Minimum Wage Policy with Optimal Taxes and Involuntary Unemployment*

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Abstract

While minimum wages are a widely used policy tool that are often justified on redistributive grounds, their desirability is highly controversial in economics. This paper derives a condition under which a binding minimum wage improves upon a (second-best) optimal tax allocation in a general model with involuntary unemployment. Importantly, I show that this condition can be expressed in terms of marginal social welfare weights and labor force participation and employment elasticities with respect to the minimum wage. The main empirical result is that the effect of the minimum wage on the welfare of low-skilled workers is captured by the ratio of the macro and micro participation responses to the minimum wage. In contrast, the macro employment response to the minimum wage – the object of interest of much of the empirical literature on the minimum wage – matters only through its effect on the government's budget constraint. Using variation in state minimum wages from 1989 to 2011, I estimate the sufficient statistics that are inputs into the desirability condition. Preliminary estimates suggest that minimum wage increases experiences in the past three decades increased the labor force participation and welfare of low-skilled women.

Keywords: Minimum wage; Sufficient statistics; Optimal policy

JEL: H21; J22; J23; J38

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1 Introduction

The minimum wage is a widely used policy tool that is often justified on the grounds that it increases the income of low-skilled workers. Yet the desirability of the minimum wage is highly controversial. In economics, the minimum wage literature can be grouped into two broad strands. The labor economics literature has focused on estimating effect of the minimum wage on labor market outcomes, notably the responses on the employment and hours of work margins.[1] Despite the dozens of studies since the important contributions of Neumark and Wascher (1992) and Card and Krueger (1994), evidence on the magnitude of the employment elasticity with respect to the minimum wage is mixed (Belman and Wolfson (2014)).[2] The mixed findings are primarily due to differences in the choice of the comparison group necessary to estimate the counterfactual level of employment (or employment flows) in the absence of a change in the minimum wage (Neumark, Salas, and Wascher (2014a,b);Dube, Lester, and Reich (2010, 2016)).[3]

A second strand of the minimum wage literature in public finance analyzes the conditions under which minimum wages complement optimal (non-linear) tax and transfer policy. Early work in this area showed that the minimum wage can not complement an optimal non-linear income tax when the labor market is competitive and labor supply choices are along the hours of work margin (Allen (1987); Guesnerie and Roberts (1987)).[4] Motivated by empirical research showing that the extensive margin is more relevant for low-skilled workers, recent work has focused on models where the primary labor supply choice is the decision about whether to enter the labor force. In these models, the case for the minimum wage is mixed and depends on assumptions about the micro-foundations of the labor market. For example, Hungerbüler and Lehmann (2009) develop a model with search frictions where workers and firms (Nash) bargain over the gross wage. They find that the minimum wage complements an optimal non-linear income tax if the worker's bargaining power is below the Hosios (1990) condition. Lee and Saez (2008, 2012) show that in a competitive labor market, a minimum wage is only desirable if unemployment due minimum wage is concentrated among those with the lowest surplus from working.[5]

An important limitation of this theoretical work is that it may be difficult to determine in practice whether the conditions under which the minimum wage is desirable are met. For example, estimates of the worker's bargaining power vary considerably (Flinn (2006), Ahn, Arcidiacono, and Wessels (2011)), and evidence on whether unemployment "efficiently rationed" is scarce (Luttmer

[1]This literature often supplements empirical findings with positive models of the labor market that are consistent with the empirical work. See Brouchu and Green (2013) and Dube, Lester, and Reich (2016) for recent examples.

[2]Belman and Wolfson (2014) also review the large empirical literature on the effects of the minimum wage on poverty and human capital accumulation.

[3]In two recent papers Sorkin (2015) and Meer and West (2015) show that short-run and long-run employment effects of the minimum wage differ. Baker, Benjamin, and Stanger (1999) is an early paper that addresses this issue.

[4]One exception is Marceau and Boadway (1994) who consider a model with both hours of work and an endogenous participation decision (see equation (35), page 77 in Marceau and Boadway (1994)). A second exception is Boadway and Cuff (2001) who show that a minimum wage combined with unemployment benefits and (effective) monitoring of job search activities can improve upon an optimal non-linear income tax allocation.

[5]A number of papers consider the efficiency and social welfare effects of the minimum wage in models where the government does not have access to taxes (Flinn (2006), Ahn, Arcidiacono, and Wessels (2011), Gravrel (2015), others).

(2007)). Moreover, these studies often do not rigorously link their theoretical results to existing empirical research.

This paper attempts to bridge the normative literature on the desirability of the minimum wage with the empirical evidence on the effects of minimum wages on labor market outcomes. In the theoretical section, I derive a condition under which the minimum wage complements a second-best optimal non-linear income tax. Importantly, I show that this "desirability condition" can be expressed expressed in terms of: (a) three labor force participation and employment elasticities with respect to the minimum wage, (b) the marginal social welfare weight of consumption for workers at the bottom of the wage distribution, and (c) the level of the (optimal) employment tax. Following several recent papers in the optimal tax and transfer literature, I do not explicitly model the labor demand side of the market or the process that determines wages. Instead, I adopt a sufficient statistics approach (Chetty (2009); Landais, Michaillat, and Saez (2015); Kroft, Kucko, Lehmann, and Schmieder (2015)), and allow the equilibrium wage and the fraction of job seekers that successfully find work (henceforth referred to as the "job finding rate") to be expressed by reduced forms. One advantage of this approach is that the desirability condition for the minimum wage is valid under various models of the labor market considered in the literature.

The theoretical section develops a model where workers vary along two dimensions. Individuals are endowed with different abilities and (unobservable) costs of searching for work. Assuming that labor markets are perfectly segmented by skill, differences in ability lead to differences in earnings for those who work.[6] This motivates the government's desire to redistribute from high to low ability individuals. Differences in search costs between workers of the same ability leads some to remain out of the labor force, whereas others choose search for a job.[7] The model admits involuntary unemployment since only a fraction $p_a \in (0, 1]$ of job seekers with skill level a successfully find a job. Therefore, for each labor market a, the number of participants may differ from the number of employed workers. In the spirit of the perturbation approach of Saez (2001), the desirability condition for the minimum wage is derived by considering whether introducing a small, binding minimum wage improves upon the optimal tax allocation.[8]

A second advantage of pursuing a sufficient statistics approach is that the desirability condition for the minimum wage can be expressed in terms of elasticities and government welfare weights. This avoids the need to estimate primitives of the model, such as parameters of individual utility functions, bargaining weights and the firm's cost functions. The sufficient statistics that are inputs into the desirability condition are: (a) the macroeconomic (macro) and microeconomic (micro) participation elasticities with respect to the minimum wage, (b) the macro employment elasticity with respect to the minimum wage, (c) the marginal social welfare weight on consumption for workers at the bottom of the earnings distribution, and (d) the level of the optimal em-

[6] Throughout the paper, I use the terms "labor market", "occupation", and "earnings level" interchangeably to refer to each of labor market.

[7] As described in detail below, I abstract from search intensity or hours of work choices, so the only labor supply decision is a binary participation choice.

[8] This approach is common in normative studies of the minimum wage. See, for example, Marceau and Boadway (1994), Lee and Saez (2008, 2012), and Hungerbüler and Lehmann (2009).

ployment tax at the bottom of the earnings distribution.[9]

The micro participation elasticity with respect to the minimum wage is the percentage increase in the labor force participation rate for workers at the bottom of the earnings distribution due to a one percent increase in the minimum wage, holding the job finding rate constant.[10] Conversely, the macro participation elasticity is the observed participation response to the minimum wage that allows for equilibrium changes in the job finding rate.[11]

The intuition for why the desirability condition for the minimum wage depends on the two labor force participation elasticities – in addition to the macro employment elasticity – is as follows. Beginning from a pre-minimum wage equilibrium, introducing a binding minimum wage increases makes low-skilled workers better off only if it increases their expected utility. Although the minimum wage increases the earnings (and consumption) of affected workers, it may lower their expected utility if the likelihood of finding a job (conditional on searching) declines. Thus, the possibility of a loss in surplus from working due to changes in the job finding rate may mitigate the gain due to higher earnings. This creates a wedge between the actual expected utility change due to the minimum wage and the one that would be experienced in the absence of a change in the job finding rate. Since the government only cares about the distribution of expected utilities, the effect of the minimum wage on expected utility of covered workers is theoretically ambiguous. However, since the participation decision also depends on the expected utility of being in the labor force, the ratio of the macro and micro participation elasticities with respect to the minimum wage corresponds exactly to the (macro) expected utility change in the government's social welfare objective.

While the ratio of the macro and micro participation responses captures the effect of introducing a binding minimum wage on the expected utility of low-skilled workers, the macro employment response to the minimum wage – the primary labor market outcome of interest in the empirical literature – only affects the government's objective through the budget constraint. Whether or not the minimum wage is desirable therefore depends on whether the sum of the social welfare term (captured by the ratio of the macro and micro participation responses) and the government revenue term (captured by the product of the macro employment response and the level of the optimal employment tax) is positive.

The theoretical analysis also shows how the level of the optimal employment tax interacts with the macro employment elasticity to either relax or tighten the government's budget constraint.[12]

[9] The level of the optimal employment tax is endogenous and may depend on welfare weights, as well as labor supply and demand responses. Since the main focus of this paper is on the minimum wage, I refer the reader to the results in Saez (2002), Lehmann, Parmentier, and Linden (2011) and Kroft, Kucko, Lehmann, and Schmieder (2015), among others, for results on the predicted sign and magnitude of the optimal employment tax at the bottom of the earnings distribution.

[10] With the job finding probability held constant, the micro participation response to the minimum wage is positive by assumption.

[11] The macro participation response does not refer to a labor supply response estimated using data from multiple locations. Rather, it corresponds to the labor force participation response that incorporates equilibrium changes in the job finding rate due to the minimum wage. The macro employment elasticity is similarly defined.

[12] Kroft, Kucko, Lehmann, and Schmieder (2015) show that the optimal tax schedule resembles an EITC at the bottom of the wage distribution when the *micro participation response to taxes* is small relative to the *macro participation response*

For example, if the optimal income tax is an in-work subsidy (such as the EITC), job losses due to the minimum wage increase government revenue, since subsidies are no longer paid to some low-skilled workers.[13] In this case, the increase in government revenue either reinforces the efficiency gains due the increase in the expected utility of low-skilled workers, or mitigates losses due to a decline in the expected utility from participating in the labor force.

The theoretical results are also related to the literature on the optimal design of redistributive policy, in particular, about the desirability of so-called indirect instruments such as the minimum wage, relative to direct income taxation. In seminal papers, Diamond and Mirrlees (1971a,b) and Atkinson and Stiglitz (1976) show that under some conditions, indirect policy tools such as minimum wages, tariffs or subsidies cannot not improve upon optimal tax allocations. Saez (2004) confirms this result in an occupational choice model when the labor market is perfectly competitive. On the other hand, Lee and Saez (2012) show that when the labor market is perfectly competitive and unemployment is efficiently rationed, a minimum wage complements an in-work subsidy such as the EITC.[14] Although the main theoretical result – that the desirability of the minimum wage can be expressed in terms of sufficient statistics – means that the desirability condition is straightforward to implement empirically, it does not provide insights into the mechanisms underling the desirability of a wage floor.

In order to address this concern, in Section 4, I show how the to interpret the desirability condition in under two models of the labor market; a static static search and matching model where wages are determined by proportional bargaining (Diamond (1982); Mortensen and Pissarides (1999)), and a competitive model with either efficient or uniform job rationing (Lee and Saez (2008, 2012)). This exercise also provides insights into the economic forces underlying the differences between the macro and micro participation responses to the minimum wage that are important inputs into the desirability condition.

As one example, in the simple search and matching framework with proportional bargaining, there is a direct relationship between the sign of the macro participation response to the minimum wage and worker's (exogenous) bargaining power. In particular, if the worker's bargaining power is lower than the elasticity of the matching function with respect to the number of workers searching (i.e. the Hosios (1990) condition), then introducing a binding minimum wage unambiguously increases the expected utility of low-skilled workers. Therefore, a positive macro participation response to the minimum wage indicates the worker's bargaining power was initially too low. If the worker's bargaining power is inefficiently low, the minimum wage may increase efficiency by correcting for congestion externalities due to too little search. In the proportional bargaining situation, an optimal non-linear income tax cannot correct for an inefficiently low bargaining power because worker's receive a fixed fraction of the surplus from a match.

to taxes. This is likely to be the case when the incidence effect of taxes on the gross wage is relatively small.

[13]If minimum wages increase employment, then the same logic applies to the argument below, but in reverse.

[14]Intuitively, if employment subsidies such as the EITC lead to lower wages through incidence effects (Rothstein (2010)), then ensuring that wages at the bottom are downward rigid using a minimum wage makes redistribution through the tax system more effective.

The theoretical model highlights the importance of estimating both the participation and employment responses to the minimum wage for optimal policy. In contrast to the large body of research on the employment effects of the minimum wage, the labor force participation margin has received relatively little attention (Luna-Alpizar (2015)). Mincer (1976) was the first to show that the unemployment effects of the minimum wage might be different than the employment effects due to flows into (or out of) non-minimum wage sectors and the labor force. Flinn (2006) estimates the primitives of a structural search and matching model and concludes that the increases to the federal minimum wage in the mid-1990s increased the value of search, which would have increased the labor force participation, employment and unemployment rates. The increase in the unemployment rate is due to the fact that the increase in the participation rate was larger than the increase in employment.[15] Ahn, Arcidiacono, and Wessels (2011) estimate that even small declines in employment due to the minimum wage may mask compositional changes in the labor force. Relative to previous work, the contribution of this paper is to shed light on roles of the participation and employment responses in evaluating the social welfare effect of the minimum wage when the government can also set income taxes optimally.

The second contribution of the paper is to estimate the effect of the minimum wage on the labor force participation rate and employment rate of low-wage workers. Using data from the CPS from 1989 to 2011, I find that a one percent increase in the minimum wage increases the macro labor force participation rate of unmarried, female high school dropouts by 0.13 percent. This estimate is relatively insensitive to several robustness checks designed to control for potential differing "pre-trends" identified by the literature. This suggests that the minimum wage increases experienced during the past three decades increased the labor force participation rate and possibly the welfare of low-skilled U.S. workers.

The remainder of the paper is structured as follows. Section 2 introduces the theoretical model. Section 3 presents the main theoretical results. To develop intuition, I first consider the conditions under which a minimum wage is desirable in environments with no taxes and one with fixed (employment) tax rates, before moving on to the full model where the government can set an optimal non-linear income tax. In Section 4, I show how to interpret the desirability condition under various models of the labor market. Section 5 describes the data used in the empirical analysis and the empirical strategy used to estimate the sufficient statistics from the theoretical model. The main results are reported and discussed in Section 6. Section 7 offers concluding remarks.

[15] Using CPS data from 1979 to 1999, Wessels (2005) estimates the effect of increases in the minimum wage on the teenage (age 16-19) labor force participation rate. However, the inclusion of the fraction of teens in the population (a labor supply determinant) and the adult unemployment rate (to control for unobservable changes in the demand for labor) make interpreting his coefficient estimates as micro or macro responses difficult. Moreover, Wessels dismisses a two-way fixed effects (state and year FE) specification that leads to a statistically insignificant effect, despite the fact that such a specification is standard in the literature.

2 Theoretical Model

2.1 Environment and Timing

The economy is populated by risk neutral agents whose size is normalized to one. Individuals are endowed different productivity levels (or earnings capacity/skills), denoted by a. The density of skills $f(a)$ is continuous and strictly positive on the interval $[a_0, a_1]$, with $0 < a_1 < a_1 \leq +\infty$. I assume the labor market is perfectly segmented by skill. An individual with productivity a produces $y(a)$ units of output if she works in a job that corresponds with her type, and nothing otherwise.[16]

Individuals also differ in their costs of searching for a job, denoted by θ. I assume that agents draw their value of search costs from a conditional distribution $M(\theta|a)$ on the semi-open interval $[0, +\infty)$, and that $M(\cdot|a)$ is strictly positive and continuously differentiable in θ. This formulation implies that at each skill level, some individuals will choose to search for a job while others will remain out of the labor force. Search costs may or may not be correlated with productivity.

I allow for involuntary unemployment by assuming that only a fraction $p_a \in (0, 1]$ of individuals that choose to search for a job successfully find work. Rather than modelling the demand side of the labor market directly, I adopt a sufficient statistics approach (Chetty (2009); Kroft, Kucko, Lehmann, and Schmieder (2015)). In particular, I allow the job finding rate in each occupation to be described by a reduced-form function of the gross wage in occupation a and the government's policy, denoted by the vector $\mathbf{\Gamma}$ and ability a. That is, $p_a = p(w_a, \mathbf{\Gamma}; a)$. The function $p(\cdot; a)$ is assumed to be twice continuously differentiable in its arguments. Similarly, I do not explicitly model the process that determines wage, and assume that the gross wage in occupation a to be described by the reduced-form $w_a = w(\mathbf{\Gamma}; a)$.

The timing of the static model is as follows. First, the government announces a tax schedule $T(\cdot)$, a non-employment benefit b, and possibly a minimum wage $\underline{w}$. Next, individuals take the government policy, gross wages and the job finding probability as given and choose whether to search for a job. As described earlier, only a fraction $p_a \in (0, 1]$ of type-a job seekers are successful. Those that are employed receive the gross wage w_a, pay taxes T_a and consume the net wage $c_a = w_a - T_a$. As the government is not assumed to be able to distinguish between the involuntarily unemployed and those that choose to remain out of the labor force. Thus, all individuals in this potentially heterogenous pool receive the non-employment benefit b.[17]

Let k_a denote the number of type-a agents that choose to search for a job, and $h_a = p_a k_a$ the number of employed workers. The unemployment rate in occupation a is $1 - p_a = \frac{k_a - h_a}{k_a}$. The number of individuals that choose to remain out of the labor force is $k_b = \int_{a_0}^{a_1} (1 - k_a) da$, and the number of non-employed agents is $h_b = \int_{a_0}^{a_a} (1 - h_a) da$.

[16] The perfect segmentation assumption is common in the literature, see for example, Lee and Saez (2008) and Hungerbüler and Lehmann (2009), among others.

[17] Given that the model is static, it seems natural to assume that the non-employment benefit cannot depend on previous labor market outcomes. The assumption that all non-employed agents receive the same welfare benefit b is also made in Hungerbüler and Lehmann (2009) and Lee and Saez (2008, 2012).

To close the model, I assume that firms earn zero profits in equilibrium. This assumption is justified in Section 4, where I show how to interpret the desirability condition for the minimum wage under different models of the labor market. These include a static search and matching model with free-entry of firms and a constant returns to scale production technology (Jacquet, Lehmann, and Linden (2014)) and perfectly competitive models with a constant returns to scale production technology (Lee and Saez (2008, 2012)).

2.2 Utility Maximization and Participation Decision

An individual of type (a, θ) enjoys utility equal to $c_a - \theta = w_a - T_a - \theta$ if she finds a job and $b - \theta$ if she searches and fails to find a job. She enjoys utility equal to b if she chooses not to search for a job. Let $U_a = p_a c_a + (1 - p_a)b$ denote the *gross* expected utility of searching for a type-a job (before search costs), and $U_b = b$ the expected utility that all non-participants receive. Let $\tau_a = \frac{T_a + b}{w_a}$ denote the employment tax rate paid by workers in occupation a and let $\Sigma_a = U_a - U_b = p_a w_a(1 - \tau_a)$ denote the gross expected surplus a type-a agent receives from participating in the labor market. This is equal to the financial gain from working, $w_a(1 - \tau_a)$ multiplied by the likelihood of finding a job. A worker with productivity a chooses to enter the labor force and search for a job if $U_a - \theta \geq U_b \Leftrightarrow \theta \leq p_a w_a(1 - \tau_a)$. Let $\hat{\theta}$ denote the cutoff value of search costs; only agents with values of θ below this cutoff enter the labor force.[18] The labor force participation rate among type-a workers is $G(\hat{\theta}|a)$. The number of participants in labor market a can be written as a continuous function of the expected surplus from participating in the labor force.

$$k_a = \hat{\mathcal{K}}_a(U_a, U_b) = G(U_a - U_b|a)f(a) = G(p_a w_a(1 - \tau_a)|a)f(a) \tag{1}$$

The number of employed individuals in occupation a is given by:

$$h_a = \hat{\mathcal{H}}(U_a, U_b) = p_a k_a = p_a G(p_a w_a(1 - \tau_a)|a)f(a) \tag{2}$$

2.3 Micro versus Macro Responses

This subsection describes the distinction between the micro and macro responses to the minimum wage wage. Suppose that the government introduces a small minimum wage $\underline{w} = w_{a_0} + dw$ just above the equilibrium wage in the lowest-skill occupation a_0. The microeconomic (partial equilibrium) effect of the minimum wage is the effect on the expected utility of searching, $U_{a_0} = p_a w_a(1 - \tau_a) + b$, holding the job finding probability p_{a_0} constant.[19] This could be possible in the short-run if production plans (and therefore labor demand) are fixed. If the minimum wage does not affect the job finding rate, workers in occupation a_0 receive a higher net wage, leading to a

[18] This set up abstracts from the search intensity decision and assumes that hours of work are fixed.

[19] I simplify the analysis by also assuming that the minimum wage does not have spillover effects onto the wages and job finding probabilities in occupations $a > a_0$.

higher expected utility. I define $\frac{dU_{a_0}}{d\underline{w}}\Big|^{\text{micro}} = p_{a_0}(1-\tau_{a_0}) - p_{a_0}w_{a_0}\frac{d\tau_{a_0}}{d\underline{w}} = p_{a_0}$.[20]

In contrast to the microeconomic (micro) response, the macroeconomic (general equilibrium) expected utility response to the minimum wage incorporates the effect of equilibrium changes in the job finding rate in occupation a_0 on the expected utility of searching in the lowest skill occupation. I define[21]:

$$\frac{dU_{a_0}}{d\underline{w}} = p_{a_0}\frac{d[w_{a_0}(1-\tau_{a_0})]}{d\underline{w}} + \frac{dp_{a_0}}{d\underline{w}}w_{a_0}(1-\tau_{a_0}) = p_{a_0}\left[1 + \frac{dp_{a_0}}{d\underline{w}}\frac{w_{a_0}(1-\tau_{a_0})}{p_{a_0}}\right] \tag{3}$$

The first term in square brackets captures the expected increase in the net wage due to a minimum wage increase, multiplied by the probability of finding a job. The second term captures the potential loss in surplus due to changes in the equilibrium job finding rate. Introducing (or increasing) a binding minimum wage increases (decreases) the expected utility of workers at the bottom of the wage distribution if the former effect dominates (is dominated by) the latter.

There is a direct link between the expected utility responses to the minimum wage and the labor force participation responses to the minimum wage. Using equation (1), the micro and macro participation responses to the minimum wage are respectively

$$\frac{dk_{a_0}}{d\underline{w}}\Big|^{\text{micro}} = \frac{d\hat{\mathcal{K}}_{a_0}}{dU_{a_0}}\frac{dU_{a_0}}{d\underline{w}}\Big|^{\text{micro}} = m(U_a - U_b|a)f(a)p_{a_0} \tag{4}$$

$$\frac{dk_{a_0}}{d\underline{w}} = \frac{d\hat{\mathcal{K}}_{a_0}}{dU_{a_0}}\frac{dU_{a_0}}{d\underline{w}} = m(U_a - U_b|a)f(a)p_{a_0}\left[1 + \frac{dp_{a_0}}{d\underline{w}}\frac{w_{a_0}(1-\tau_{a_0})}{p_{a_0}}\right] \tag{5}$$

The difference between the two responses is that the latter incorporates the effect of equilibrium changes to the job finding rate on the number of labor force participants in occupation a_0. Changes to the job finding rate attenuates the effect of an increase in the net wage for workers in occupation a_0. This could occur, for example, if hiring rates decrease, as in Brouchu and Green (2013) and Dube, Lester, and Reich (2016). Therefore if $\frac{dp_{a_0}}{d\underline{w}} < 0$, the macro participation response to the minimum wage will be smaller than the micro participation response. Since the labor force participation decision depends only on the gap in the expected utility between searching and remaining out of the labor force, the macro participation response is positive if and only if the macro expected utility response to the minimum wage is positive. From equations (3) and (5), we see that

$$sign\left(\frac{dU_{a_0}}{d\underline{w}}\right) = sign\left(\frac{dk_{a_0}}{d\underline{w}}\right)$$

[20]To see this, note that (evaluating the derivative at the initial wage w_{a_0}) $\frac{d\tau_{a_0}}{d\underline{w}} = -\frac{T_{a_0}+b}{[w_{a_0}]^2} = -\frac{\tau_{a_0}}{w_{a_0}}$. This holds the occupation-specific tax liability T_a constant.

[21]Note that all derivatives are evaluated at the pre-minimum wage level w_{a_0}.

In the analysis that follows, it will be useful to express the desirability condition for the minimum wage in terms of elasticities. Let $e^m_{a_0} = \frac{\underline{w}}{k_{a_0}} \frac{dk_{a_0}}{d\underline{w}}\Big|^{\text{micro}}$ denote the micro participation elasticity with respect to the minimum wage, and $e_{a_0} = \frac{\underline{w}}{k_{a_0}} \frac{dk_{a_0}}{d\underline{w}}$ the macro participation elasticity. Also, let $\eta_{a_0} = \frac{\underline{w}}{h_{a_0}} \frac{dh_{a_0}}{d\underline{w}}$ denote the macro employment elasticity, the percentage change in the number of individuals working in occupation a_0 when the minimum wage increases by one percent.

2.4 Government

I assume that the government evaluates outcomes according to an increasing and strictly concave social welfare function of the expected utilities. This implies that the government redistributes from high productivity to low productivity workers, but does not insure agents that search for a job and are unsuccessful. The social welfare function is

$$SW(U_b, \{U_a\}_{a\in[a_0,a_1]}) = \int_{a_0}^{a_1} \left(\int_0^{\hat{\theta}} \Phi(U_a - \theta)m(\theta|a)d\theta + \Phi(U_b)(1 - M(\theta|a)) \right) f(a)da \tag{6}$$

where $\Phi'(.) > 0$ and $\Phi''(.) < 0$. The government chooses the policy vector $\mathbf{\Gamma}$ to maximize (6) subject to the following budget constraint:

$$bh_b + E = \int_{a_0}^{a_1} T_a h_a da \Leftrightarrow b + E = \int_{a_0}^{a_1} \tau_a w_a h_a da \tag{7}$$

where $E \geq 0$ is exogenous public expenditures. The second equality is obtained by noting that $h_b = 1 - \int_{a_0}^{a_1} h_a$. I describe how taxes are set in section 3.2. Let λ denote the Lagrange multiplier associated with the budget constraint.[22]

Let g_a denote the marginal social welfare weight of consumption for workers in occupation a, expressed in terms of the marginal cost of public funds λ. Intuitively, g_a captures the marginal social value of increasing consumption for workers in occupation a relative to the marginal value of resources to society. Using $h_a = p_a k_a$, the marginal social welfare weight is

$$g_a = \frac{\int_0^{\hat{\theta}} \Phi'(U_a - \theta)m(\theta|a)d\theta f(a)p_a}{\lambda h_a} = \frac{\int_0^{\hat{\theta}} \Phi'(U_a - \theta)m(\theta|a)d\theta f(a)}{\lambda k_a} \tag{8}$$

Similarly, the marginal social welfare weight of consumption for the non-employed is

$$g_b = \frac{1}{h_b} \left(\frac{\Phi'(U_b)}{\lambda} + \int_{a_0}^{a_1} g_a k_a (1 - p_a) da \right)$$

Assuming that U_a is increasing in a, as would be the case if the job finding rate and gross wage is increasing in a, then the marginal social welfare weights g_a are strictly decreasing in a.

[22]By the assumptions on $\Phi(.)$, the government's budget constraint is binding at the optimum.

Moreover, assuming that $U_{a_0} \geq U_b \geq 0$ in equilibrium (so that some type-a_0 individuals enter the labor force), the concavity of Φ implies that $g_b > g_{a_0}$.

3 The Desirability of the Minimum Wage

3.1 The Desirability of the Minimum Wage with no Taxes

I begin the analysis by characterizing the conditions under which the minimum wage is desirable when the government cannot set taxes or a welfare benefit. Although unrealistic, this case is an interesting benchmark that has been considered by Ahn, Arcidiacono, and Wessels (2011), Flinn (2006), Lee and Saez (2008, 2012), Gravrel (2015), among others. Compared to these papers, I derive a condition under which the minimum wage is desirable in terms of sufficient statistics, namely the macro and macro labor force participation elasticities and the marginal social welfare weight of consumption for workers in occupation a_0. The no taxes case also helps build intuition for the desirability condition in the full model.

Let $(w_a^{LF}, p_a^{LF}, k_a^{LF})_{a \in [a_0, a_1]}$ denote the wages, job finding probabilities and number of searchers in the laissez-faire (no government intervention) equilibrium.[23] Before describing how the introduction of the minimum wage affects the government's objective (6), I introduce two assumptions on the laissez-faire wage distribution.

Assumption 1: In the laissez-faire equilibrium, the gross wage w_a^{LF} (and therefore the net wage $c_a^{LF} = w_a^{LF}$) is continuous and strictly increasing in a.

Assumption 1 is relatively weak and is satisfied in many models of the labor market. For example, in the competitive model where workers are paid their marginal product ($w_a^{LF} = a$), Assumption 1 is satisfied because of the assumptions on $f(a)$.[24]

Assumption 2: Small minimum wage changes do not have spillover effects on the equilibrium wage or job finding rate in adjacent occupations. Specifically, $\frac{dw_a}{d\underline{w}} = \frac{dp_a}{d\underline{w}} = \frac{dU_a}{d\underline{w}} = 0$ for all $a > a_0$.

Empirical evidence on the spillover effects of the minimum wage is mixed. For example, Lee (1999) argues that spillover effects are potentially large. However, Autor, Manning, and Smith (2016) argue that Lee's finding may be biased upwards, and using an instrumental variables strategy find smaller spillover effects (especially for females). In the Canadian setting, Fortin and Lemieux (2015) and Campoletti (2015) find that spillover effects of the minimum wage are small. One interpretation of this evidence is that spillover effects tend to reinforce the argument for the minimum wage by increasing the earnings of those higher up in the wage distribution. Although the presence of spillover effects is not central to the argument in this paper, I discuss how relaxing this assumption affects the main result in Section 3.3.

The government's problem is simply to decide whether to introduce a binding minimum wage

[23] I assume that such a unique equilibrium exists.

[24] Alternatively, suppose that wages are determined by Nash bargaining or proportional bargaining, and that the worker's type-specific bargaining power, β_a is a continuous and non-decreasing function of a. Then then the gross wage is $w_a = \beta_a \cdot a$, satisfying Assumption 1.

just above the equilibrium wage at the lowest skill level, $\underline{w} = w_{a_0}^{LF} + dw$.[25] Specifically, the government sets a wage floor equal to to $\underline{w}$ for all $a \in [a_0, a_0 + da]$ for some small da. I assume that dw is small relative to da so that bunching induced by the minimum wage has second-order effects on the government's objective. Considering the marginal welfare effect of a local policy change is similar in spirit to the perturbation approach in Lee and Saez (2008, 2012), and in the optimal income taxation literature (Saez (2001, 2002); Lehmann, Parmentier, and Linden (2011)).

Proposition 1. *With no taxes and transfers, and if Assumptions 1 and 2 hold, introducing a small, binding minimum wage at the bottom of the laissez-faire wage distribution, $\underline{w} = w_{a_0}^{LF} + dw$, increases social welfare if and only if*

$$\frac{e_{a_0}}{e_{a_0}^m} g_{a_0} h_{a_0} > 0 \tag{9}$$

Moreover, $sign\left(\frac{dSW}{d\underline{w}}\right) = sign\left(\frac{dU_{a_0}}{d\underline{w}}\right) = sign\left(\frac{dk_{a_0}}{d\underline{w}}\right)$.

Proof: Evaluated at the laissez-faire equilibrium values $(w_{a_0}^{LF}, p_{a_0}^{LF}, U_{a_0}^{LF})$, the macro expected utility response to the minimum wage in occupation a_0 is (where I drop the superscripts for esthetic reasons)

$$\frac{dU_{a_0}}{d\underline{w}} = p_{a_0}\left[1 + \frac{dp_{a_0}}{d\underline{w}}\frac{c_{a_0}}{p_{a_0}}\right]$$

From (6) and (8), the value that the government places on this expected utility change (in monetary terms) is

$$dSW^{LF} = g_{a_0} h_{a_0} dU_{a_0} d\underline{w} = g_{a_0} h_{a_0}\left[1 + \frac{dp_{a_0}}{d\underline{w}}\frac{c_{a_0}}{p_{a_0}}\right] d\underline{w}$$

Using 4 and 5, the term in square brackets corresponds exactly to the ratio of the macro and micro labor force participation responses to the minimum wage: $\frac{dk_{a_0}}{d\underline{w}} = [1 + \frac{dp_{a_0}}{d\underline{w}}\frac{c_{a_0}}{p_{a_0}}] \left.\frac{dk_{a_0}}{d\underline{w}}\right|^{\text{micro}}$. Substituting this ratio and using the definitions for the elasticities e_{a_0} and $e_{a_0}^m$ leads to (9). The second part of the proposition follows immediately. □

The intuition underlying Proposition 1 is the following. In the absence of taxes and transfers (and under Assumptions 1 and 2), introducing a binding minimum wage is desirable if and only if it increases the expected utility of workers in occupation a_0. This effect that is captured by (3). The minimum wage has potentially offsetting effects on U_{a_0}. On the one hand, the reform increases the net financial gain from working, $c_{a_0}^{LF} = w_{a_0}^{LF}$. This change increases the expected utility of participating in the labor force by $p_{a_0}^{LF} \cdot dc_{a_0}^{LF} = p_{a_0}^{LF} \cdot d\underline{w} > 0$. On the other hand, the increase in the

[25]If $U_{a_0} = 0$, (i.e. those with a very low value of a are not employable), the the analysis in the following sections would carry through if the minimum wage was introduced just above the equilibrium wage for the lowest participating skill level $\underline{a} \in (a_0, a_1)$.

gross wage may affect the job finding probability. This change increases the expected utility of participating by $dp^{LF}_{a_0} \cdot c^{LF}_{a_0}$. If the sum of these two changes is positive (negative), then the introduction of a minimum wage increases (decreases) the expected utility of workers in occupation a_0. Since the labor force participation decision also depends on the expected utility of searching, this familiar trade-off is captured by the ratio of the macro and micro participation responses to the minimum wage. Moreover, the sign of this ratio depends only on the sign of $\frac{dk_{a_0}}{d\underline{w}}$ because the micro participation response is positive by assumption.

If the introduction of the minimum wage increases (decreases) the expected utility from searching in occupation a_0, then some individuals will be induced to enter (exit) the labor force, lowering (increasing) the cutoff value $\hat{\theta}$. However, since $d\underline{w}$ is small, those that switch between participating and remaining out of the labor force have no first-order effects on welfare (by the envelope theorem).

Proposition 1 shows that the ratio of the macro and micro participation responses is a sufficient statistic for the macro expected utility response to the minimum wage for workers in occupation 1. Surprisingly, the participation response to the minimum wage has received much less attention in the empirical literature. In Section 6 below, I provide evidence that the state minimum wage increases in recent decades as led to higher labor force participation rates for those at the bottom of the wage distribution.

3.2 The Desirability of the Minimum Wage with Fixed Tax Rates

This section extends the analysis by introducing taxes and transfers. Consider the situation where the government can observe job the gross wage and sets taxes and the non-employment benefit based on earnings. Specifically, the government assigns a tax liability $T_a = T(w_a)$ for individuals that report the wage w_a (and hence work in occupation a) and a non-employment benefit b for those that report no earnings. The net wage (after-tax earnings) is $c_a = w_a - T_a$ for those that report w_a and $c_b = b$ for the non-employed. Workers in occupation a pay taxes if $T_a > 0$; they receive a subsidy if $T_a < 0$.

A type-a worker chooses to participate in the labor market if and only if $\theta \leq p_a[w_a - T_a - b]$. Using the definition for τ_a and (1), the number of searchers in occupation a is $k_a = M[p_a w_a(1 - \tau_a)]f(a)$.

To see how the introduction of taxes affects the desirability of the minimum wage, it is easier to first consider the case when employment tax rates $\bar{\tau}_a$ are exogenously fixed, and the non-employment benefit b automatically adjusts to satisfy the budget balance condition (7) when the minimum wage is introduced. This case mirrors the analysis in Section 4.2 in Lee and Saez (2008). Let $(w^{\bar{\tau}}_a, p^{\bar{\tau}}_a, k^{\bar{\tau}}_a)_{a\in[a_0,a_1]}$ denote the wages, job finding probabilities and number of searchers in the fixed tax rate equilibrium.[26] Similar to the analysis in the previous section, the following assumption guarantees that agents report wages that correspond to their type.

[26] I assume that such a unique equilibrium exists. Fully specifying the demand side of the market would provide conditions on the primitives of the model required for such an equilibrium to exit.

Assumption 3: In the fixed-tax rate equilibrium, the gross wage $w_a^{\bar{\tau}}$ and the net wage $c_a^{\bar{\tau}}$ are continuous and strictly increasing in a.

Assumption 3 implies that marginal tax rates are less than 100 percent. Introducing a small minimum wage just above the equilibrium wage at the lowest skill level, $\underline{w} = w_{a_0}^{LF} + dw$ has the following effect on social welfare.

Proposition 2. *With fixed tax rates, and if Assumptions 2 and 3 hold, introducing a small, binding minimum wage at the bottom of the wage distribution, $\underline{w} = w_{a_0}^{\bar{\tau}} + dw$, increases social welfare if and only if*

$$\frac{e_{a_0}}{e_{a_0}^m} g_{a_0} + \bar{\tau}_{a_0} + \bar{\tau}_{a_0} \eta_{a_0} > 0 \tag{11}$$

The proof is presented in Appendix A.1.

When $\bar{\tau}_{a_0} = 0$, condition (11) reduces to (9) (Proposition 1). Moreover, equations (9) and (11) correspond respectively to the conditions in Propositions 1 and 2 in Lee and Saez (2008), for the case when there are no wage spillovers.[27] In addition to the effect on the expected utility of workers in occupation a_0, Proposition 2 shows that the introduction of a small, binding minimum wage leads to two fiscal effects. Recall that each worker in occupation a_0 pays $w_{a_0} \bar{\tau}_{a_0}$ in taxes. Thus, the first fiscal effect, captured by the second term in (11), is the (mechanical) increase in government revenues due to each worker paying $\bar{\tau}_{a_0} d\underline{w}$ in additional taxes. However, the introduction of the minimum wage will also change the level employment due to demand $\frac{dp_{a_0}^{\bar{\tau}}}{d\underline{w}}$ and participation responses $\frac{dk_{a_0}^{\bar{\tau}}}{d\underline{w}}$. These responses lead to a change in the level of employment, ultimately affecting the revenue the government collects from workers in occupation a_0. These behavioral responses lead to a $w_{a_0}^{\bar{\tau}} \bar{\tau}_{a_0} dh_{a_0}^{\bar{\tau}} = w_{a_0}^{\bar{\tau}} \bar{\tau}_{a_0} \eta_{a_0} \underline{w}^{-1} \approx \bar{\tau}_{a_0} \eta_{a_0}$ increase in revenue per-worker.

Introducing a minimum wage increases the government's objective if the sum of the two fiscal effects, $\bar{\tau}_{a_0} + \bar{\tau}_{a_0} \eta_{a_0}$, and the social welfare effect $\frac{e_{a_0}}{e_{a_0}^m} g_{a_0}$ is positive. In the fixed tax rate case, a positive macro participation response (i.e. a positive e_{a_0}) is not sufficient for the minimum wage to be desirable. Rather, the marginal welfare effect of the minimum wage depends on the (fixed) employment tax rates and three elasticities: the macro and micro participation responses and the macro employment response to the minimum wage. Interestingly, the macro employment elasticity, the object of attention for much of the empirical research on the effects of the minimum wage, affects of the government's (constrained) objective function only through the budget constraint. As in (9), the ratio of the macro and micro participation elasticities captures the effect of the minimum wage on the expected utility (and therefore the welfare) of workers at the bottom of the wage distribution.

Proposition 2 also illustrates an interesting interaction between the employment tax rate at the bottom of the wage distribution and the desirability of minimum wage legislation. Workers in occupation a_0 face a negative (positive) tax rate if $\bar{\tau}_{a_0} < 0$ ($\bar{\tau}_{a_0} > 0$). As a result, the minimum wage may tighten or relax the government's budget constraint. For example, if the workers in

[27] In Lee and Saez (2008), assuming that wages are fixed means that $\frac{dw_2}{d\underline{w}} = 0$, so that the g_2 and τ_2 terms drop out.

occupation a_0 are face a negative tax rate, then job losses due to the minimum wage will increase government revenue, as fewer individuals receive a subsidy. Indeed, for certain values of the elasticities and the tax rate (11) shows that it is possible for the minimum wage to both increase the expected utility of workers in occupation a_0 (and therefore the number of searchers), while also lowering employment and relaxing the government's budget constraint. Conversely, employment increases due to the minimum wage will tighten the government's budget constraint if $\bar{\tau}_{a_0} < 0$. The following section shows that a similar interaction affects the desirability of the minimum wage when the government sets income tax rates optimally.

3.3 The Desirability of the Minimum Wage with Optimal Taxes and Transfers

This section extends the analysis by considering whether introducing a minimum wage is desirable when the government can set taxes and transfers optimally. Unlike the previous section with fixed tax rates, the government chooses a tax liability T_a for each occupation to maximize (6) subject to (7) and workers reporting the gross wage that corresponds to their skill type. After setting tax rates optimally, I derive a condition under which introducing a minimum wage improves upon the optimal-tax allocation using the perturbation approach described earlier.

Assumption 4: (a) An optimal tax equilibirum exists and is unique; (b) At the optimum, the gross wage $w_a = w(\mathbf{\Gamma}; a)$ and the net wage $c_a = w_a - T_a$ are continuous and strictly increasing in a. Moreover, $w(\mathbf{\Gamma}; a)$ is a continuously differentiable function in its argument, $\mathbf{\Gamma}$.

Assumption 4 states that the optimal tax schedule leads to a separating equilibrium. Let $(b^*, \{T_a^*, w_a^*, c_a^*, k_a^*, p_a^*\}_{a\in[a_0,a_1]})$ denote the equilibrium non-employment benefit, occupation-specific tax liabilities, wages, consumption, number of labor market participants and job finding probabilities. In this equilibrium agents with productivity a find it optimal to report the (unique) wage w_a^* that corresponds with their type. In equilibrium, type-a workers pay T_a^* in taxes and consume $c_a^* = w_a^* - T_a^*$. All agents that are not employed consume b^*.

The focus of this paper is not in the shape of the optimal income tax schedule. However, the previous section shows that optimality condition for the minimum wage requires knowing the *level* of the employment tax at the bottom of wage distribution. As a result, invoking Assumption 4 seems practical for the purposes of this paper. Moreover, assuming that a separating equilibirum exists and is unique is standard in papers that adopt a sufficient statistics approach (Chetty (2009); Landais, Michaillat, and Saez (2015); Kroft, Kucko, Lehmann, and Schmieder (2015)).[28]

The following lemma (proved in Appendix A.2) characterizes the optimal tax and benefit system.

Lemma 1. *If Assumptions 2 and 4 hold, the optimal employment tax at the bottom of the wage distribution is given by*

[28] The conditions under which the optimal income tax leads to a separating equilibirum exists is discussed in Chroné and Laroque (2011) for the case of a competitive labor market, and in Lehmann, Parmentier, and Linden (2011) in the case of a search and matching economy. Other papers assume that the labor market is perfectly segmented by skill, that hours of work are fixed, and that workers can only work at their maximal productivity in order to rule out pooling (Saez (2002); Lee and Saez (2008); Jacquet, Lehmann, and Linden (2014)).

$$\frac{\tau_{a_0}}{1-\tau_{a_0}} = \frac{1 - \frac{\pi_{a_0}}{\pi^m_{a_0}} g_{a_0}}{\zeta_{a_0}} \tag{12}$$

where $\pi^m_{a_0} = -\frac{w_{a_0}(1-\tau_{a_0})}{k_{a_0}} \frac{dk_{a_0}}{dT_{a_0}}\Big|^{micro}$, $\pi_{a_0} = -\frac{w_{a_0}(1-\tau_{a_0})}{k_{a_0}} \frac{dk_{a_0}}{dT_{a_0}}$, *and* $\zeta_{a_0} = -\frac{w_{a_0}(1-\tau_{a_0})}{h_{a_0}} \frac{dh_{a_0}}{dT_{a_0}}$ *correspond to the micro and macro participation and macro employment elasticities with respect to (own) taxes in occupation* a_0 *respectively.*

Moreover, the following equality holds at the optimal tax allocation $\int_{a_0}^{a_1} g_a h_a + g_b h_b = 1$.

The first part of Lemma 1 is due to Kroft, Kucko, Lehmann, and Schmieder (2015), and states that the optimal employment tax rate at the bottom of the wage distribution is given by an inverse elasticity rule.[29] The optimal tax rate at is decreasing in the ratio of the macro and micro participation elasticities with respect to taxes, the marginal social welfare weight for workers in occupation a_0, and the macro employment elasticity with respect to taxes. As shown by Kroft, Kucko, Lehmann, and Schmieder (2015), given that ζ_{a_0} is positive by assumption, the optimal employment tax rate at the bottom of the wage distribution is negative if the numerator of (12) is negative (i.e. $g_{a_0} > \frac{\pi^m_{a_0}}{\pi_{a_0}}$). In this case, the optimal income tax for the lowest skilled workers resembles an EITC.

As in the previous section, after optimizing the tax system, the government considers introducing a binding minimum wage $\underline{w} = w^*_a + dw$, just above the equilibrium wage at the bottom of the skill distribution. Proposition 3 shows that optimality condition for the minimum wage depends on the marginal social welfare weight g_{a_0}, the three participation and employment elasticities with respect to the minimum wage and the optimal employment tax rate for workers in occupation a_0.

Proposition 3. *If Assumptions 2 and 4 hold, introducing a small, binding minimum wage at the bottom of the optimal tax wage distribution,* $\underline{w} = w^*_{a_0} + dw$, *increases social welfare if and only if*

$$\frac{e_{a_0}}{e^m_{a_0}} g_{a_0} + \left(\frac{1 - \frac{\pi_{a_0}}{\pi^m_{a_0}} g_{a_0}}{\zeta_{a_0} + 1 - \frac{\pi_{a_0}}{\pi^m_{a_0}} g_{a_0}} \right) \eta_{a_0} > 0 \tag{13}$$

The proof is presented in Appendix A.3. A heuristic proof for Proposition 3 is as follows. The introduction of a minimum wage leads to a "social welfare" effect and two fiscal effects.

Social Welfare Effect: Analogous to the no taxes and fixed tax rate cases, the introduction of a minimum wage increases the value of SW if it increases the expected utility of searching in occupation a_0. When income tax rates are set optimally, introducing a minimum wage increases (decreases) the expected utility of participating if the sum $\left[p^*_{a_0} + dp^*_{a_0} \cdot [w_{a_0}(1-\tau^*_{a_0})]\right] d\underline{w}$ is positive (negative). Given that the labor force participation decision also depends on the expected utility from searching in occupation a_0, the term in square brackets can be replaced by the ratio of the macro and micro participation responses to the minimum wage.

[29] The assumption that occupations are perfectly segmented by skill implies that within each labor market, individuals only differ by their search costs.

If the minimum wage increases (decreases) the expected utility of searching in occupation a_0, some individuals will be induced to enter (exit) the labor force. By the envelope theorem, these agents are marginal and do not have a first-order effect on the government's objective. Therefore, the social welfare effect is equal to the ratio of the macro and micro participation elasticities with respect to the minimum wage multiplied by the marginal social welfare weight on workers in occupation a_0.

Mechanical Fiscal Effect: The introduction of a binding minimum wage leads to two (mechanical) fiscal effects. Recall that the government's budget constraint is $b + E = \int_{a_0}^{a_1} (T_a + b) h_a da = \int_{a_0}^{a_1} w_a \tau_a h_a da$. Thus, the government's revenue from workers in occupation a is equal to the total gross earnings by workers, $w_a h_a$, multiplied by the employment tax rate paid by workers in that occupation, τ_a. The first mechanical effect arises because the gross wage of each worker in occupation a_0 increases by $d\underline{w}$. This causes the government's revenue to increase by $\tau_{a_0} h_{a_0} d\underline{w}$. However, since $\tau_{a_0} = \frac{T_a + b}{w_{a_0}}$, a higher gross wage leads to a lower employment tax rate which lowers government revenue by $\tau_{a_0} h_{a_0} d\underline{w}$. Thus, the two mechanical fiscal effects cancel out.[30] This is in contrast to the case in Section 3.2 where employment tax rates were (exogenously) fixed.

Employment Effect: As described earlier, the introduction of the minimum wage also leads to labor demand and participation responses that affect the level of employment. This employment effect increases government revenue by $w_{a_0}^* \tau_{a_0}^* dh_{a_0} \approx \tau_{a_0}^* \eta_{a_0}$ for a small $d\underline{w}$. Substituting the optimal employment tax formula for $\tau_{a_0}^*$ from (12) gives the second term in (13).

The interaction between the sign of the employment tax rate at the bottom of the wage distribution and the desirability of the minimum wage is also apparent from an examination of (13). The sign of the fiscal effect $\tau_{a_0}^* \eta_{a_0}$ depends on both the sign of the employment elasticity with respect to the minimum wage and the optimal tax rate. For example, if $g_{a_0} > \frac{\pi_{a_0}^m}{\pi_{a_0}}$ and $\eta_{a_0} < 0$, then the minimum wage leads to lower employment, but improves the government's fiscal position because workers in occupation a_0 receive a net transfer. Ultimately, inequality (13) makes clear that there is no a-priori case for (or against) the minimum wage in a this general model with involuntary unemployment. Rather, the desirability of the minimum wage is an empirical question that depends on the signs and magnitudes of the labor force participation *and* employment responses to taxes and the minimum wage that may vary across locations and over time (Kopczuk (2005); Lee and Saez (2008)).

It is helpful to understand how relaxing Assumption 2 affects the desirability condition for the minimum wage (13). To simplify the analysis, Assumption 2 rules out spillover effects due to the minimum wage. In practice, the minimum wage may increase the earnings (and possibly the job finding rates) of those higher up in the wage distribution (Lee (1999); Autor, Manning, and Smith (2016)). Suppose that the minimum wage uniformly increases the wages of those in occupations $a \in (a_0, a']; \;\; a' < a_1$. In particular, suppose that the all wages in this interval increased from w_a

[30] This assumes that the occupation a_0 tax liability, T_{a_0} is held fixed. If $T_a = T(w_a)$ is affected by a minimum wage increase, then inequality (13) reduces to $[1 - T'_{a_0}] \frac{e_{a_0}}{e_{a_0}^m} g_{a_0} + T'_{a_0} + \tau_{a_0}^* \eta_{a_0}$, where T'_{a_0} is the marginal tax rate (evaluated at $w = w_{a_0}^*$) and $\tau_{a_0}^*$ is given by (12) (i.e. Lemma 1).

to $w_a + \Delta$. For simplicity, assume also that the job finding probabilities were not affected by this increase (i.e. $\frac{dp_a}{d\underline{w}} = 0$; $a \in (a_0, a']$).

Allowing for such spillover effects would increase the expected utility of searching in occupations $a \in (a_0, a']$ and introduce two additional terms into inequality (13). The first term would capture the the increase in social welfare due to the higher expected utility from searching and would be equal to $\Delta \int g_a h_a da$. The increased expected utility would induce some individuals that were initially indifferent between participating and remaining out of the labor force to begin searching. Since these individuals are marginal, the change in their labor market status has no first-order effect on SW. The second term would capture the change in the government's budget constraint due to the increased participation (and therefore employment) in occupations $a \in (a_0, a']$. This "spillover employment effect" would be equal to $\Delta \int \tau_a \eta_a h_a da$ (where $\eta_a = \frac{dh_a}{dw_a} \frac{w_a}{h_a}$). In this case, the ratio of the macro and micro participation responses to the minimum wage in occupation a_0 would still capture the effect of the minimum wage on the expected utility of type-a_0 workers.

4 Links Between the Optimal Minimum Wage Formula and Various Models of the Labor Market

The previous analysis shows that when the government can set taxes optimally, the effect of the minimum wage on the expected utility of covered workers is captured by the ratio of the macro and micro participation elasticities with respect to the minimum wage. A positive macro participation response indicates that the expected utility of workers in occupation a_0 increases with the minimum wage. However, whether introducing a minimum wage is desirable also depends on its effect on the government budget constraint. An advantage of adopting a sufficient statistics approach is that the desirability condition for the minimum wage (inequality (13)) is valid under different assumptions about the micro-foundations of the labor market. In this section, I show how various restrictions on the model lead to special cases that have been examined in the previous literature.

Apart from showing that the optimality condition is robust to different micro-foundations of the labor market, this analysis provides insights into the economic forces driving the micro and macro participation and employment responses to the minimum wage. The two broad classes of models I consider are search and matching models with constant returns to scale (CRS) and wage bargaining (Hungerbüler and Lehmann (2009)), and competitive models of the labor market (Lee and Saez (2008); Lee and Saez (2012)).

4.1 Search and Matching Models with Wage Bargaining

The presence of search frictions in the labor market and firms with wage setting power is sometimes used to justify the minimum wage. To develop intuition for the economic forces underlying inequality (13), I consider a simple search and matching economy with a constant returns to scale production technology where wages are determined by proportional bargaining (Diamond (1982); Mortensen and Pissarides (1999)).

Let $y_a = a$ denote the output from a match in occupation a so that the firm's surplus is $a - w_a$. Creating a vacancy requires paying a fixed cost equal to χ_a. The number of vacancies created by firms in occupation a is v_a and the number of searchers (participants) is k_a. The constant returns to scale matching function $h_a = H_a(v_a, k_a)$ determines the number of employed workers. The probability that a vacancy is matched with a worker is $q_a(\rho_a) = \frac{H_a}{v_a} = H_a(1, \rho_a^{-1})$, where $\rho_a = \frac{v_a}{k_a}$ denotes the labor market tightness. Assuming free entry implies that $q_a(a - w_a) = \chi_a$ in equilibrium (i.e. firms earn zero profits in equilibrium). Conditional on searching, the probability that a worker matches with a firm is $p_a(\rho_a) = \frac{H_a}{k_a} = \rho_a q_a = q_a^{-1}(\frac{\chi_a}{a-w_a})\frac{\chi_a}{a-w_a}$, where $q^{-1}(\cdot)$ is the inverse of the firm's matching probability function. Finally, let $\mu_a = \frac{dH_a}{dk_a}\frac{k_a}{h_a}$ denote the elasticity of the matching function with respect to the mass of searchers in occupation a.

Wages are determined by proportional bargaining between workers and firms. The worker's ex-post surplus from a match is $c_a - b = w_a - T_a - b$. Let β_a denote the exogenous bargaining share for workers in occupation a; β_a may vary across occupations or may be the same for all occupations.[31] In this setting, the bargained wage in occupation a is:

$$w_a = \beta_a a + (1 - \beta_a)(T_a + b) \tag{14}$$

Thus, wages in occupation a do not depend on the output, wages or taxes in other occupations, providing a micro-foundation for the no Assumption 2.[32] It is useful to rewrite the government's optimization problem as one in which the government seeks to maximize social welfare (equation (6)) subject to the economy's resource constraint.

$$bh_b + \int_{a_0}^{a_1} c_a h_a da = b + \int_{a_0}^{a_1} (c_a - b) h_a da = \int_{a_0}^{a_1} a h_a da - \int_{a_0}^{a_1} v_a \chi_a da \tag{15}$$

In Appendix A.4 I show that the effect of the minimum wage on the job finding rate for occupation a_0 is $\frac{dp_{a_0}}{d\underline{w}} = -\frac{(1-\mu_{a_0})p_{a_0}}{\mu_{a_0}(1-\beta_{a_0})(a-T_{a_0}-b)} < 0$. This implies that $\frac{dU_{a_0}}{d\underline{w}} = p_{a_0}\left[\frac{\mu_{a_0}-\beta_{a_0}}{\mu_{a_0}(1-\beta_{a_0})}\right]$. Therefore, introducing a minimum wage increases the expected utility of workers in occupation a_0 if and only if $\beta_{a_0} < \mu_{a_0}$; that is if worker's bargaining power is below the Hosios (1990) condition.[33]

[31] Assuming that $\beta_a = \beta(a)$ is continuous and (weakly) increasing in a provides a micro-foundation for Assumption 1 in Section 3.1.

[32] There several differences between the model proposed here and the model Hungerbüler and Lehmann (2009). One difference is that the benefit individuals derive from being out of the labor force (analogous to search costs) is constant across all individuals in Hungerbüler and Lehmann (2009), so that all individuals above some skill level participate. In contrast, the present model allows search costs to vary across individuals with the same skill level. This leads to differences in labor supply between individuals with the same productivity, as well as differences across skill groups. A second difference is that I adopt a sufficient statistics approach. Hungerbüler and Lehmann (2009) explicitly model the labor demand side of the market and assume that wages are determined by Nash bargaining. As described below, the sufficient statistics approach allows me to derive a desirability condition for the minimum wage in terms of elasticities and marginal social welfare weights, rather than the structural primitives of a model.

[33] With fixed wages, introducing a binding minimum wage above the equilibrium wage increases the expected utility of workers in occupation a_0 (leading to a positive macro participation response) if $g_{a_0} > 1$ and the elasticity of the matching function with respect to the number of job seekers (μ_{a_0}) is sufficiently large.

Substituting $\frac{dU_{a_0}}{d\underline{w}}$ into the optimality condition for the minimum wage in the simple search and matching economy gives:

$$\frac{1}{\lambda}\left.\frac{d\mathscr{L}^*}{d\underline{w}}\right|_{w\,=\,w^*} = \left[\frac{\mu_{a_0}-\beta_{a_0}}{\mu_{a_0}(1-\beta_{a_0})}\right] g_{a_0} h^*_{a_0} - h^*_{a_0} + \left[a_0 - (c^*_{a_0} - b^*)\right]\frac{dh^*_{a_0}}{d\underline{w}} - \chi^*_{a_0}\frac{dv^*_{a_0}}{d\underline{w}} \tag{16}$$

The introduction of a minimum wage $\underline{w} = w^*_{a_0} + dw$ has three effects on the government's objective. The first term is the social welfare effect for workers in occupation a_0. If worker's bargaining power is below the Hosios (1990) condition (i.e. $\beta_{a_0} < \mu_{a_0}$), the introduction of a minimum wage increases the expected utility of workers in occupation a_0. The intuition for this is as follows. When $\beta_{a_0} < \mu_{a_0}$, the (optimal tax) equilibrium wage and labor force participation are inefficiently low. Introducing a binding minimum wage improves the effective bargaining power of workers and increases labor force participation, making it easier for firms to match with workers. Thus, if $\beta_{a_0} < \mu_{a_0}$, introducing a minimum wage leads to efficiency gains.

The assumption that wages are determined by proportional bargaining implies that the optimal income tax cannot correct for the congestion externality that arises because of an inefficiently low worker's bargaining power (Jacquet, Lehmann, and Linden (2012, 2014)). This is because under the proportional bargaining solution, workers receive a fixed fraction of the match surplus; this fraction cannot be altered by "corrective" taxation. This is in contrast to the Nash bargaining solution, where the gross wage depends on the marginal tax rate. Boone and Bovenberg (2002) show that in the Nash bargaining situation, setting a negative marginal tax rate increases worker's effective bargaining power, leading to a higher after-tax wage and greater labor force participation. Thus, whether the minimum wage can correct for congestion externalities in a search an matching model depends on the wage bargaining process.[34]

The second term in (16) captures the effect on the government's objective when more of the economy's resources are allocated to workers in occupation a_0. Introducing a binding minimum wage leads to $h_{a_0}dw$ more resources being allocated to workers in occupation a_0, tightening the economy's resource constraint. The third and fourth terms capture the effect on net output. If the minimum wage leads a decline in employment, $[a_0(1-\beta_{a_0}) + \beta_{a_0}(T_{a_0} + b)]dh_{a_0}$ units of output are lost. However, fewer resources are spent on creating vacancies: $\chi_{a_0}dv_{a_0}$ units of output are saved due to the creation of fewer vacancies. These opposing effects imply that the effect of the minimum wage on net output is ambiguous, a point made in a similar setting by Hungerbüler and Lehmann (2009) (pages 473-474).

Using the the equation for the firm's match rate and the free-entry condition, the final term in equation (16) simplifies to $\chi_{a_0}\frac{dv^*_{a_0}}{d\underline{w}} = \frac{a_0 - w^*_{a_0}}{q^*_{a_0}}(q^*_{a_0}\frac{dh^*_{a_0}}{d\underline{w}} - h^*_{a_0}\frac{dq^*_{a_0}}{d\underline{w}}) = (a_0 - w^*_{a_0})\frac{dh^*_{a_0}}{d\underline{w}} - h^*_{a_0}$. Substituting this into (16) and using the relationship between $\frac{dU_{a_0}}{d\underline{w}}$ and the ratio of the macro and micro

[34] It is important to note that a minimum wage may also be desirable when wages are determined by Nash bargaining if the government uses taxation to redistribute income in addition to restoring efficiency. This is because progressive taxation requires a positive marginal tax rate, whereas correcting for an inefficiently low bargaining power requires a negative marginal tax rate. Due to this conflict, the optimal policy may involve a positive marginal tax rate to redistribute from high to low-ability workers, together with a minimum wage, as in Hungerbüler and Lehmann (2009).

participation elasticities with respect to the minimum wage shows the link between (13) and the desirability condition for the minimum wage in this simple search and matching economy.

$$\frac{1}{\lambda h^*_{a_0}} \left.\frac{d\mathscr{L}^*}{d\underline{w}}\right|_{w=w^*} = \left[\frac{\mu_{a_0} - \beta_{a_0}}{\mu_{a_0}(1-\beta_{a_0})}\right] g_{a_0} + (T^*_a + b^*)\frac{dh^*_{a_0}}{d\underline{w}} \equiv \frac{e_{a_0}}{e^m_{a_0}} g_{a_0} + \tau^*_{a_0}\eta_{a_0}\frac{w^*_{a_0}}{\underline{w}}$$

The following proposition (proof in Appendix A.4) illustrates the link between the the effect of the introduction of a binding minimum wage on the macro participation response and the worker's bargaining power.

Proposition 4. *In a search and matching economy where wages are determined by proportional bargaining the macro participation response to the minimum wage is positive if and only if $\beta_a < \mu_a$ (i.e. the worker's exogenous bargaining power is lower than the Hosios (1990) condition).*

Proposition 4 shows that there is a direct link between the worker's bargaining power, the elasticity of the matching function with respect to the number of searchers and the macro participation response to the minimum wage. A positive macro participation response indicates that the bargaining share of workers in occupation a_0 is inefficiently low. This result is useful for policy makers because the minimum wage is often justified on the grounds that worker's bargaining power is too low (Flinn (2006); Hungerbüler and Lehmann (2009)). However, estimating worker's bargaining share may be difficult in practice, and estimates in the literature vary widely (Flinn (2006); Ahn, Arcidiacono, and Wessels (2011), others). Proposition 4 shows that it is not necessary to estimate β_{a_0} directly to determine whether introducing (or raising) the minimum wage increases the expected utility of low-wage workers.[35]

Although the minimum wage can increase worker's effective bargaining power, it is a blunt policy tool. It would be better to set $\beta_{a_0} = \mu_{a_0}$ directly. In the absence of the ability to set worker's bargaining share directly, Proposition 4 shows that the minimum wage can increase welfare if the efficiency gains due to correcting the congestion externality is at least as large as the potential fiscal costs.

4.2 Competitive Labor Market Models

In a perfectly competitive labor market, firms and workers take the market wage as given and prior to government intervention, all individuals that are willing to work at the market wage are employed. The general model introduced in Section 2 collapses to the perfectly competitive model when search frictions are assumed away ($p_a = 1$). In this case, the (expected) utility from participating in labor market for a type-a worker is $c_a - \theta$. The no spillovers assumption (Assumption 2) can be micro-founded by assuming that the production technology is linear.[36] Let

[35]The macro and micro participation responses to the minimum wage are equal if $\beta_{a_0} = 0$ or if $\beta_{a_0} = 1$.

[36]I maintain this assumption throughout subsection 4.2. Together with the assumption that occupations are skill-specific implies that for each labor market, the production function is $y(a)h_a$. Firm's profits are equal to $\Pi = (y(a) - w^{CE}_a)h^{CE}_a - t_{\Pi_a}$. I further assume that the production technology exhibits decreasing returns to scale and that the government can impose a lump-sum tax on profits so that firms earn zero profits in equilibrium.

$(b^{CE}, \{T_a^{CE}, w_a^{CE}, c_a^{CE}, h_a^{CE}\}_{a\in[a_0,a_1]})$ denote the equilibrium non-employment benefit, occupation-specific tax liabilities, wages, consumption, number of workers in the optimal tax competitive equilibrium.

Efficient rationing: Introducing a binding minimum wage in a perfectly competitive labor market leads to job rationing; some job seekers willing to work at the minimum wage are unable to find work. To illustrate the link between the present paper and the occupational choice models in Lee and Saez (2008, 2012), I adopt their efficient rationing (i.e. workers that lose their jobs due to the minimum wage are those with the lowest private surplus from working). The efficient rationing assumption implies that marginal workers induced to enter the labor force due to the minimum wage do not displace workers with a higher surplus that are already in the market. Since the likelihood of finding a job does not change for inframarginal workers, the macro and micro participation responses to the minimum wage are equal in the competitive model with efficient rationing. Differentiating the government's objective function with respect to $\underline{w}$ yields[37]

$$\frac{1}{\lambda}\left.\frac{d\mathscr{L}^{CE}}{d\underline{w}}\right|_{w\,=\,w^{CE}} = g_{a_0}h_{a_0}^{CE} + (T_{a_0}^{CE} + b^{CE})\frac{dh_{a_0}^{CE}}{d\underline{w}} \tag{17}$$

Further assuming that wages are fixed with respect to taxes implies that the optimal employment tax rate (equation (12)) reduces to $\frac{\tau_a}{1-\tau_a} = \frac{1-g_a}{\zeta_a}$ for all a, a result first derived by Saez (2002). Introducing a binding minimum wage increases social welfare only if (assuming dw is small so that $w_{a_0}^{CE}/\underline{w} \approx 1$):

$$g_{a_0}h_{a_0} + \left[\frac{1-g_{a_0}}{\zeta_{a_0}}\right]\eta_{a_0}w_{a_0}^{CE}\frac{h_{a_0}}{\underline{w}} \approx g_{a_0} + \left[\frac{1-g_{a_0}}{\zeta_{a_0}+1-g_{a_0}}\right]\eta_{a_0} > 0 \tag{18}$$

Thus, in the competitive model with efficient rationing, the desirability of the minimum wage depends only on the marginal social welfare weight and the macro employment responses to taxes and the minimum wage. Individuals in occupation a_0 with a positive surplus from working keep their job and enjoy dw more income, a gain the government values at g_{a_0} per worker. Moreover, if $g_{a_0} > 1$, the optimal employment tax at the bottom of the wage distribution is negative. Given that the employment elasticity η_{a_0} is negative in the competitive model, $g_{a_0} > 1$ is a sufficient condition for the minimum wage to be desirable, assuming the efficient rationing assumption holds. This is equivalent to the the result in Proposition 2 in Lee and Saez (2012), for the special case when the production technology is linear.[38]

Uniform rationing: Next, I consider the more realistic case where job rationing is unform (i.e. that all workers in occupation a_0 have face an equal likelihood of losing their job due to the minimum wage). With uniform rationing, the inequality for the desirability of the minimum

[37]By the envelope theorem, those that lose their jobs due to the minimum wage (captured by the macro employment elasticity η_{a_0}) are just indifferent between working and not, and so have no first order effect on the social welfare function. These employment losses only affect the government's objective through changes in the budget constraint.

[38]Lee and Saez (2012) assume that there are a discrete number of occupations in the economy and that individuals differ only in the disutility of work. Since I assume that labor markets are perfectly segmented by skill, within an occupation, individuals only differ along the search cost dimension, similar to Lee and Saez (2012)

wage depends on the interpretation of the parameter θ. If θ is a fixed cost paid by all market participants upon entering the labor force (as is the case in the general model described earlier), then the condition for the desirability of the minimum wage is identical to (13). To see this, first note that the introduction of a minimum wage will encourage some individuals to enter the labor force. Let $\tilde{\theta}$ denote the new cutoff value for θ.[39] Although these agents are marginal, they may displace some of those that are initially working. If θ is a fixed cost borne by all individuals that enter the labor force (regardless of whether or not they find a job), then all market participants receive the same gross surplus from working.

Prior to the introduction of the minimum wage, the (gross) expected utility of participating in the labor force is $U_a^{CE} = p_a^{CE} c_a^{CE} + (1 - p_a^{CE}) b^{CE} = c_a^{CE}$ (since $p_a^{CE} = 1$). Now suppose that the government introduces a small, binding minimum wage, just above the equilibrium wage in occupation a_0, $\underline{w} = w_{a_0}^{CE} + dw$.[40] This reform has the following effect on the expected utility of those (initially) working in occupation a_0: $dU_a = p_a dc_a d\underline{w} + dp_a [c_a - b] d\underline{w} = p_a \left[1 + dp_a \frac{c_a - b}{p_a}\right] d\underline{w}$. Since the macro participation response is: $\frac{dk_{a_0}}{d\underline{w}} = g(U_a - U_b | a) f(a) \frac{dU_a}{d\underline{w}} = \left[1 + \frac{dp_{a_0}}{d\underline{w}} \frac{c_a - b}{p_{a_0}}\right] \cdot \left.\frac{dk_{a_0}}{d\underline{w}}\right|^{\text{micro}}$, the ratio of the macro and micro participation responses to the minimum wage corresponds exactly to the effect of introducing a minimum wage on the expected utility of workers in occupation a_0. The condition for the desirability of the minimum wage is given by Proposition 3.

On the other hand, if θ is a (fixed) cost of work, borne only by those that find a job, then the introduction of a minimum wage may lead to first-order welfare losses due to those with a lower (ex-post) surplus from *working* displacing those with a higher surplus. Proposition 5 shows that the desirability of the minimum wage depends on an additional term that captures the welfare loss due to this allocative inefficiency.[41]

Proposition 5. *Suppose that the labor market is perfectly competitive, unemployment due to the minimum wage is uniformly rationed, $g_{a_0} > 1$, and that the parameter θ is the cost of working (paid only by those that find a job). Then, introducing a small, binding minimum wage just above the (optimal tax) competitive equilibrium wage at the bottom of the earnings distribution, $\underline{w} = w_{a_0}^{CE} + dw$, is desirable if*

$$\frac{g_{a_0}^u}{g_{a_0}} < \frac{k_{a_0}}{e_{a_0} - \eta_{a_0}} \tag{19}$$

where $g_{a_0}^u = \int_0^{c_{a_0} - b} [\Phi(c_{a_0} - \theta) - \Phi(b)] g(\theta|a) d\theta f(a) / \lambda k_{a_0}$ is the welfare weight on the marginal unemployment losses in occupation a_0 due the minimum wage.

The proof is presented in Appendix A.5.

[39]The new cutoff value is $\tilde{\theta} = \tilde{p}_{a_0}(\underline{w} - T_{a_0}) + (1 - \tilde{p}_{a_0}) b$, where $\tilde{p}_{a_0} \in (0, 1]$ is the new uniform job finding probability ($\tilde{p}_{a_0} < p_{a_0}^{CE} = 1$; $p_{a_0}^{CE}$ is the equilibrium job finding rate prior to the introduction of the minimum wage). The minimum wage induces individuals with values of $\theta \in [\hat{\theta}, \tilde{\theta}]$ to enter the labor force.

[40]I drop the supercripts for esthetic reasons.

[41]Equation (19) uses the formula for the optimal employment tax, $\frac{T_{a_0} + b}{w_{a_0}^{CE}} = \frac{1 - g_{a_0}}{\zeta_{a_0} + 1 - g_{a_0}}$, the fact that the change in the job finding probability can be written as $\frac{dp_{a_0}}{d\underline{w}} = \frac{p_{a_0}}{\underline{w}} (\eta_{a_0} - e_{a_0})$, and that $w_{a_0}^{CE} / \underline{w} \approx 1$ when the minimum wage change, dw is small.

The intuition for Proposition 5 is as follows. When θ is the cost of working, introducing a minimum wag leads to two social welfare effects and one fiscal effect. The first social welfare effect is the increase in the government's objective due to higher earnings (and therefore consumption) for workers in occupation a_0. The value that the government places on this change is g_{a_0} per worker in occupation a_0. The second social welfare effect is the loss in welfare due to some workers with a low surplus from working displacing those that value working more. The value that the government places on this loss (in monetary terms) is $g^u_{a_0} k_{a_0}$ multiplied by the change in the likelihood that existing workers find a job under the new policy, $-[e_{a_0} - \eta_{a_0}]$. The fiscal effect is the change in the government's revenue due to the unemployment generated by the minimum wage. If $g_{a_0} > 1$, then workers in occupation a_0 receive a subsidy, so a decline in the level of employment relaxes the budget constraint. Introducing a minimum wage is desirable if the sum of these effects is positive.

Proposition 5 shows that the sufficient statistics that determine the desirability of the minimum wage in the uniform rationing case are: the macro participation and employment elasticities with respect to the minimum wage, the government's marginal social welfare weight on consumption for workers in occupation a_0, the governemnt's welfare weight on unemployment losses and the optimal employment tax at the bottom of the wage distribution.

5 Data and Empirical Strategy

The theoretical analysis shows that evaluating the effect of the minimum wage on social welfare requires estimates of the macro and micro participation responses to the minimum wage, in addition to the macro employment response to the minimum wage. This section describes the data and empirical strategy used to estimate these responses using U.S. data from the past three decades.

5.1 Data

Individual-level information on labor force participation and employment is obtained from the monthly Outgoing Rotation Group (MORG) from the Current Population Survey (CPS) extracted from the NBER web site.[42] The advantage of the CPS relative to administrative datasets used in recent minimum wage studies, such as the Business Dynamics Statistics (BDS), Quarterly Workforce Indicators (QWI) and the Quarterly Census of Employment and Wages (QCEW), is that it provides information on both employment and labor force participation behavior. I pool MORG cross-sections for the 1989 to 2011 period (inclusive). I select 1989 as the first year of the sample because there is little variation in minimum wages across states before 1989. The MORG data contain information on labor market participation, employment, hours worked, earnings, date of the interview and demographic information individuals and their families. Each observation in the data represents a person-month-year.

[42] http://www.nber.org/cps/.

The government's objective in the theoretical model is to choose the optimal minimum wage and tax policy to maximize social welfare. In the model, prospective workers searching in occupation a_0 are the lowest-skilled workers in the economy. To focus on this group, I select as the analysis sample, unmarried individuals with a high school degree or less between the ages of 16 and 45 that report not being in school full time.[43] Perhaps most importantly, I exclude teens that are in school full time from this sample. The optimal redistributive policy for this group is a mix of a net liability and possibly the minimum wage. Since dependents, including teenagers that are in school full time, are not the intended recipients of tax and transfer policies, excluding this group seems consistent with the theoretical model where all agents are single-person households. Section 6.2 discusses how the inclusions of teens affects interpretation of the marginal welfare effects of the minimum wage.

For each month they are in the survey, CPS respondents are asked about their labor force status. Specifically, respondents are asked about their labor force activities in the week prior to the interview, known as the 'reference week'. I classify an individual as employed if they report being employed and at work, or if they report being employed but temporarily absent from work during the reference week (i.e. due to an illness). A respondent is classified as a participant in the labor force if they report being employed or if they report being unemployed and searching for work. For individuals that are employed, I also construct an hourly wage variable by dividing a respondent's reported weekly earnings by the number of hours they report working in a typical week. The hourly wage variable is used primarily to analyze the extent to which a state's prevailing minimum wage is binding on various demographic groups. The CPS data are merged to quarterly state minimum wage data from Dube, Lester, and Reich (2016). All dollar amounts are inflated to 2010 dollars using the national urban consumer price index.

Table 1 (column 1) reports summary statistics for the sample of unmarried individuals with a high school degree or less in the CPS MORG. Column 2 restricts the sample to unmarried women between the ages of 16 and 45 with a high school degree or less. There are several differences between columns 1 and 2. Women with low education are less likely to be white and earn lower wages than similarly skilled men. However, women and men face similar unemployment rates in this sample. In particular, the unemployment rate for women is $\frac{0.70-0.62}{0.70} = 0.11$, compared to $\frac{0.77-0.67}{0.77} = 0.13$ for men. Columns 3 and 4 show how the summary statistics vary between women with less than a high school diploma and those with a diploma or a GED and those less than a high school diploma. Women with less than a high school degree are much less likely to be in labor force and face higher unemployment rates. The unemployment rate is $\frac{0.55-0.44}{0.55} = 0.20$ for high school dropouts, compared to $\frac{0.78-0.70}{0.78} = 0.10$ for women with a high school diploma. This suggests that search frictions may be relatively more important for the low-skilled job-seekers.

[43] I exclude married (including common-law) individuals to be consistent with the theoretical section that abstracts from labor force participation decisions for two-person households.

Table 1: Summary Statistics (1989-2011)

Variable	(1) Men & Women	(2) Women Only	(3) Female HS Dropouts	(4) Female HS Graduates
Age	28.54	29.03	27.36	29.85
	(8.43)	(8.65)	(9.11)	(8.30)
White	0.74	0.70	0.70	0.70
	(0.44)	(0.46)	(0.46)	(0.46)
Black	0.22	0.25	0.25	0.26
	(0.41)	(0.43)	(0.43)	(0.44)
Labor Force Participation	0.77	0.70	0.55	0.78
	(0.42)	(0.46)	(0.50)	(0.41)
Employment	0.67	0.62	0.44	0.70
	(0.47)	(0.49)	(0.50)	(0.46)
Real minimum wage	6.92	6.91	6.92	6.90
	(0.69)	(0.68)	(0.69)	(0.67)
Hourly wage (mean)	13.38	12.21	9.83	12.93
	(8.70)	(8.11)	(10.31)	(7.16)
Hourly wage $p = 10\%$	7.17	6.92	6.30	7.21
Hourly wage $p = 25\%$	8.66	8.24	7.32	8.70
Hourly wage $p = 50\%$	11.40	10.55	8.69	11.33

Notes: The sample in column 1 is all unmarried individuals between the ages of 16 and 45 (inclusive) that are not in school full-time in the CPS MORG between the years of 1989 and 2011. Beginning in column 2, the sample is restricted to women. Wages are only reported for those that are working in the CPS. All dollar amounts are inflated to 2010 dollars.

5.2 Empirical Strategy

The first challenge in estimating the participation and employment responses to the minimum wage is defining labor markets that approximate the theoretical model. Following the empirical literature that estimates the labor supply responses to taxes, I assume that labor markets can be defined by an individual's education (e), state (s) and year (t) (Rothstein (2010); Kroft, Kucko, Lehmann, and Schmieder (2015)). The implicit assumption underlying this definition of labor markets is that individuals with the same education attainment and living in the same state and year are viewed as substitutes by employers. Consequently, individuals within the same labor market face the similar wages and net tax liability.

The second step in estimating the participation and employment responses to the minimum wage is obtaining an empirical specification that is motivated by the theoretical model. I assume that the labor force participation decision for individual i in labor market $m = (e, s, t)$ depends only on the wage, job-finding rate and tax liability in labor market m. Moreover, I assume that the job finding rate and wage in occupation (labor market) m depends only on the minimum wage and (average) tax liability in occupation m. Under these assumptions, the labor force participation decision for individual i in labor market m can be written as follows.

$$\mathcal{K}_{i(m)} = \hat{\mathcal{K}}_{i,e,s,t}(p_{e,s,t}(\underline{w}, T_{e,s,t}), w_{e,s,t}(\underline{w}, T_{e,s,t}), T_{e,s,t})$$

Taking a linear approximation to $\mathcal{K}_{i(m)}$ gives the baseline estimating equation:

$$k_{i(m)} = \beta^K \underline{w} + (\alpha^K + \gamma^K) T_m + X'_{i(m)} \Lambda^K + \delta_s + \delta_t + u_{i(m)} \tag{20}$$

where $\beta^K = \left[\frac{\partial \hat{\mathcal{K}}_{i(m)}}{\partial p_m} \frac{\partial p_{i(m)}}{\partial \underline{w}} + \frac{\partial \hat{\mathcal{K}}_{i(m)}}{\partial w_m} \frac{\partial w_m}{\partial \underline{w}} \right]$, $\gamma^K = \left[\frac{\partial \hat{\mathcal{K}}_{i(m)}}{\partial p_m} \frac{\partial p_{i(m)}}{\partial T_m} + \frac{\partial \hat{\mathcal{K}}_{i(m)}}{\partial w_m} \frac{\partial w_m}{\partial T_m} \right]$ and $\alpha^K = \frac{\partial \hat{\mathcal{K}}_{i(m)}}{\partial T_m}$. The parameter β captures the macro participation response to the minimum wage in occupation m. $X'_{i(m)}$ are individual characteristics and δ_s and δ_t are state and year fixed effects respectively. Finally, α^K is the micro participation response to taxes and $\alpha^K + \gamma^K$ is the macro participation response to taxes for occupation m.

In order to focus on the effect of the minimum wage on labor market outcomes, I drop T_m from the estimating equation (20). Assuming that changes in the average tax liability are uncorrelated with changes in the minimum wage, this should not affect the estimates for β^K.[44] Thus, defining $e_{i(m)} = (\alpha^K + \gamma^K) T_m + u_{i(m)}$, the estimating equation for the participation response to the minimum wage can be written as follows.

$$k_{i(m)} = \beta^K \underline{w} + X'_{i(m)} \Lambda^K + \delta_s + \delta_t + e_{i(m)} \tag{21}$$

Similarly, the estimating equation for the employment rate is[45]

$$h_{i(m)} = \beta^H \underline{w} + X'_{i(m)} \Lambda^H + \delta_s + \delta_t + \nu_{i(m)} \tag{22}$$

6 Results [PRELIMINARY AND INCOMPLETE]

6.1 Main Results

Coming soon.

6.2 Robustness Checks and Sensitivity to Sample Selection

Coming soon.

7 Concluding Remarks and Extensions

This paper considers the conditions under which introducing a binding minimum wage can improve upon an allocation with a second-best optimal income tax. I develop a theoretical model that allows for involuntary unemployment where individuals differ along two dimensions: ability and search costs. After setting an optimal non-linear income tax, a social-welfare maximizing government considers whether to introduce a small, binding minimum wage above the equilibrium

[44] I show how the inclusion of a proxy for T_m affects the estimate for β in Section 6.2. I proxy for the tax liability faced by low-skilled workers with the level of the state EITC.

[45] where $\beta^H = \frac{\partial p_{i(m)}}{\partial \underline{w}} + \beta^K$, $\gamma^H = \frac{\partial p_{i(m)}}{\partial T_m} + \gamma^K$, $\alpha^H = \alpha^K$, and $\nu_{i(m)} = (\alpha^H + \gamma^H) T_m + \epsilon_{i(m)}$.

wage at the bottom of the earnings distribution. I derive a desirability condition for the minimum wage that can be expressed in terms of marginal social welfare weights and labor force participation and employment elasticities with respect to the minimum wage. Although I adopt a sufficient statistics approach, I show that the desirability condition for the minimum wage is valid under a number of models of the labor market commonly used in the normative analysis of minimum wage legislation minimum. The main insight from the model is that the effect of the minimum wage on the welfare of workers at the bottom of the wage distribution is captured by the ratio of the macro and micro participation responses to the minimum wage. The macro employment response to the minimum wage, the object of interest of much of the empirical literature on the minimum wage, matters only through its impact on the government's budget constraint.

Using state minimum wage variation from 1989 to 2011 I estimate the sufficient statistics that are inputs into the desirability condition. Preliminary estimates indicate that a 10 percentage point increase in the minimum wage increases the labor force participation rate of unmarried women with a high school diploma or less by XX percent. This estimate is relatively robust to controls for underlying pre-trends in state labor market conditions commonly used in the literature.

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A Theoretical Appendix

A.1 Proof of Proposition 2 (See page 14)

Since $U_a = p_a(c_a - b) + b = p_a w_a(1-\tau_a) + b$ the government's objective function with fixed tax rates is

$$\mathscr{L}^{\bar{\tau}} = SW^{\bar{\tau}} - \lambda(b + E - \int_{a_0}^{a_1} w_a \bar{\tau}_a h_a da)$$

where

$$SW^{\bar{\tau}} = \int_{a_0}^{a_1} \left[\int_0^{\hat{\theta}} \Phi(p_a^{\bar{\tau}} w_a^{\bar{\tau}}(1-\bar{\tau}_a) + b - \theta) m(\theta|a) d\theta + \Phi(b)[1 - M(\hat{\theta}|a)] \right] f(a) da$$

Differentiating this with respect to the minimum wage (holding tax rates fixed) and evaluating at the fixed tax rate equilibrium wage yields

$$\frac{1}{\lambda}\frac{d\mathscr{L}^{\bar{\tau}}}{d\underline{w}} = \frac{1}{\lambda}\int_0^{\hat{\theta}} \Phi'(.) g(\theta|a_0) d\theta f(a) \left[p_{a_0}^{\bar{\tau}}(1-\bar{\tau}_{a_0}) + \frac{dp_{a_0}^{\bar{\tau}}}{d\underline{w}} w_{a_0^{\bar{\tau}}}(1-\bar{\tau}_{a_0}) \right] + \bar{\tau}_{a_0} + w_{a_0}^{\bar{\tau}} \bar{\tau}_{a_0} \frac{dh_{a_0}^{\bar{\tau}}}{d\underline{w}}$$

Using the definition for $g_{a_0} = \frac{p_{a_0} \int_0^{\hat{\theta}} \Phi'(U_{a_0} - \theta) m(\theta|a_0) d\theta f(a_0)}{\lambda h_{a_0}}$ (where the superscript $\bar{\tau}$ has been suppressed for esthetic purposes)

$$\frac{1}{\lambda}\frac{d\mathscr{L}^{\bar{\tau}}}{d\underline{w}} = g_{a_0}\lambda h_{a_0}(1-\bar{\tau}_{a_0}) \left[1 + \frac{dp_{a_0}}{d\underline{w}} \frac{w_{a_0}}{p_{a_0}} \right] + \bar{\tau}_{a_0} + w_{a_0}\bar{\tau}_{a_0}\frac{dh_{a_0}}{d\underline{w}}$$

Since number of participants in the type-a_0 occupation is $k_{a_0}^{\bar{\tau}} = M(p_{a_0}^{\bar{\tau}} w_{a_0}^{\bar{\tau}}(1-\bar{\tau})|a_0) f(a_0)$, the micro and macro participation responses to the minimum wage are respectively

$$\left.\frac{dk_{a_0}}{d\underline{w}}\right|^{\text{micro}} = m(p_{a_0}^{\bar{\tau}} w_{a_0}^{\bar{\tau}}(1-\bar{\tau}_{a_0})|a_0) f(a_0) p_{a_0}(1-\bar{\tau}_{a_0})$$

$$\frac{dk_{a_0}}{d\underline{w}} = m(p_{a_0}^{\bar{\tau}} w_{a_0}^{\bar{\tau}}(1-\bar{\tau}_{a_0})|a_0) f(a_0) p_{a_0}^{\bar{\tau}}(1-\bar{\tau}_{a_0}) \left[1 + \frac{dp_{a_0}^{\bar{\tau}}}{d\underline{w}} \frac{w_{a_0}^{\bar{\tau}}}{p_{a_0}^{\bar{\tau}}} \right] = \left[1 + \frac{dp_{a_0}^{\bar{\tau}}}{d\underline{w}} \frac{w_{a_0}^{\bar{\tau}}}{p_{a_0}^{\bar{\tau}}} \right] \cdot \left.\frac{dk_{a_0}}{d\underline{w}}\right|^{\text{micro}}$$

Substituting the ratio of the macro and micro participation responses and using the elasticity definitions yields:

$$\frac{1}{\lambda h_{a_0}^{\bar{\tau}}}\frac{d\mathscr{L}^{\bar{\tau}}}{d\underline{w}} = \frac{e_{a_0}}{e_{a_0}^m} g_{a_0} + \bar{\tau}_{a_0} + w_{a_0}^{\bar{\tau}} \bar{\tau}_{a_0} \eta_{a_0} \underline{w}^{-1}$$

For $d\underline{w}$ small, $\frac{w_{a_0}^{\bar{\tau}}}{\underline{w}} \approx 1$, which leads to equation (11).

A.2 Proof of Lemma 1 (See page 15)

In the full model, the government sets a non-linear tax schedule $T(.)$ and a benefit b for non-employed individuals. The optimal non-linear income tax solves an adverse selection problem. The government would like to set taxes optimally to redistribute income from high-ability to low-ability workers, and possibly to "correct" labor market inefficiencies. The government's ability to redistribute income is constrained by (a) its budget constraint (7), and (b) a first-order incentive constraint. The latter states that the optimal tax schedule must incentivize a type-a individual to report the wage/earnings that corresponds to her type, w_a.[46] Formally, the government's problem is

$$\max_{b,\{T_a,c_a,w_a,U_a\}_{a\in[a_0,a_1]}} \int_{a_0}^{a_1} \left(\int_0^{\hat{\theta}} \Phi[U_a-\theta]g(\theta|a)d\theta + \Phi[U_b](1-M(\hat{\theta}|a)) \right) f(a)da \tag{1}$$

subject to

$$b+E \leq \int_{a_0}^{a_1} (T_a+b)h_a da \tag{2}$$

$$\dot{c}(a) \geq 0; \quad \dot{w}(a) \geq 0 \tag{3}$$

where U_{a_0} (and c_{a_0}) and U_{a_1} and (c_{a_1}) are free.

Assumption 4 guarantees that an optimal-tax equilibrium exists and that the monotonicity condition holds with a strict inequality ($\frac{dw(.;a)}{da} > 0$). Assumption 4 also implies that the first-order incentive condition, $\frac{dc_a}{da} > 0$, also holds with a strict inequality at the optimum (i.e. marginal tax rates are less than 100 percent at the optimum).

Since Assumption 4 guarantees that a unique, separating equilibirum exists, characterizing the optimal employment tax amounts to choosing a number b and a tax liability for each occupation, $T_a, \;\; a \in [a_0, a_1]$, to solve the following "relaxed problem".

$$\mathscr{L} = \int_{a_0}^{a_1} \left\{ \int_0^{\hat{\theta}} \Phi[U_a-\theta]m(\theta|a)d\theta + \Phi[U_b](1-M(\hat{\theta}|a)) \right\} f(a)da - \lambda b - \lambda E + \lambda \int_{a_0}^{a_1} (T_a+b)h_a$$

Using Assumptions 2 and 4, the first-order condition with respect to T_a is:

$$0 = \frac{1}{\lambda}\frac{d\mathscr{L}}{dT_a} = \frac{1}{\lambda}\int_0^{\hat{\theta}} \Phi'[.]m(\theta|a)d\theta f(a)p_a \left(\frac{dw_a}{dT_a} - 1 + \frac{dp_a}{dT_a}\frac{w_a(1-\tau_a)}{p_a} \right) + h_a + (T_a+b)\frac{dh_a}{dT_a}$$

[46]For now, I ignore the (strict) monotonicity constraint, which states that the gross wage must be (strictly) increasing in ability at the optimum. In the general model, the gross wage w_a may be affected by tax liability in occupation a, T_a. In previous sections, Assumptions 1 and 3 guarantee that wages are strictly increasing in productivity.

$$0 = g_a h_a \left(\frac{dw_a}{dT_a} - 1 + \frac{dp_a}{dT_a} \frac{w_a(1-\tau_a)}{p_a} \right) + h_a + (T_a + b) \frac{dh_a}{dT_a} \tag{4}$$

The micro and macro participation responses to taxes are respectively

$$\left. \frac{dk_a}{dT_a} \right|^{\text{micro}} = m(U_a - U_b|a) f(a) \left. \frac{dU_a}{dT_a} \right|^{\text{micro}} = -m(U_a - U_b|a) f(a) p_a$$

$$\frac{dk_a}{dT_a} = m(U_a - U_b|a) f(a) \frac{dU_a}{dT_a} = m(U_a - U_b|a) f(a) p_a \left[\frac{dw_a}{dT_a} - 1 + \frac{dp_a}{dT_a} \frac{w_a(1-\tau_a)}{p_a} \right]$$
$$= - \left[\frac{dw_a}{dT_a} - 1 + \frac{dp_a}{dT_a} \frac{w_a(1-\tau_a)}{p_a} \right] \cdot \left. \frac{dk_a}{dT_a} \right|^{\text{micro}}$$

Substituting the term in square brackets into the first order condition gives

$$0 = -g_a h_a \frac{\frac{dk_a}{dT_a}}{\left. \frac{dk_a}{dT_a} \right|^{\text{micro}}} + h_a + (T_a + b) \frac{dh_a}{dT_a}$$

Let $\pi_a^m = -\frac{w_a(1-\tau_a)}{k_a} \left. \frac{dk_a}{dT_a} \right|^{\text{micro}}$ denote the micro participation elasticity with respect to (own) taxes, $\pi_a = -\frac{dk_a}{dT_a} \frac{w_a(1-\tau_a)}{k_a}$ the macro participation elasticity with respect to taxes, and $\zeta_a = -\frac{dh_a}{dT_a} \frac{w_a(1-\tau_a)}{h_a}$ the macro employment elasticity with respect to taxes. Applying the elasticity definitions and using the definition of τ_a leads to equation (12).

$$\frac{\tau_a}{1-\tau_a} = \frac{1 - \frac{\pi_a^m}{\pi_a^m} g_a}{\zeta_a}$$

For the second part of the lemma, first take the first order condition with respect to b

$$0 = \frac{1}{\lambda} \int_{a_0}^{a_1} \left(\int_0^{\hat{\theta}} \Phi'(.) m(\theta|a) d\theta \frac{dU_a}{db} + \Phi'(U_b)[1 - M(\hat{\theta}|a)] \right) f(a) da - 1 + \int_{a_0}^{a_1} \left(h_a + (T_a + b) \frac{dh_a}{db} \right) da$$

$$0 = \int_{a_0}^{a_1} \frac{\int_0^{\hat{\theta}} \Phi'(.) m(\theta|a) d\theta}{\lambda} \frac{dU_a}{db} f(a) da + \int_{a_0}^{a_1} \frac{\Phi'(U_b)[1 - M(\hat{\theta}|a)]}{\lambda} f(a) da - 1 + \int_{a_0}^{a_1} h_a da + \int_{a_0}^{a_1} (T_a + b) \frac{dh_a}{db} da$$

$$0 = \int_{a_0}^{a_1} \frac{\int_0^{\hat{\theta}} \Phi'(.)m(\theta|a)d\theta p_a}{\lambda} \left(\frac{1-p_a}{p_a} + \frac{dw_a}{db} + \frac{dp_a}{db}\frac{w_a(1-\tau_a)}{p_a} \right) f(a)da + \int_{a_0}^{a_1} \frac{\Phi'(U_b)[1-M(\hat{\theta}|a)]}{\lambda} f(a)da$$
$$-1 + \int_{a_0}^{a_1} h_a da + \int_{a_0}^{a_1} (T_a+b)\frac{dh_a}{db} da$$

$$0 = \int_{a_0}^{a_1} g_a h_a \left(\frac{1-p_a}{p_a} + \frac{dw_a}{db} + \frac{dp_a}{db}\frac{w_a(1-\tau_a)}{p_a} \right) da + \int_{a_0}^{a_1} \frac{\Phi'(U_b)[1-M(\hat{\theta}|a)]}{\lambda} f(a)da$$
$$-h_b + \int_{a_0}^{a_1} (T_a+b)\frac{dh_a}{db} da$$

Using the definition of $g_b = \frac{1}{h_b}\left(\int_{a_0}^{a_1} \frac{\Phi'(U_b)[1-M(\hat{\theta}|a)]}{\lambda} f(a)da + \int_{a_0}^{a_1} g_a k_a (1-p_a) da \right)$ and rearrangin yields

$$0 = \int_{a_0}^{a_1} g_a h_a \left(\frac{dw_a}{db} + \frac{dp_a}{db}\frac{w_a(1-\tau_a)}{p_a} \right) da + \int_{a_0}^{a_1} g_a k_a (1-p_a) da + g_b h_b - \int_{a_0}^{a_1} g_a k_a (1-p_a) da$$
$$-h_b + \int_{a_0}^{a_1} (T_a+b)\frac{dh_a}{db} da$$

$$0 = \int_{a_0}^{a_1} g_a h_a \left(\frac{dw_a}{db} + \frac{dp_a}{db}\frac{w_a(1-\tau_a)}{p_a} \right) da + g_b h_b - h_b + \int_{a_0}^{a_1} (T_a+b)\frac{dh_a}{db} da \tag{5}$$

Summing 4 across all a and subtracting 5 gives:

$$0 = \int_{a_0}^{a_1} g_a h_a \left(\frac{dw_a}{dT_a} - 1 + \frac{dp_a}{dT_a}\frac{w_a(1-\tau_a)}{p_a} - \frac{dw_a}{db} - \frac{dp_a}{db}\frac{w_a(1-\tau_a)}{p_a} \right) da + \int_{a_0}^{a_1} h_a da - g_b h_b + h_b$$
$$+ \int_{a_0}^{a_1} (T_a+b)\frac{dh_a}{dT_a} da - \int_{a_0}^{a_1} (T_a+b)\frac{dh_a}{db} da$$

$$0 = \int_{a_0}^{a_1} g_a h_a \left(\frac{dw_a}{dT_a} - \frac{dw_a}{db} + \frac{w_a(1-\tau_a)}{p_a}\left[\frac{dp_a}{dT_a} - \frac{dp_a}{db} \right] \right) da - \int_{a_0}^{a_1} g_a h_a da + \int_{a_0}^{a_1} h_a da - g_b h_b + h_b$$
$$+ \int_{a_0}^{a_1} (T_a+b)\left[\frac{dh_a}{dT_a} - \frac{dh_a}{db} \right] da \tag{6}$$

In the absence of income effects, an uniform increase in all tax liabilities combined with a simultaneous uniform decrease in the non-employment benefit (i.e. $dT_a = -db \;\; \forall\, a$) will have no effect on labor force participation decisions, gross wages and therefore the job finding probabilities. Thus, for all a $\frac{dw_a}{dT_a} = \frac{dw_a}{db}$, $\frac{dp_a}{dT_a} = \frac{dp_a}{db}$, and $\frac{dh_a}{dT_a} = \frac{dh_a}{db}$. As a result, (6) reduces to

$$0 = -\int_{a_0}^{a_1} g_a h_a da + \int_{a_0}^{a_1} h_a da - g_b h_b + h_b$$

$$\int_{a_0}^{a_1} g_a h_a da + g_b h_b = 1$$

****

Let γ be the co-state variable associated with the state variable c_a and λ the multiplier associated with the budget constraint.

Note that since $c_a = w_a - T_a = w_a(1-\tau_a) + b$ and $\tau_a = \frac{T_a + b}{w_a}$, we have that:

$$\frac{dc_a}{da} = (1-\tau_a)\frac{dw_a}{da} - w_a\frac{d\tau_a}{da} = (1-\tau_a)\frac{dw_a}{da} + \tau_a\frac{dw_a}{da} - w_a\frac{1}{w_a}\frac{dT_a}{da} = \frac{dw_a}{da} - \frac{dT_a}{da}$$

I use optimal control techniques to characterize the optimal income tax schedule, with c as the state variable and T as the control variable. The Hamiltonian writes

$$H(T,b,c,\lambda,\gamma,a) = \int_0^{\hat{\theta}} \Phi[p_a(c_a - b) + b - \theta]m(\theta|a)d\theta f(a) + \Phi[b](1 - M(\hat{\theta}|a))f(a) + \lambda(T_a + b)h_a + \gamma\frac{dc_a}{da}$$

The first order condition with respect to T is:

$$0 = \frac{1}{\lambda}\frac{dH}{dT_a} = \frac{1}{\lambda}\int_0^{\hat{\theta}} \Phi'[.]m(\theta|a)d\theta f(a)p_a\left[\frac{dw_a}{dT_a} - 1 + \frac{dp_a}{dT_a}\frac{w_a(1-\tau_a)}{p_a}\right] + h_a + (T_a + b)\frac{dh_a}{dT_a} + \frac{\gamma_a}{\lambda}\frac{d^2c_a}{dadT_a}$$

Using the definitions of g_a and letting $\eta_a = -\frac{c_a - b}{h_a}\frac{dh_a}{dT_a}$ denote the macro employment elasticity with respect to own taxes yields:

$$0 = \frac{1}{\lambda h_a}\frac{dH}{dT_a} = g_a\left[\frac{dw_a}{dT_a} - 1 + \frac{dp_a}{dT_a}\frac{w_a(1-\tau_a)}{p_a}\right] + 1 - \frac{T_a + b}{c_a - b}\zeta_a + \frac{\gamma_a}{\lambda}\frac{d^2c_a}{dadT_a}$$

By Assumption 4, the optimal tax equilibrium leads to no bunching, so that the multiplier on the first-order incentive constraint, $\gamma_a = 0$, for all a. In this case, the occupation a macro participation response to (own) taxes is $\frac{dk_a}{dT_a} = m(U_a - U_b|a)f(a)\frac{dU_a}{dT_a} = -\left[\frac{dw_a}{dT_a} - 1 + \frac{dp_a}{dT_a}\frac{w_a(1-\tau_a)}{p_a}\right]\frac{dk_a}{dT_a}\Big|^{\text{micro}}$. Substituting the term in square brackets for the ratio of the macro and micro participation responses to taxes and rearranging leads to equation (12), proving the first part of Lemma 1.

$$\frac{\tau_a}{1-\tau_a} = \frac{1 - \frac{\pi_a}{\pi_a^m}g_a}{\zeta_a}$$

Note that the optimal tax formula uses the following definitions for the micro participation elasticity with respect to taxes $\pi_a^m = -\frac{c_a - b}{k_a} \frac{dk_a}{dT_a}\Big|^{\text{micro}}$, and the macro participation elasticity with respect to taxes, $\pi_a = -\frac{c_a-b}{k_a}\frac{dk_a}{dT_a}$.

For completeness, note that the first order condition with respect to c_a verifies

$$\frac{\dot{\gamma}_a}{\lambda} = -\frac{1}{\lambda}\frac{dH}{dc_a} = -\frac{1}{\lambda}\int_0^{\hat{\theta}} \Phi'(.)m(\theta|a)d\theta f(a)p_a\left[1 + \frac{dp_a}{dc_a}\frac{w_a(1-\tau_a)}{p_a}\right] - (T_a+b)\frac{dh_a}{dc_a}$$

$$\Rightarrow \frac{\dot{\gamma}_a}{\lambda} = -g_a h_a\left[1 + \frac{dp_a}{dc_a}\frac{w_a(1-\tau_a)}{p_a}\right] - (T_a+b)\frac{dh_a}{dc_a}$$

Since $\frac{dp_a}{dc_a} = \frac{dp_a}{dw_a} - \frac{dp_a}{dT_a}$, we can write the above equation as follows.

$$\Rightarrow \frac{\dot{\gamma}_a}{\lambda} = -g_a h_a\left[1 + \frac{dp_a}{dw_a}\frac{w_a(1-\tau_a)}{p_a}\right] + g_a h_a\frac{dp_a}{dT_a}\frac{w_a(1-\tau_a)}{p_a} - (T_a+b)\frac{dh_a}{dc_a}$$

Let $\sigma_a^D = -\frac{c_a-b}{p_a}\frac{dp_a}{dT_a}$ denote the (macro) labor demand elasticity with respect to taxes and $\delta_a^c = \frac{c_a-b}{h_a}\frac{dh_a}{dc_a}$ the (macro) employment elasticity with respect to consumption.

$$\Rightarrow \frac{\dot{\gamma}_a}{\lambda h_a} = -g_a\left(\frac{e_a}{e_a^m} + \sigma_a^D\right) - \frac{\tau_a}{1-\tau_a}\delta_a^c$$

A.3 Proof of Proposition 3 (See page 16)

The Lagrangian function for the government's optimization problem is

$$\mathscr{L}^* = SW^*(U_b^*, \{U_a^*\}_{a\in[a_0,a_1]}) - \lambda\int_{a_0}^{a_1}(b^* + E - w_a^*\tau_a^*h_a^*)da$$

where

$$SW^*(U_b^*, \{U_a^*\}_{a\in[a_0,a_1]}) = \int_{a_0}^{a_1}\left(\int_0^{\hat{\theta}}\Phi(U_a^*-\theta)m(\theta|a)d\theta + \Phi(U_b^*)(1-M(\theta|a))\right)f(a)da$$

Differentiating $\mathscr{L}^*$ with respect to $\underline{w}$ gives:

$$\frac{d\mathscr{L}^*}{d\underline{w}} = \int_0^{\hat{\theta}}\Phi'[.]m(\theta|a_0)d\theta f(a_0)\frac{dU_{a_0}^*}{d\underline{w}} + \lambda w_{a_0}^* h_a^*\frac{d\tau_{a_0}^*}{d\underline{w}} + \lambda\tau_{a_0}^*h_{a_0}^* + \lambda w_{a_0}^*\tau_{a_0}^*\frac{dh_{a_0}^*}{d\underline{w}}$$

$$\frac{d\mathscr{L}^*}{d\underline{w}} = \int_0^{\hat{\theta}}\Phi'[.]m(\theta|a_0)d\theta f(a_0)p_{a_0}^*\left[1 - \tau_{a_0}^* - w_{a_0}^*\frac{d\tau_{a_0}^*}{d\underline{w}} + \frac{dp_{a_0}^*}{d\underline{w}}\frac{w_{a_0}^*(1-\tau_{a_0}^*)}{p_{a_0}^*}\right]$$
$$+\lambda w_{a_0}^* h_{a_0}^*\frac{d\tau_{a_0}^*}{d\underline{w}} + \lambda\tau_{a_0}^*h_{a_0}^* + \lambda w_{a_0}^*\tau_{a_0}^*\frac{dh_{a_0}^*}{d\underline{w}}$$

Since $\left.\frac{d\tau^*_{a_0}}{d\underline{w}}\right|_{w\,=\,w^*} = -\frac{T^*_a + b^*}{[w^*_{a_0}]^2} = -\frac{\tau^*_{a_0}}{w^*_{a_0}}$, we have that

$$\frac{d\mathscr{L}^*}{d\underline{w}} = g_{a_0} h^*_{a_0} \lambda \left[1 - \tau^*_{a_0} + w^*_{a_0} \frac{\tau^*_{a_0}}{w^*_{a_0}} + \frac{dp^*_{a_0}}{d\underline{w}} \frac{w^*_{a_0}(1-\tau^*_{a_0})}{p^*_{a_0}} \right] - \lambda w^*_{a_0} h^*_{a_0} \frac{\tau^*_{a_0}}{w^*_{a_0}} + \lambda \tau^*_{a_0} h^*_{a_0} + \lambda w^*_{a_0} \tau^*_{a_0} \frac{dh^*_{a_0}}{d\underline{w}}$$

$$\frac{d\mathscr{L}^*}{d\underline{w}} = g_{a_0} h^*_{a_0} \lambda \left[1 + \frac{dp^*_{a_0}}{d\underline{w}} \frac{w^*_{a_0}(1-\tau^*_{a_0})}{p^*_{a_0}} \right] + \lambda w^*_{a_0} \tau^*_{a_0} \frac{dh^*_{a_0}}{d\underline{w}}$$

Substituting the ratio of the macro and micro participation elasticities with respect to the minimum wage for the term in square brackets, $\eta_{a_0} = \frac{dh_{a_0}}{d\underline{w}} \frac{\underline{w}}{h_{a_0}}$, and the optimal tax formula from Lemma 1 yields

$$\frac{1}{\lambda h_{a_0}} \frac{dSW^*}{d\underline{w}} = \frac{e_{a_0}}{e^m_{a_0}} g_{a_0} + \frac{w^*_{a_0}}{\underline{w}} \left(\frac{1 - \frac{\pi_{a_0}}{\pi^m_{a_0}} g_{a_0}}{\zeta_{a_0} + 1 - \frac{\pi_{a_0}}{\pi^m_{a_0}} g_{a_0}} \right) \eta_{a_0}$$

For a small $d\underline{w}$, $\frac{w^*_{a_0}}{\underline{w}} \approx 1$, leads to (13).

A.4 Proof of Proposition 4 (See page 21)

Proof: Risk-neutrality and proportional bargaining imply that $\frac{dw_a}{d\underline{w}} = \frac{dp_a}{d\underline{w}} = \frac{dU_a}{d\underline{w}} = 0$ for all $a > a_0$. Therefore, the expected utility from participating in occupation $a > a_0$ is not affected by a minimum wage increase. Differentiating $\mathscr{L}^*$ with respect to $\underline{w}$ leads to

$$\frac{1}{\lambda} \frac{d\mathscr{L}^*}{d\underline{w}} = \left(1 + \frac{dp_{a_0}}{d\underline{w}} \frac{w_{a_0}(1-\tau_{a_0})}{p_{a_0}} \right) g_{a_0} h_{a_0} + w_{a_0} \tau_{a_0} \frac{dh_{a_0}}{d\underline{w}} \tag{7}$$

Under the free-entry condition, $q_a = \frac{\chi_a}{a - w_a}$ for all a. The derivative of q_a with respect to w_a implies that

$$\frac{dq_a}{q_a} = \frac{dw_a}{a - w_a} \tag{8}$$

Equation (8) shows that introducing a small minimum wage $\underline{w} = w^*_a + dw$, increases the firm's matching probability. Introducing a minimum wage decreases the ex-post (after matching) surplus for the firm, leading to fewer vacancies being created. Since $a - w_a$ decreases, the firm's matching probability must increase to satisfy the free-entry condition. The presence of fewer vacancies in the market causes the remaining firms to match at a higher rate. The derivative of $p_a = \rho_1 q_1(\rho_1)$ with respect to w_a implies that for all a

$$\frac{dp_a}{p_a} = \frac{dq_a}{q_a} + \frac{d\rho_a}{\rho_a} = \frac{dw_a}{a - w_a} + \frac{d\rho_a}{\rho_a} \tag{9}$$

The elasticity of the firm's matching probability with respect to the labor market tightness (ρ_a) is:

$$\frac{dq_a}{d\rho_a}\frac{\rho_a}{q_a} = \frac{d\left[H_a(1, \frac{1}{\rho_a})\right]}{d\rho_a}\frac{\rho_a}{q_a} = \frac{dH_a}{dk_a}\frac{d(1/\rho_a)}{d\rho_a}\frac{\rho_a}{q_a} = \mu_a p_a(-1)\frac{1}{\rho_a^2}\frac{\rho_a}{q_a} = -\mu_a \tag{10}$$

Equation (10) implies that $\frac{d\rho_a}{\rho_a} = -\frac{1}{\mu_a}\frac{dq_a}{q_a}$. Substituting this into equation (9) leads to

$$\frac{dp_a}{p_a} = \frac{dw_a}{a - w_a} - \frac{1}{\mu_a}\frac{dq_a}{q_a} = -\frac{dw_a}{a - w_a}\frac{1 - \mu_a}{\mu_a} \tag{11}$$

Rearranging, solving for $\frac{dp_{a_0}}{d\underline{w}}$ and substituting w_{a_0} from equation (14) gives:

$$\left.\frac{dp_{a_0}}{d\underline{w}}\right|_{w\,=\,w^*_{a_0}} = -\frac{(1 - \mu_{a_0})p^*_{a_0}}{\mu_{a_0}(1 - \beta_{a_0})(a - T^*_{a_0} - b^*)} \tag{12}$$

Substituting (12) for $\frac{dp_{a_0}}{d\underline{w}}$, the macro employment elasticity with respect to the minimum wage $\eta_{a_0} = \frac{dh_{a_0}}{d\underline{w}}\frac{\underline{w}}{h_{a_0}}$, and the optimal employment tax formula into equation (7) gives:

$$\frac{1}{\lambda h_{a_0}}\left.\frac{d\mathscr{L}^*}{d\underline{w}}\right|_{w\,=\,w^*} = \left(\frac{\mu_{a_0} - \beta_{a_0}}{\mu_{a_0}(1 - \beta_{a_0})}\right)g_{a_0} + \left(\frac{1 - \frac{\pi_{a_0}}{\pi^m_{a_0}}g_{a_0}}{\zeta_{a_0} + 1 - \frac{\pi_{a_0}}{\pi^m_{a_0}}g_{a_0}}\right)\frac{w^*_{a_0}}{\underline{w}}\eta_{a_0} \tag{13}$$

Since $\frac{dU_{a_0}}{d\underline{w}} = \frac{\mu_{a_0} - \beta_{a_0}}{\mu_{a_0}(1-\beta_{a_0})}$, introducing a binding minimum wage increases the expected utility of participating in occupation a_0 if and only if $\beta_{a_0} < \mu_{a_0}$. From the link between the macro expected utility response and the macro labor force participation response to the minimum wage, we have that $\frac{dk_{a_0}}{d\underline{w}} = \frac{\mu_{a_0} - \beta_{a_0}}{\mu_{a_0}(1-\beta_{a_0})} \cdot \left.\frac{dk_{a_0}}{d\underline{w}}\right|^{\text{micro}}$, confirming that $\frac{dk_{a_0}}{d\underline{w}}$ is positive if and only if $\beta_{a_0} < \mu_{a_0}$, proving the proposition.

□

A.5 Proof of Proposition 5 (See page 23)

In the competitive model where θ is the cost of working (paid only by those that find a job), the social welfare function can be written as follows.

$$SW(U_b, \{U_a\}_{a\in[a_0,a_1]}) = \int_{a_0}^{a_1} \left\{ p_a \int_0^{\hat{\theta}} \Phi(c_a - \theta)m(\theta|a)d\theta + (1-p_a)\int_0^{\hat{\theta}} \Phi(b)m(\theta|a)d\theta \right.$$
$$\left. +\Phi(b)(1-M(\theta|a)) \right\} f(a)da$$

where the new terms capture the fact that only a fraction of those that want to work will find a job under the minimum wage. Evaluating at $p_a = p_a^{CE} = 1$, as is the case in the competitive equilibrium, leads to SW collapsing to equation (6). Differentiating the objective with respect to $\underline{w}$ (and evaluating at the competitive equilibrium wage and job finding probability) yields:

$$\frac{d\mathscr{L}^{CE}}{d\underline{w}} = p_{a_0}^{CE}\int_0^{\hat{\theta}} \Phi'(.)m(\theta|a_0)d\theta f(a_0) + \frac{dp_{a_0}}{d\underline{w}}\int_0^{\hat{\theta}} \Phi(c_{a_0} - \theta)m(\theta|a_0)d\theta f(a_0) + p_{a_0}^{CE}\Phi(c_{a_0} - \hat{\theta})m(\hat{\theta}|a_0)f(a_0)\frac{d\hat{\theta}}{d\underline{w}}$$
$$-\frac{dp_{a_0}}{d\underline{w}}\int_0^{\hat{\theta}} \Phi(b)m(\theta|a_0)d\theta f(a_0) + (1-p_{a_0}^{CE})\Phi(b)m(\hat{\theta}|a_0)f(a_0)\frac{d\hat{\theta}}{d\underline{w}} - \Phi(b)m(\hat{\theta}|a_0)f(a_0)\frac{d\hat{\theta}}{d\underline{w}}$$
$$+\lambda(T_{a_0} + b)\frac{dh_{a_0}}{d\underline{w}}$$

But when evaluating at $p_{a_0}^{CE} = 1$, the fifth term drops out. By the envelope theorem, those that enter (or exit) the labor force have no first order effects on social welfare, so the third and sixth terms also cancel out, leading to

$$\frac{d\mathscr{L}^{CE}}{d\underline{w}} = \int_0^{\hat{\theta}} \Phi'(.)m(\theta|a_0)d\theta f(a_0) + \frac{dp_{a_0}}{d\underline{w}}\int_0^{\hat{\theta}} \Phi(c_{a_0} - \theta)m(\theta|a_0)d\theta f(a_0) - \frac{dp_{a_0}}{d\underline{w}}\int_0^{\hat{\theta}} \Phi(b)m(\theta|a_0)d\theta f(a_0)$$
$$+\lambda(T_{a_0} + b)\frac{dh_{a_0}}{d\underline{w}}$$

Applying the definition of g_{a_0} and collecting the $\frac{dp_{a_0}}{d\underline{w}}$ terms yields:

$$\frac{d\mathscr{L}^{CE}}{d\underline{w}} = g_{a_0}\lambda h_{a_0} + \frac{dp_{a_0}}{d\underline{w}}\left[\int_0^{\hat{\theta}} \Phi(c_{a_0} - \theta)m(\theta|a_0)d\theta f(a_0) - \int_0^{\hat{\theta}} \Phi(b)m(\theta|a_0)d\theta f(a_0)\right]$$
$$+\lambda(T_{a_0} + b)\frac{dh_{a_0}}{d\underline{w}}$$

Note that the derivative $\frac{dp_{a_0}}{d\underline{w}}$, can be written as follows: $\frac{dh_{a_0}}{d\underline{w}} = \frac{dp_{a_0}}{d\underline{w}}k_{a_0} + \frac{dk_{a_0}}{d\underline{w}}p_{a_0} \Rightarrow \frac{dp_{a_0}}{d\underline{w}} = \frac{1}{k_{a_0}}\left[\frac{dh_{a_0}}{d\underline{w}} - \frac{dk_{a_0}}{d\underline{w}}p_{a_0}\right] = \frac{1}{k_{a_0}}\left[\eta_{a_0}\frac{h_{a_0}}{\underline{w}} - e_{a_0}\frac{h_{a_0}}{\underline{w}}\right] = \frac{p_{a_0}}{\underline{w}}\left[\eta_{a_0} - e_{a_0}\right]$. Substituting this and dividing both sides by λ yields:

$$\frac{1}{\lambda}\frac{d\mathscr{L}^{CE}}{d\underline{w}} = g_{a_0}h_{a_0} - p_{a_0}[e_{a_0} - \eta_{a_0}]\frac{\int_0^{\hat{\theta}}\left[\Phi(c_{a_0}-\theta) - \Phi(b)\right]m(\theta|a_0)d\theta f(a_0)}{\lambda k_{a_0}} + (T_{a_0} + b)\frac{dh_{a_0}}{d\underline{w}}$$

Let $g^u_{a_0} = \frac{\int_0^{\hat{\theta}}\left[\Phi(c_{a_0}-\theta) - \Phi(b)\right]m(\theta|a_0)d\theta f(a_0)}{\lambda k_{a_0}}$ denote the welfare weight on marginal unemployment losses (due to the minimum wage). Since for a small dw, $w^{CE}_{a_0}/\underline{w} \approx 1$, we have that.

$$\frac{1}{\lambda}\frac{d\mathscr{L}^{CE}}{d\underline{w}} = g_{a_0}h_{a_0} - p_{a_0}[e_{a_0} - \eta_{a_0}]g^u_{a_0} + w^{CE}_{a_0}\tau_{a_0}\eta_{a_0}\frac{h_{a_0}}{\underline{w}}$$

$$\frac{1}{\lambda h_{a_0}}\frac{d\mathscr{L}^{CE}}{d\underline{w}} \approx g_{a_0} - \frac{p_{a_0}}{h_{a_0}}[e_{a_0} - \eta_{a_0}]g^u_{a_0} + \tau_{a_0}\eta_{a_0}$$

$$\frac{1}{\lambda h_{a_0}}\frac{d\mathscr{L}^{CE}}{d\underline{w}} \approx g_{a_0} - \frac{[e_{a_0} - \eta_{a_0}]}{k_{a_0}}g^u_{a_0} + \left[\frac{1 - g_{a_0}}{\zeta_{a_0} + 1 - g_{a_0}}\right]\eta_{a_0}$$

If $g_{a_0} > 1$, the last term is positive. This is because workers in occupation a_0 receive a subsidy, and employment losses due to the minimum wage relax the government's budget constraint. Thus, if $g_{a_0} > 1$, introducing a minimum wage is desirable if the sum of the first two terms is positive. A sufficient (but not necessary) for this sum to be positive is

$$g_{a_0}k_{a_0} > [e_{a_0} - \eta_{a_0}]g^u_{a_0}$$

$$\frac{g^u_{a_0}}{g_{a_0}} < \frac{k_{a_0}}{e_{a_0} - \eta_{a_0}}$$