productivity SA firm: when $\check{p} \leq q \leq p$.

WP vs. WP

In the equilibrium, if the p-productivity firm selects WP then it must be the case that all less productive firms also select WP. The problem facing the firm is thus very similar to the problem considered by Bontemps et al. (2000). The optimal WP wage offer, $w_{PP}(p)$, maximizes the expected profit from the WP contract:

$$\pi^P(p) = \underbrace{[p - w_{PP}(p)]}_{\text{rent per worker}} \underbrace{\ell(p)}_{\text{labor supply}} . \tag{A.2}$$

Bontemps et al. (2000) show that when all firms WP and make optimal wage choices, constrained efficiency of labor flows arrives and labor supply is pinned down by firms' productivity type. The intuition is that more productive firms prefer to post higher wages, and continuity of $\Gamma(p)$ yields the required one-to-one mapping between $w_{PP}(p)$ and p.[24] To see that the result extends to the separating equilibrium, I must verify that the mapping remains one-to-one in the presence of (more productive) SA firms. All SA firms can outbid the highest profitable posted wage of the most productive WP firm and the measure of $\check{p}$ WP firms is zero since $\Gamma(p)$ is continuous.

WP firms' maximization problem can be solved by applying the envelope theo-

[23] Note that $k \to \kappa$ when $\mu \to 0$.

[24] See Burdett and Mortensen (1998) and Bontemps et al. (2000) for details and proofs of these results.

rem and solving the implied differential equation:

$$w_{PP}(p) = p - [1 + \kappa_1 \bar{\Gamma}(p)]^2 \int_{w_{UP}}^{p} [1 + \kappa_1 \bar{\Gamma}(x)]^{-2} dx \qquad \text{for } p < \check{p}, \tag{A.3}$$

where w_{UP} is the reservation wage of a worker to accept a job at a WP firm from unemployment.

SA vs. WP

Returning to optimal wages for workers transitioning between contract types, what remains is to solve for the value of employment in a WP firm. Integration of equation 2.2 by parts gives

$$V^P(w_{PP}(q)) = \frac{w_{PP}(q)}{\mu+\delta} + \frac{1}{\mu+\delta} \int_q^{\check{p}} \frac{k_1[\Gamma(\check{p}) - \Gamma(x)]}{1 + k_1[\Gamma(\check{p}) - \Gamma(x)]} \frac{dw_{PP}(x)}{dx} + \frac{\delta V^U}{\mu+\delta}.$$

Plugging in V^P and V^C appropriately and manipulating gives the result:

$$\begin{aligned} w_{PA}(q,p) = w_{PP}(q) - k_1 \bar{\Gamma}(\check{p}) \big[\check{p} - w_{PP}(q) - \int_q^{\check{p}} \tfrac{k_1[\Gamma(\check{p})-\Gamma(x)]}{1+k_1[\Gamma(\check{p})-\Gamma(x)]} \tfrac{dw_{PP}(x)}{dx}\big] \\ - k_1 \int_{\check{p}}^{p} \bar{\Gamma}(x) dx \text{ for } q \le \check{p} \le p. \end{aligned} \tag{A.4}$$

Hiring out of unemployment

Finally, the reservation wages for entering employment at a WP firm or a SA firm of type-p are pinned down as the wages equate the value of unemployment to the value of employment in the relevant firm. The value of unemployment (when all firms follow the proposed strategies) can be written as:

$$w_{UP} = b + (k_0 - k_1) \int_{w_{UP}}^{\check{p}} \frac{[\Gamma(\check{p}) - \Gamma(x)]}{1 + k_1[\Gamma(\check{p}) - \Gamma(x)]} \frac{dw_{PP}(x)}{dx}. \tag{A.5}$$

Meanwhile, when entering employment in a SA firm, reservation wages depend on the SA firm's productivity, p. However, since the value of unemployment equals the value of employment in the least productive WP firm, $V^U = V^P(\underline{p})$, and the SA firm selects the wage to match the value of the best-to-date outside option, the schedule of reservation wages is equal to the wage required to hire from the least productive firm: $w_{UC}(p) = w_{PC}(\underline{p}, p)$.

A.3 Proof of Claim 5

The claim is proved if every firm's wage bill weakly rises. Consider a small increase in the costs of SA. Firms are of three types: always WP, switch from SA to WP, always SA. Wage bills for always WP firms are clearly unaffected by the change in threshold productivity induced by the change in cost (note that I am considering an identical set of active firms). Switching firms strictly increases their wage bill, as before the cost increase they paid for the right to SA, but after the cost increase they prefer to pay larger WP wage bills to evade the higher cost of SA. The third category of firms requires heavier lifting.

First, note that the wages of workers with best-to-date outside option a WP are set under schedule $w_{PA}(q,p)$ in each p-productivity SA firm and that the mass of such workers in each firm is $\ell(\check{p})$. Also, note that $\frac{d^2 w_{PA}(q,p)}{dq d\check{p}} < 0$. So the change in the wage bill associated with these workers when $\check{p}$ rises is at least

$$\frac{dw_{PA}(\check{p},p)}{d\check{p}} = [1 + k_1\bar{\Gamma}(\check{p})]\frac{dw_{PP}(\check{p})}{d\check{p}} + k_1 d\Gamma(\check{p})[\check{p} - w_{PP}(\check{p})] > 0.$$

Meanwhile, each p-productivity firm can lower the wage paid to employees whose best-to-date outside option was a SA firm before the cost increase and is now a WP firm. This reduces the p-productivity firms wage bill by $d\ell(\check{p})[\check{p} - w_{PP}(\check{p})]$.

The increase in the schedule of wages for workers with best-to-date outside option a WP firm dominates, which is made explicit by noting that $\frac{dw_{PP}(\check{p})}{d\check{p}} = \frac{2k_1 d\Gamma(\check{p})}{1+k_1\bar{\Gamma}(\check{p})}[\check{p} - w_{PP}(\check{p})]$ and $d\ell(\check{p}) = \frac{2k_1 d\Gamma(\check{p})}{1+k_1\bar{\Gamma}(\check{p})}\ell(\check{p})$.

Since the wage bill for every productivity firm weakly rises, the total wage bill rises.

A.4 Wage distributions

The distribution of wages can be derived by aggregating across firms within sectors and then by aggregating across sectors.

Distribution of wages within firms

Within WP firms the distribution of wages is a mass at the posted wage. For SA firms, however, wages are disperse. Following the discussion in section 2.4 and Postel-Vinay and Robin (2002a, pg. 999-1001), the distribution of wages within SA firms can be expressed as a function of the distribution of employees best-to-date outside options:

$$G(w|p) = \frac{\ell(q(w,p))}{\ell(p)} = \begin{cases} \mathbb{1}_{w \geq w_{PP}(p)}, & \text{if } p < \check{p} \\ \left[\frac{1+\kappa_1[1-\Gamma(p)]}{1+\kappa_1[1-\Gamma(q(w,p))]}\right]^2, & \text{if } \check{p} \leq p \end{cases}$$

Aggregate distribution of wages

Within the WP sector, the wage distribution can be expressed as:

$$G(w|WP) = L(q(w_{PP}))$$

where $q(w_{PP})$ is the productivity of the firm that optimally posts wage w_{PP}. Meanwhile, within the SA sector, the wage distribution can be expressed as:

$$G(w|SA) = \int_{w(\underline{p},\bar{p})}^{w} G(w|p)dL(p) + L(\underline{p})G(w|\underline{p}).$$

Summing the two gives the aggregate wage distribution.

B Estimation strategy

B.1 Identifying Assumption

I deliberately avoid use of proxies for establishment quality in estimation of parameters since these are, at best, measured with significant noise. The estimation strategy employed relies only on transition and duration data, which is recorded with high fidelity in the German register. Identification requires one assumption:

Assumption 1. *There exists some ranking of establishments such that if $p' > p$ then a worker given the opportunity to work for a firm with ranking p or p' always chooses to work for the firm with ranking p'.*

This assumption provides a more hands-off approach to the issue of measuring firm quality; however, it is still reasonably restrictive. Identification requires that some ranking of firm quality exists, is agreed upon by all observed workers, and that workers climb the job-ladder described by this ranking at every available opportunity. These assumptions preclude explicit wage-tenure contracts and heterogeneous preferences.

B.2 Moments

Estimation targets means of the following variables:

d_{ei}	*average duration of the initial employment spell*
d_{pi}	*average duration of the initial pay level*
$jcen_i$	*fraction with first employment spell censored*
jtj_i	*fraction with job-to-job transition ends first employment spell*
jn_i	*fraction with nonemployment ends first employment spell*
r_i	*fraction with at least one positive wage change in the initial employment spell*
$\mathbb{E}[w]$	*mean wages*
$\mathbb{V}[w]$	*wage variance*

Constructing these variables requires a definition of what constitutes a job-to-job transition and what constitutes a pay-gain. For the primary analysis, I define a job-to-job transition as any change of employer with 7 days or fewer of intervening nonemployment and no overlapping registration as unemployed with the Employment Agency. An on-the-job pay gain is defined as a mid-year pay change that results in a nominal increase in average daily pay.

B.3 Estimation procedure

I estimate the model by minimizing the sum of squared errors between simulated and empirical moments. Simulations contain a cross-section of 10,000 simulated employment histories. Workers' simulated initial firm is drawn from the theoretical steady state distribution of workers across firms given some parameter choice for λ_1 and δ:

$$L(p) = \Gamma(p)/(1 + \kappa_1 \bar{\Gamma}(p)).$$

Workers' best-to-date outside option is drawn from the theoretical steady state distribution of outside offers given incumbent employer type:

$$G(q|p) = \frac{\ell(q)}{\ell(p)} = \left(\frac{1 + k_1[1 - \Gamma(q)]}{1 + k_1[1 - \Gamma(p)]} \right)^2.$$

One year (365 days) of employment history is then simulated for each worker. Each day, each worker may receive a new job opportunity, or may become separated from her employer. These events occur with Poisson probabilities λ_1 and δ to be estimated.[25]

If the worker receives a new job opportunity, it may come from a more productive firm than the incumbent – with $[1 - \Gamma(p)]$ probability – or a more productive firm than the best-to-date outside option – with $[1 - \Gamma(q)]$ probability. Note again that the functional form of Γ is irrelevant so long as it is a differentiable *c.d.f.* In the first case, I record a job-to-job transition. If the second case but not the first holds – if the new draw quality is between q and p – the worker may experience an on-the-job raise; however, the raise occurs only if incumbent employer quality is above some threshold, $\check{p}$. Note that cardinal value of the threshold, $\check{p}$, is not substantive. $s_F \in [0, 1]$ is the share of employers with quality greater than this threshold. $s_w = \delta s_F/(\delta + \lambda_1[1 - s_F])$ is the parameter of interest, which is to be estimated.

In estimation, I simulate moments and match these to empirical moments. Simulation allows me to integrate out the impact of firm quality on transition and pay change, matching aggregate incidence of these as opposed to employee-level incidence. From the year of simulated data, I compute the fraction that experience no employment change, job-to-job transition, job-to-nonemployment transition followed by re-employment, job-to-censored-non-employment transition, and on-the-job pay gain in the initial employer. Note that the first four of these sum to one by construction. I also record simulated initial employment and initial pay level durations. Finally, I simulate the mean and variance of initial wages, which are the analogues to the empirical moments recorded in table 1. To take into account censoring in the data, I censor simulations at the maximum social security contribution. This affects the mean and variance of initial wages and the incidence and duration of on-the-job pay gain.[26]

I estimate an optimally weighted, over-identified model, minimizing the sum of squared errors between simulated moments and empirical moments:

$$\hat{\theta} = argmin_{\theta} \left\{ m(\theta)' W m(\theta) \right\}, \tag{B.1}$$

[25]At daily frequency and plausible parameter values it is extremely unlikely to simulate more than one shock per day. Daily is, however, only an approximation to the continuous time process in which the model is formulated.

[26]Simulations are conducted in MATLAB. All code is available upon request. Contact by email: Cynthia.L.Doniger@frb.gov.

where $m(\theta) = \frac{1}{n}\sum_i [S(\theta) - d(x_i)]$ is the vector of moment conditions. $S(\theta)$ is the vector of simulated moments conditional on θ and $d(x_i)$ is the vector of data for each observation. W is the optimal weight matrix.

Estimation is conducted in two steps. First, I minimize the sum of squared errors with W as the identity matrix and estimate the optimal weight matrix as:

$$\hat{W} = \hat{\Omega}^{-1} = \left[\frac{1}{n}\sum_i (S(\theta) - d(x_i))'(S(\theta) - d(x_i)\right]^{-1}. \tag{B.2}$$

I then repeat the minimization using the estimated weighting matrix.

Standard errors are computed by means of numerical derivatives. The variance covariance matrix is:

$$N\mathbb{V}(\hat{\theta}) = (D'\Omega D)^{-1}(D'W\Omega WD)(D'\Omega D)^{-1} \tag{B.3}$$

where $D = \frac{dm(\cdot)}{d\theta}|_{\theta=\theta_0}$. Numerical derivatives are computed from a 1 percent deviation around the point estimate of the parameter value. Since the optimal weight matrix is such that $W = \Omega^{-1}$, equation B.4 simplifies to:

$$N\mathbb{V}(\hat{\theta}) = (D'\Omega D)^{-1}. \tag{B.4}$$

Empirical moments and covariances are computed in STATA. Simulation and estimation are conducted in MATLAB. Optimal values are found using `fminsearch`.