Urban Accounting for Geographic Concentration of Skills and Welfare Inequality

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Abstract

Using Jones (2014) generalized division of labor (GDL) accounting of productivity, we extend Desmet and Rossi-Hansberg (2013) urban accounting model to account for geographic concentration of skill and welfare inequality between high-skill and low-skill workers. We show that high-skill workers would be less geographically concentrated absent location heterogeneity with respect to productivity and amenity fundamentals and excess frictions. Welfare gap would narrow when variation in fundamental amenity or excess friction is eliminated, but widen when productivity fundamental is equalized. We further show that agglomeration economies both for productivity and amenity would affect skill concentration and welfare inequality in the same direction. This study sheds light on the causes of increased skill concentration in across US metropolitan areas and the implications for welfare inequality.

Key words: skill concentration, welfare inequality, urban attributes, externalities

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1. Introduction

Skill concentration refers to concentration of high-skill people in a small number of smart cities. As E. Moretti (2012) classifies, nowadays there exists "The Three Americas": Brain hubs, traditional industries, and the ones in the middle. Top-ten smartest cities gauged by the percentage of college graduates of local labor force in United States are Seattle (53%), San Francisco (50%), Raleigh (50%), Washington, D.C. (45%), Austin (44%), Minneapolis (43%), Atlanta (43%), Boston (41%), San Diego (40%) and Lexington (40%) (Johnson, 2012). Except college towns, Raleigh and Lexington, eight out of the ten are extremely large metropolises. If high-skill workers are more likely to be tempted by desirable amenities, for instance, natural amenities such as cooler summer, warmer winter and closer proximity to coastal areas, and social amenities such as theaters, cafés, museums, and shopping centers, it helps draw a picture of prosperity in more amenable cities. These suggest that high-skill workers are overrepresented in where urban productivity is soaring, quality of life is high, and costs of living are as well immensely large. These kinds of cities are called "Superstars" in Gyourko, Mayer, and Sinai (2006). However, there has been no full accounting for the variation of high-skill concentration across cities by the three urban characters so far: efficiency, amenities, and frictions.

Skill concentration is concurrent with wage inequality between the high-skill and low-skill workers. Wage premium is increasing with city size (Baum-Snow, Freedman, & Pavan, 2014; Baum-Snow & Pavan, 2012), and it is rising with high-skill concentration (Davis & Dingel, 2012, 2013). Two main hypotheses document the reasons behind. Under demand-driven hypothesis, skill-biased technical change (Autor, Katz, & Kearney, 2004), complementarity between skilled workers (Behrens, Duranton, & Robert-Nicoud, 2010; Giannetti, 2003; Venables, 2011), and complementarity between cities and skills (Berry & Glaeser, 2005; Glaeser & Mare, 2001; Glaeser & Resseger, 2010) . Based upon supply-shift hypothesis, the young and educated are more likely to work in productive cities so as to gain learning-in-cities (Fu & Liao, 2012; Glaeser, 1999; Glaeser & Mare, 2001; Lucas, 2004); Moreover, recent studies eloquently document that cities better-off in amenity disproportionally attract more high-skill workers (Gyourko et al., 2006; Lee, 2010). Other studies pointing out that industry-specific skill intensity as one demand force and breakdown of low-skill protective labor market institutions such as labor unions are arguments for surging skill premium[1].

It is intriguing to investigate how location fundamentals together with agglomeration economies account for welfare inequality between the two skill groups, since nominal wage is insufficient to capture one's consumption capacities such as goods and amenities under budget constraint. Up to now, we are unaware of cross-city decomposition of welfare inequality by local fundamentals and externalities in urban literature. Moretti (2008) weighs the relative importance between skill demand force and skill supply shift in higher educated as well as expensive cities during 1980 and 2000. The high housing costs could reflect consumption of amenities, or could offset the high productivity. He expounds that welfare inequality, in terms of real wage gap, is narrower once demand force outweighs supply shift. Although Moretti (2008) is not full accounting of welfare inequality either over time or across locations, this pioneer study motivates to answer the question, how location attributes account for geographic concentration of high-skill and welfare inequality between the two skill cohorts.

High-skill workers refer to those who have obtained college degrees and above, instead low-skill workers are those without college degrees and below. Based on Jones (2014) Generalized Division of Labor (GDL) framework, high-skill productivity depends on location advantage and local skill mix; the latter also determines the productivity of low-skill workers. Moretti (Enrico Moretti, 2004a; E. Moretti, 2004b) finds that earnings of the less educated are raised by the increasing supply of college graduates. He also accentuates that share of college graduates is positively correlated with wage of high-school graduates (E. Moretti, 2012). Glaeser's review of Moretti's book illustrates that taco stand worker earns quite different in Visalia (Visalia-Porterville, CA) and Menlo Park (San Francisco-Oakland-Fremont, CA) (Glaeser, 2013). That geographic pattern of the greatest minds as well as the lower educated both appealing to large cities like New York is expounded by Eeckhout, Pinheiro, and Schmidheiny (2010). Winters (2012) illustrates that low-skill workers benefit from locating near the high-skill by improving their labor force participation and employment. Pereira-Lopez and Soloaga (2013) find consistent evidence of imperfect substitution between different skills in Mexico metropolitan areas. Low-skill workers are more and more employed with higher payments in large cities,

[1] Katz and Autor (1999) and Goldin and Katz (2007) offer comprehensive surveys of literature on nominal wage inequality.

suggesting that low-skill must be more productive by nearing high-skill workers (Fu & Hao, 2014; Jones, 2014). Indeed, New York City is home to investment bankers and busboys, San Francisco to Internet entrepreneurs and grocery clerks, Boston to biomed engineers and the janitors who pick up their offices (Emily Badger, The Atlantic, 2014). Lindley and Machin (2014) document the labor market polarization, which results in faster employment growth in high skill occupations, but also in a higher demand for low wage workers in low skill occupations.

Jones (2014) GDL framework delineates imperfect substitution association between high-skill workers and low-skill ones, therefore low-skill productivity, i.e. low-skill wage when assuming an identical human capital service flow, entirely depends on local skill mix, especially when the engine of economic growth shifts to high technologies, scarcity of low-skill workers makes themselves even more productive. Quality of human capital investment by high-skill workers is highly heterogeneous across cities, which are accounted for by the division of labor. In an urban context, more location advantage and higher density in terms of city size and city skill-share lowers coordination costs and boost high-skill productivity (Fu & Hao, 2014). We define skill-share as the high-skill population divided by low-skill population within a city. Through the lens of coordination costs, agglomeration economies do not directly benefit the low-skill, instead, stronger agglomeration economies generates higher productivity for high-skill workers and shifts low-skill workers demand curve rightward due to the imperfect substitution. This is to say, externalities generated by urban size and skill-share which we call "productivity externality", do good to both the two skill groups, but in different manners[2].

As the elasticity of substitution changes, for example, lowers, the demand curve for high-skill is even more downward sloping, in which case, the relative output price between the two skill groups decline and low-skill workers are demanded more. Therefore, the substitution elasticity also plays a role to shape distribution of skill-share. In addition, observed large supply of higher educated workers over the last three decades (1980-2010) for instance, enhances the scarcity of low-skill supply, and could have significant impacts on the distribution of skill-share as well as the welfare inequality. In other words, the relative decline of the non-college-degree population

[2] There is abundant literature on role of agglomeration economies to urban workers without classification of skill-specific benefits.

in the U.S. economy offers another explanation for skill concentration and welfare inequality from aggregate labor supply side.

Besides, skill concentration and welfare inequality when workers have skill-specific preferences to urban amenities are elucidated. Amenities are becoming a more important determinant of where people choose to live (Rappaport, 2007, 2008; Rappaport & Sachs, 2003). Glaeser, Kolko, and Saiz (2001) also find that high amenity cities have grown faster than low amenity ones because the demand for living in cities has risen for reasons beyond rising wages. In the *Places Rated Almanac* (Savageau & D'Agostino, 2000) where Quality of Living (QOL) rankings are listed, many large cities have quite better scores regardless of high costs of livings. Besides Natural amenities, social amenities such as theaters are more abundant with more concentration of workers. Rappaport (2008) demonstrates that, "Increases in density from very low levels may similarly increase quality of life. For example, moving from low to moderate density might facilitate social interaction, allow for greater product variety, and support the provision of public goods"[3]. That more abundant social amenity is due to higher density in large cities is "amenity externality", another impact on location choices and welfare inequality.

Gyourko et.al (2006) depict "Superstar" cities that are both highly productive and desirable in amenities. High-skill workers living in these cities earn high wage, pay more housing rents but largely due to consumption of amenities. The well-being inequality gap could be wider with the presence of "Superstar" cities, relative to an urban system without these kinds of cities, particularly when high-skill premium over amenities is positive. Lee (2010) provides consumption-side explanation for urban wage premium, when high-skill workers have a bigger willingness-to-pay (WTP) for urban amenities than their counterparts within the city, wage premiums are decreasing in skill levels. Handbury (2012) finds that higher income households pay more for higher quality grocery than lower income ones. Amenities directly have impacts on skill concentration and welfare inequality; moreover, amenities play an indirect role to skill concentration and welfare inequality through productivity due to "Amenity effect" proposed in Fu and Hao (2013). Since unequal tastes for urban amenities affect location choices of different skills in unequal manners, by which urban size and skill composition are determined, urban

[3] That increase in density decreases quality of life such as traffic, pollution, and other non-priced sources of congestion are not concerned.

productivity then is influenced.

Frictions deter people to be more productive and consume desirable amenities. As cities grow, they become more and more congested and living in a large city no doubt increases the distance between residences and workplaces, workers would have to spend a larger proportion of time on commuting between residences and workplaces if they pay for lower rents and live further. When a local government is less efficient, revenues collected from workers in the form of income tax for instance, will not be efficiently spent on public services. Urban friction, rising with population size, labor loss per mile on road and urban government inefficiency, also is one factor that directly and indirectly determine skill concentration and welfare inequality due to "Friction effect" (Fu and Hao, 2013).

Productivity, amenities, frictions, skill concentration and welfare inequality are jointly determined in a spatial equilibrium. Given location fundamentals and the presences of agglomeration economies, urban performances such as skill mix, wages, and welfare of each skill group are mapped. Each group's welfare is gained by goods and services consumption using disposable earnings, consumption of local amenities, but lost due to congestion-caused housing rents, commuting costs between residence and workplace, and local government inefficiency. Urban size accounting model of Desmet and Rossi-Hansberg (D-RH) (2013) is extended to a two-skill accounting model, decomposing skill mix, wage distribution, and welfare inequality into three location attributes (location advantage, fundamental amenity, and excessive frictions) and two types of agglomeration economies (productivity externality and amenity externality).

We perform comparative statics of welfare inequality and skill concentration in a two-city example, with respect to urban attributes, externalities, elasticity of substitution, labor loss per mile, and aggregate skill supply. We find that skill concentration and welfare inequality depends on not only heterogeneities in productivity and amenities, but also the association between efficiency and amenities. Heterogeneity in productivity and amenities both lead to more skill concentration in one city, no matter positive or negative correlation between productivity and amenities. Welfare inequality can be either widened or narrowed under different scenarios of heterogeneous amenities, including the four cases of only heterogeneous amenities, four cases of positive relation between productivity and amenities, and four cases of negative relation.

We utilize U.S. Metropolitan areas of year 2005 in counterfactual exercises to investigate roles played by urban attributes and externalities to skill concentration and welfare inequality. We find that large cities are productive, better-off in fundamental amenity, and doing a little bit worse in controlling excessive frictions. The geographic concentration of skill indicated by mean difference of skill-share between top 50 percentile and bottom 50 percentile cities show that both heterogeneous fundamental amenity and full amenities (in terms of exogenous and endogenous components together) respectively cause more skill concentration. Either fundamental productivity (i.e. location advantage) or efficiency (i.e. both location advantage and endogenous productivity externalities) in U.S. cities brings about larger skill concentration. Heterogeneous excessive frictions result in further skill concentration. Location advantage contributes the greatest to skill concentration.

Shutting down heterogeneity in productivity weakens skill concentration and widens welfare gap, which means in reality there are more skill concentration and narrower welfare inequality on account of productivity distribution. This could be due to imperfect substitution. Skill concentration is conducive to low-skill workers because labor demand of them is enlarged. Fundamental amenity and excessive frictions individually enlarges welfare gap in reality. We also find that the two types of externalities, productivity externality and amenity externality together influences skill concentration and welfare inequality in the same direction. If skill concentration is enhanced ascribing to the total externality, welfare gap is expanded, vice versa. More specifically, total externality enhances skill concentration and welfare inequality when equalizing fundamental amenity and excess friction, but lessens skill concentration and welfare gap when equalizing location advantage.

This paper also closely relates to Diamond (2012) that accounts for changes of welfare inequality and skill concentration by changes of wage, rents and amenities in U.S. cities from 1980 to 2000. Diamond (2012) adopts wage as gauge of productivity (in terms of exogenous Bartik shock and endogenous externalities) although productivity is also determined by city size and skill mix. In her equation (4) and (5), productivity of respective high-skill and low-skill workers is raised in places with more of their skill group if elasticity of substitution is bigger than 1. Diamond (2012) demonstrates that high-skill and low-skill workers have segregated location choices: high-skill workers choose to migrate to productive, amenable although congested cities, instead low-skill workers prefer less productive, disamenable but less crowded ones. She argues that shipping and distribution offer good pay for the low-skill, however on one hand, large concentration of low-skill workers moves their payments to an extremely low level along the labor supply curve; on the other hand, scarcity-caused productivity improvement due to imperfect substitution in those large and productive cities drives up low-skill wage which attracts low-skill workers. These two reasons make her equilibrium unstable after all. Based on imperfect substitution, this paper validates how co-location between low-skill workers and high-skill ones takes effect on skill concentration and welfare inequality. Moreover, that low-skill workers are pushed out of high-amenity places as proposed by Diamond (2012) and thus welfare gap widens is contaminated by the presences of "Superstar" cities where both high-skill and low-skill workers are productive, amenities are more consumed by high-skill workers, but relative small amenity consumption outweighs benefits from being more productive for low-skill workers, producing higher chances for wider welfare gap even though different skill cohorts co-locate. One thing we should be aware of is that we do not take into account of heterogeneous housing costs for high-skill and low-skill workers. However, heterogeneity in housing costs consolidate co-location because, low-skill workers are more likely to stay in expensive cities when they can pay a lower price for smaller units or lower quality housing services.

Both Moretti (2008) and Diamond (2012) help clarify the channels through which urban attributes shape skill mix and welfare inequality, nonetheless our extended urban accounting framework is superior to Moretti (2008) and Diamond (2012). It reconciles the conflict of welfare inequality between these two studies[4]. We inherit the discussion in Moretti (2008), except demand-pull force works for both high-skill workers and low-skill workers, and supply-push force due to amenities is especially stronger for the high-skill; in addition, "Amenity effect" indirectly determines the spatial equilibrium. This is what deviates from Moretti (2008) in which supply shock of amenities has no channel to increase low-skill wage as well as high-skill wage. We do conclude that nominal wage gap in every city should be smaller than welfare inequality as documented in Diamond (2012), as long as high-skill premium of amenities is positive in each

[4] Demand shock in Brinkman (2014) is decomposed into both industry-specific technology change and skill-specific technology change in his theoretical model; unsurprisingly, demand shock is still the dominant in determining concentration of high-skill in large cities conforming to Moretti (2008), although Brinkman does not touch welfare inequality question.

city. This paper provides a basis for analyzing welfare inequality change and skill concentration change over the past decades in U.S. cities.

Contributions of this paper are significant. First, it is the first time to perform urban accounting for skill concentration and welfare inequality across locations. The decomposition approach helps complement previous research on skill concentration and wage inequality across local labor market in a general equilibrium. We utilize mean difference of skill-share between the top 50 percentile cities and the bottom 50 percentile ones to address skill concentration. Two types of externalities are introduced, and their separate effect on skill concentration and welfare inequality are illustrated. Second, this paper provides insight for future studies related to welfare inequality. Utilizing this framework one could investigate how skill mix and welfare inequality gap change from 1980 to the recent time when cities become less alike. Third, for place-based policy that induces spatially heterogeneous distortion (Kline & Moretti, 2013) and national policy to which heterogeneous spaces respond differently (Albouy & Hanson, 2014), impacts on behavioral mobility responses of heterogeneous skill groups and social welfare inequality could be expounded through the novel lens of skill accounting offered in this paper. Fourth, this study documents the association between skill concentration and welfare inequality. By and large, strengthened skill concentration is associated with magnified welfare inequality; however, equalizing either urban efficiency or location advantage documents a negative relation between skill concentration and welfare inequality, because decreased skill concentration is unable to fulfill more low-skill demand by imperfect substitution. Fifth, we find large cities are in general more amenable (in terms of either fundamental amenity or full amenity), which is contrary to D-RH (2013). In line with Savageau (2000) and Sperling and Sander (2004) Sperling and Sander (2004), our study also finds that many large cities score quite favorably in overall "livability" in spite of their high cost-of-living.

Before we get to start the welfare inequality analyses, there are two issues dealt with in advance. One issue we should note is urban accounting framework of D-RH (2013) upon which our model is based has some mistakes in need of corrections. Doing so is to pave the foundation for comparison between analyses of homogeneous skill and heterogeneous skill. Another issue needed to be coped with is that productivity is not fully accounted for by "Solow residual" utilized in D-RH (2013). In Solow's accounting framework, total wage is convex to Total Factor

Productivity (TFP). While with Jones GDL approach, more even distribution of productivity yields higher total wages for high-skill and low-skill workers, since high-skill total wage and low-skill total wage is respectively concave to productivity. Therefore, it is welfare gain instead of loss when urban size distribution is more even.

Organization of the paper is as follows. Section 2 improves D-RH (2013) urban accounting model with homogeneous workers. Section 3 builds an extended spatial equilibrium with heterogeneous skills based upon D-RH (2013) and Jones (2014), and numerically examines the mechanism in a 2-city example. Section 4 testifies the impacts of location attributes and externalities using U.S. metropolitan areas in 2005. Section 5 concludes.

2. *The Improved Urban Size Accounting*

2.1 Benchmark model modification

Urban outcome such as city size is shaped by positive effects from productivity, amenities, as well as costs and frictions arising from congestions. This section re-performs accounting exercises in D-RH (2013) (i.e. D-RH) to pave the ground for heterogeneous workers analysis.

We need to set up an alternative utility function. The reason of doing so hinges on the mistake of the budget constraint (i.e. equation(3)) in D-RH (2013), which intends to show that disposable income subtracted by income loss in transportation is used to consume goods. The constraint, however, is

$$c_{it} = w_{it}h_{it}\left(1-\tau_{it}\right) - R_{it} - T_{it}$$

where $\left(R_{it}+T_{it}\right)$ does not capitalize the loss due to distance to Central Business District (CBD) into income loss. Even if it equals to $\kappa(N_{it}/\pi)^{\frac{1}{2}}$ in which κ should have denoted commuting costs per mile in their point of view, we are not sure what is the measuring unit of κ, dollar or penny? Confusion caused by this problem would lead to different results with different measuring units of wage. The value of income loss should conform to earned wage income, but a parameter is unable to take up the role. Besides, inconsistency is found between above budget constraint and local government expenditure function in D-RH (2013), which is

$$G\left(h_{it}w_{it}, TC_{it}\right) = g_{it}h_{it}w_{it}\kappa TC_{it}$$

If κ is commuting costs per mile, it is sufficient to denote income losses on road by solely $g_{it}\kappa TC_{it}$, which is the amount that urban government spends on building and maintaining urban infrastructure.

In fact, κ captures labor loss per mile instead of labor income loss per mile on road. And the correct form of budget constraint should have been

$$c_{it} = w_{it}h_{it}(1-\tau_{it}) - w_{it}h_{it}(R_{it}+T_{it})$$

Then

$$c_{it} = (1-\theta)y_{it}\left((1-\tau_{it})-(R_{it}+T_{it})\right) \tag{2.1}$$

And with the utility function $U_{it} = \log c_{it} + \varphi \log(1-h_{it}) + \gamma_{it}$, and First Order Conditions (FOC) with respect to consumption [c_{it}] and working hour [h_{it}], we get that

$$\varphi \frac{c_{it}}{1-h_{it}} = w_{it}\left((1-\tau_{it}) - R_{it} - T_{it}\right) \tag{2.2}$$

Combing equation (2.1) with (2.2), we obtain that

$$\varphi \frac{(1-\theta)y_{it}\left((1-\tau_{it})-(R_{it}+T_{it})\right)}{(1-\theta)y_{it}/h_{it}\left((1-\tau_{it})-R_{it}-T_{it}\right)} = 1-h_{it}$$

which essentially is

$$\varphi \frac{(1-\theta)y_{it}\left((1-\tau_{it})-(R_{it}+T_{it})\right)}{(1-\theta)y_{it}\left((1-\tau_{it})-R_{it}-T_{it}\right)} = \frac{1-h_{it}}{h_{it}} \tag{2.3}$$

Equation (2.3) implies that working hour is a constant, and $h_{it} = 1/(1+\varphi)$. In addition, this largely simplify the utility function to be

$$U_i = \log c_i + \gamma_i$$

$$\text{s.t.} \quad c_i = w_i(1-\tau_i - R_i - T_i)$$

Time dimension has no substantial effect either here or in D-RH (2013) except physical capital is rated universally in steady state, and we suppress the time subscript for convenience. Welfare losses are due to frictions. Frictions cause income losses, less income lead to less consumption, less consumption generates lower utility. Income loss is paid by the form of labor tax. We simplify disposable income as wage after paying labor tax at a new rate of $\hat{\tau}$, so the budget constraint is

$$\text{s.t.} \quad c_i = w_i e^{-\hat{\tau}_i}$$

From the equalization of above two budget constraints,

$$e^{-\hat{\tau}_i} \equiv 1 - \tau_i - R_i - T_i$$

If $\tau_i + R_i + T_i$ is sufficiently small, then

$$\hat{\tau}_i = \tau_i + R_i + T_i \tag{2.4}$$

Deviated from D-RH (2013), the new $\hat{\tau}$ comprehensively captures the rate of income loss by frictions. The first component of frictions is the outcome of standard city size effect. All cities are monocentric that all residents within a city work in CBD and live in residential areas around the CBD. To make sure that a point of location within a city is indifferent from any other points within the city, a person lives closer to CBD should pay higher rent to bid for the location, because lower commuting costs occurred when travelling to CBD. Imagine one lives at the fringe of a city paying no housing rent, as population increases and city size expands, it will cost more for him to commute since distance to CBD is farther. Imagine you live at the closest place to CBD, as population increases and city size expands, more people will compete with him for his location to avoid higher commuting costs, which results in higher bid rent. Let the radius of a monocentric city be $\bar{d}_i$, and the per mile labor loss due to commuting is κ, then commuting-caused proportion of labor loss from a distance of d to CBD is

$$T(d) = \kappa d$$

The summation of commuting-caused proportion and renting-caused proportion of labor loss at distance d to CBD is

$$R_i(d) + T(d) = 0 + T(\bar{d}_i) = \kappa \bar{d}_i$$

Total population equals to the total housing units provided by the city,

$$N_i = \pi \bar{d}_i^2$$

then

$$R_i(d)+T(d)=\kappa\left(\frac{N_i}{\pi}\right)^{\frac{1}{2}} \quad \text{for } \forall \quad d\,. \tag{2.5}$$

The second component of frictions is caused by urban government inefficiency. Local government provides public services. A lot more effort by urban government must be taken in providing services when it is inefficient. Revenues come from the collected labor taxes in total. When government expenditure equals government revenue,

$$g_i w_i \kappa TC_i = \tau_i N_i w_i$$

where g_i is the "excess friction" representing urban government inefficiency.

Total commuting is

$$TC_i = \int_0^{\bar{d}_i} (2\pi d^2)dd = \frac{2}{3}\pi^{-\frac{1}{2}} N_i^{\frac{3}{2}}$$

therefore labor wedge rate (labor income tax rate in general) is

$$\tau_i = g_i \kappa \frac{2}{3}\left(\frac{N_i}{\pi}\right)^{\frac{1}{2}} \tag{2.6}$$

From equation (2.4), (2.5), and (2.6), we get

$$\tau_i + R_i(\text{d}) + T(\text{d}) = g_i \kappa \frac{2}{3}\left(\frac{N_i}{\pi}\right)^{\frac{1}{2}} + \kappa\left(\frac{N_i}{\pi}\right)^{\frac{1}{2}}$$

and then a more comprehensive version of labor tax rate is

$$\hat{\tau}_i = \hat{g}_i \kappa \left(\frac{N_i}{\pi}\right)^{\frac{1}{2}} \tag{2.7}$$

$$\hat{g}_i = \frac{2}{3} g_i + 1$$

So far we get to see a clearer picture of labor tax rate. It captures the extent of frictions from two sources: one is the extent of standard friction caused by congestion embodied by N_i; the other is the extent of excess frictions coming from urban government inefficiency. The improvement based on D-RH (2013) allows us to clarify the relation between labor tax rate and the extent of urban inefficiency, and the relation between labor tax rate and the extent of congestion. Moreover, equation (2.7) has important implication for welfare analysis. A clearer picture between total friction and city size is $\hat{\tau}_i N_i = \hat{g}_i \kappa \pi^{-\frac{1}{2}} N_i^{\frac{3}{2}}$, with the vertical axis being total friction and horizontal axis being city size.

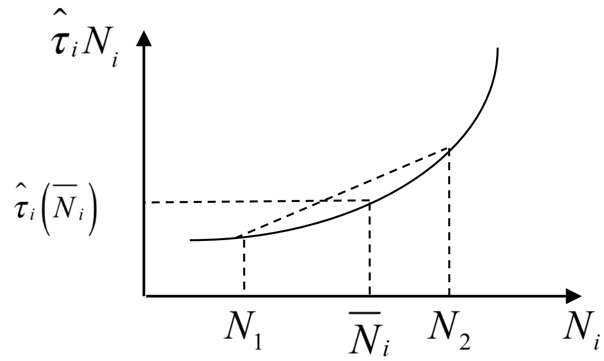


Figure: The convexity between total friction and urban size from model

Total friction is a convex function of city size N_i, which means

$$\hat{\tau}_i(\overline{N}_i) < \frac{\hat{\tau}_1(N_1) + \hat{\tau}_2(N_2)}{2}$$

Its curvature matters for welfare because when city size distribution is more dispersed, total friction tends to become larger, and more social welfare is forfeited.

Take log of equation (2.7),

$$\log \hat{\tau}_i - \frac{1}{2}\log N_i = constant + \log \hat{g}_i \tag{2.8}$$

where

$$constant = \log \kappa - \frac{1}{2}\log \pi$$

The deviation comes from excess frictions. After running regression on constant, error terms which are $\log \hat{g}_i$ could be used to derive κ,

$$\kappa = \exp\left(constant + \frac{1}{2}\log \pi \right)$$

From the budget constraint, the new rate of labor wedge is

$$\hat{\tau}_i = -\ln \frac{c_i}{w_i}$$

where c_i is consumption per person, $c_i = C_i / total\, pop_i$, C_i is aggregate consumption, w_i is per working labor personal income.

Production is launched by productivity shifter, physical capital, and labor inputs since working hour is a constant,

$$Y_i = A_i K_i^{1-\alpha} N_i^{\alpha} \quad (2.9)$$

Wage is derived from the First Order Condition of output with respect to labor inputs,

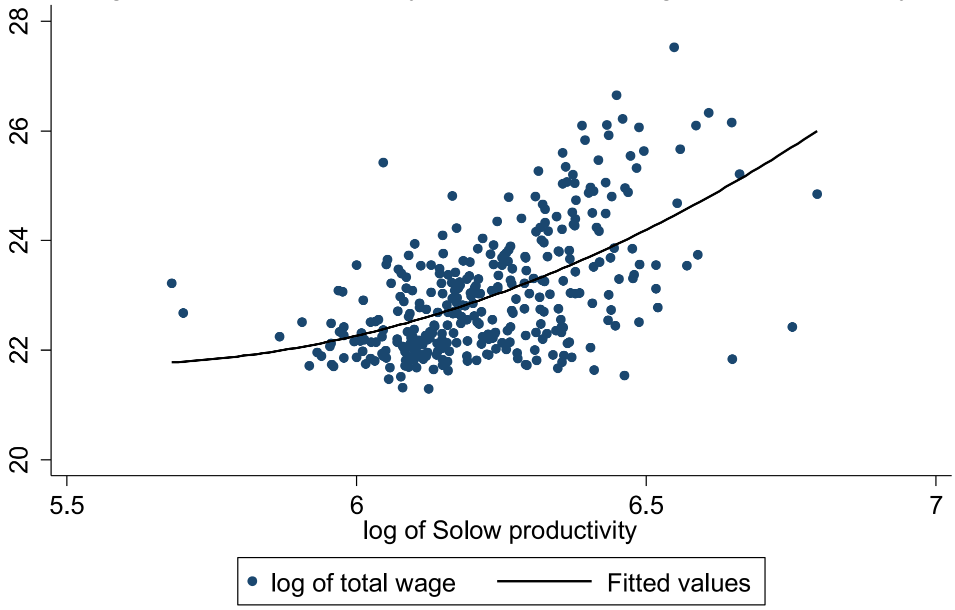
$$w_i = A_i^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} \alpha \quad (2.10)$$

and productivity shifter is

$$A_i = y_i^{\alpha} \left(\frac{r}{1-\alpha} \right)^{1-\alpha} \quad (2.11)$$

where y_i is per capita output, r is the rental rate of physical capital. Even though D-RH (2013) do not adopt the measurement by equation (2.11) of productivity[5], they argue that this model-based calculation is highly correlated with empirical measure using actual data of physical capital stock at 0.9. Figure 2.1 provides evidence of convex relation between total wage and Solow residual productivity by 340 cities in 2005. This means uneven distribution of productivity boosts total wage and facilitates welfare gains.

Figure 2.1: The convexity between total wage and productivity



Source: 2005 Bureau of Economic Analysis and 2005 CPS-MORG Files, 340 cities.

[5] The correlation between productivity shifter obtained by equation (11) and the one measured in Fu and Hao (2013) is as high as 0.88. Fu and Hao (2013) adopts the same methodology as in D-RH (2013) to calculate the productivity shifter.

The indirect utility indifferent across cities is

$$\overline{U} = \log\left(w_i e^{-\tau}\right) + \rho_i$$

In equilibrium, indirect utility is determined when labor market clears by

$$N = \sum_{i=1}^{I} N_i$$

City size is now accounted for by three factors: productivity shifter A_i, amenity index ρ_i, and excess frictions g_i, as below

$$N_i = \left(\frac{\frac{1}{\alpha}\log A_i + \log\left(\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} \alpha\right) + \rho_i - \overline{U}}{\left(\frac{2}{3} g_i + 1\right)\kappa} \right)^2 \pi \tag{2.12}$$

Then amenity index is backed out from equation (2.12) in the equilibrium, which is

$$\rho_i = \left(\frac{N_i}{\pi}\right)^{\frac{1}{2}} \hat{g}_i \kappa - \frac{1}{\alpha}\log A_i - \log\left(\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} \alpha\right) + \overline{U} \tag{2.13}$$

2.2 Parameter calibration

All parameters adopted in the following counterfactual analyses have the same values as in D-RH (2013), except the one representing labor losses per mile. To match the actual city size distribution based on the model, the value of κ=0.001 instead of 0.02 as in D-RH (2013). The reason might be that data we use is from year 2005 instead of year 2005-year 2008. In the latter case, a larger number of large cities are exposed to severe urban congestions such that in equation (2.8), the constant term is escalated due to that rate of labor tax should be higher because of severer congestions. Nevertheless, intuitions would not change using either of the two samples. Other parameters are the same as in D-RH (2013).

2.3 Results

Results and analyses of the cases without and with agglomeration economies are analogous to what D-RH (2013) have already documented. Any of the three sources, urban efficiency, amenity and excess friction is of great importance in decomposing urban size distribution. Welfare alternation is modest with large reallocations in each counterfactual scenario.

3. *Accounting for Skill Concentration and Welfare Inequality*

3.1 Benchmark model

It is more intriguing to investigate whether or not more alike cities contribute to more geographic concentration of skill and narrower welfare inequality gap between the high-skill and the low-skill, since more and more highly educated workers are clustering in productive and amenable cities. The understanding of skill heterogeneity is based on discussions in Fu and Hao (2014). High-skill workers productivity is dependent on location advantage and lower coordination costs in thicker labor market where urban size is larger and skill-share is higher. Low-skill productivity relies on the skill composition too, since imperfect substitution generates more demand of them.

Based on above homogeneous urban accounting model, we present the case of heterogeneous skill. Welfare is gained by consumption of a capacity of goods and services and local amenities; the budget constraint imposes that disposable income after local tax payment can be used to consume goods and services. Local friction is represented in the form of local income tax, $\hat{\tau}_i$. For low-skill workers, each of them get utility

$$U_{1i} = \log c_{1i} + \log \rho_{1i} \qquad \text{s.t.} \quad c_{1i} = w_{1i} e^{-\hat{\tau}_i}$$

For high-skill workers, each of them get utility

$$U_{2i} = \log c_{2i} + \log \rho_{2i} \qquad \text{s.t.} \quad c_{2i} = w_{2i} e^{-\hat{\tau}_i}$$

where $\rho_2 > \rho_1$ if high-skill workers have a larger WTP for urban amenities, i.e. high-skill premium for urban amenities is positive.

Then rate of labor tax from the two budget constraints is

$$\hat{\tau}_i = -\ln \frac{c_{1i}L_{1i} + c_{2i}L_{2i}}{w_{1i}L_{1i} + w_{2i}L_{2i}} = -\ln \frac{C_i}{total\,income_i} = -\ln \frac{c_i}{average\;income_i} \tag{3.1}$$

where C_i is aggregate consumption; c_i is per capita consumption. Therefore $\hat{\tau}_i$ is the same as in the homogenous case. The proportion of losses on wage due to commuting and renting is identical for either of the two skilled groups' residents who live at the same distance to the CBD.

But amount of income losses is larger for higher income workers even if they live at the same distance to CBD as lower income ones.

All workers live within the city circle. Imagine high-skill workers all live in the arc area that is L_{2i}/N_i proportional to the whole circle, because each of them occupies 1 housing unit. Similarly for low-skill workers, their occupied arc area has the proportion of L_{1i}/N_i to the whole circle. City size or area size of the circle is the sum of both two skilled groups, $N_i = L_{1i} + L_{2i}$.

Then total commuting of low-skill workers is

$$TC_{1i} = \int_0^{\bar{d}_i} (2\pi d^2 \frac{L_{1i}}{N_i})dd = \frac{2}{3}\pi^{-\frac{1}{2}} N_i^{\frac{1}{2}} L_{1i}$$

Then total commuting of high-skill workers is

$$TC_{2i} = \int_0^{\bar{d}_i} (2\pi d^2 \frac{L_{2i}}{N_i})dd = \frac{2}{3}\pi^{-\frac{1}{2}} N_i^{\frac{1}{2}} L_{2i}$$

Local government face that government expenditure equals government revenue,

$$g_i\left(w_{1i}\kappa TC_{1i} + w_{2i}\kappa TC_{2i}\right) = \tau_i\left(L_{1i}w_{1i} + L_{2i}w_{2i}\right)$$

where κTC_{1i} is labor losses for total commuting of low-skill workers, $w_{1i}\kappa TC_{1i}$ is income losses for total commuting of low-skill workers, similarly for high-skill ones. Then rate of labor wedge is

$$\tau_i = g_i \kappa \frac{2}{3}\left(\frac{N_i}{\pi}\right)^{\frac{1}{2}}$$

At any point within the city, labor losses due to commuting and renting is the same for high-skill and low-skill workers,

$$R_i(\mathrm{d}) + T(\mathrm{d}) = \kappa(\frac{N_i}{\pi})^{\frac{1}{2}}$$

Same as the improvement we make in the homogenous case, new labor tax rate $\hat{\tau}_i$ is defined as

$$e^{-\hat{\tau}_i} \equiv 1 - \tau_i - R_i - T_i$$

where $\hat{\tau}_i = \tau_i + R_i + T_i$ if $\tau_i + R_i + T_i$ is sufficiently small.

Therefore,

$$\hat{\tau}_i = \hat{g}_i \kappa (\frac{N_i}{\pi})^{\frac{1}{2}} \tag{3.2}$$

$$\hat{g}_i = \frac{2}{3} g_i + 1$$

Analogously, we could retrieve the estimations of excess friction $\hat{g}_i$ and κ.

For both low-skill workers and high-skill workers, their indirect utility functions are respectively

$$\overline{U}_1 = \log\left(w_{1i} e^{-\hat{\tau}_i}\right) + \log \rho_{1i} \tag{3.3}$$

$$\overline{U}_2 = \log\left(w_{2i} e^{-\hat{\tau}_i}\right) + \log \rho_{2i} \tag{3.4}$$

It includes complete factors that account for utility: wage that is promoted by more extensive division of labor, the comprehensive labor tax rate that represents the extent of income losses from standard frictions and excess frictions, and skill-specific preferences for urban amenities.

Production function is defined as in Fu and Hao (2013),

$$Y_i = K_i^{1-\alpha} H_i^{\alpha} \tag{3.5}$$

Human capital stock is

$$H_i = \left[\left(h_1 L_{1i}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(h_{2i} L_{2i}\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.6}$$

when the elasticity of substitution is lower, low-skill productivity is enhanced because of scarcity caused by larger demand. More productive cities with concentration of high-skill workers should see more presence of low-skill workers.

First Order Conditions are

$$w_1 = K^{1-\alpha} \alpha H^{\alpha-1} G_1 \, \mathrm{h}_1 \tag{3.7}$$

$$w_2 = K^{1-\alpha} \alpha H^{\alpha-1} G_2 \, \mathrm{h}_2 \tag{3.8}$$

$$r = \frac{\partial Y}{\partial K} = \left(1-\alpha\right) K^{-\alpha} H^{\alpha} \tag{3.9}$$

Subscripts that indicate city i are expressed for convenience. Then low-skill wage is

$$w_1 = \alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} h_1^{1-\frac{1}{\varepsilon}} L_1^{-\frac{1}{\varepsilon}} H^{\frac{1}{\varepsilon}} = \alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} h_1 \left(1+\left(\frac{\mathrm{h}_2}{\mathrm{h}_1}\right)^{1-\frac{1}{\varepsilon}}\left(\frac{L_2}{L_1}\right)^{1-\frac{1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} \tag{3.10}$$

High-skill wage is

$$w_2 = w_1\left(\frac{h_2}{h_1}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{L_1}{L_2}\right)^{\frac{1}{\varepsilon}} \tag{3.11}$$

High-skill quality is

$$h_2 = h_1\left(\frac{w_2}{w_1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(\frac{L_2}{L_1}\right)^{\frac{1}{\varepsilon-1}} \tag{3.12}$$

High-skill productivity is as well the urban productivity since low-skill quality is assumed to be invariant across locations. And low skill productivity is low-skill wage.

If one assumes $\rho_2 = \rho_1^{\omega}$, then

$$\frac{w_1}{w_2} = e^{\bar{U}_1-\bar{U}_2}\frac{\rho_2}{\rho_1} \quad \text{and} \quad \frac{\rho_2}{\rho_1} = \rho_1^{(\omega-1)}$$

Substitute above two equation into (3.10) and (3.11), we get

$$w_1 = \alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} h_1\left(1+\left(\frac{\rho_1^{(\omega-1)}\exp\left(\bar{U}_1-\bar{U}_2\right)}{h_1}h_2\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} \tag{3.13}$$

$$w_2 = \frac{\alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} h_1}{\rho_1^{(\omega-1)}\exp\left(\bar{U}_1-\bar{U}_2\right)}\left(1+\left(\frac{\rho_1^{(\omega-1)}\exp\left(\bar{U}_1-\bar{U}_2\right)}{h_1}h_2\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} \tag{3.14}$$

If elasticity of substitution $\varepsilon < 2$ (in fact $\varepsilon = 1.6$ in estimations of Fu and Hao (2013), equation (3.13) says that low-skill wage w_1 is a concave function of urban productivity h_2, ceteris paribus; moreover due to high-skill wage is proportional to low-skill wage, high-skill wage w_2 is also a concave function of urban productivity h_2. The data gives that $\rho_1^{(\omega-1)}\exp\left(\bar{U}_1-\bar{U}_2\right)\in(0,1)$, so high-skill wage should have a steeper slope with respect to productivity, shown as below

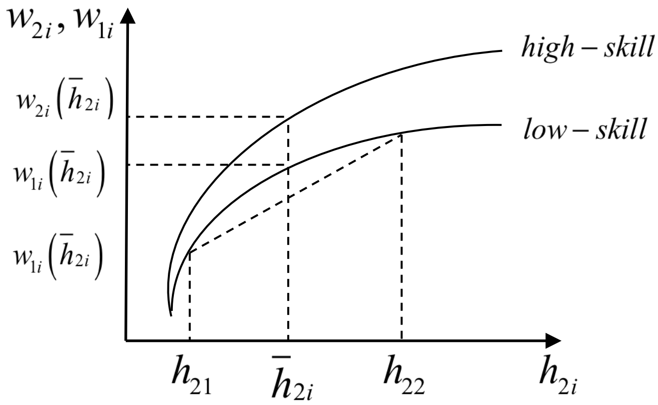


Figure: The concavity between wage and productivity from model

Concavity means

$$w_{2i}\left(\bar{h}_{2i}\right) > \frac{w_{21}\left(h_{21}\right) + w_{22}\left(h_{22}\right)}{2} \quad \text{and} \quad w_{1i}\left(\bar{h}_{2i}\right) > \frac{w_{11}\left(h_{21}\right) + w_{12}\left(h_{22}\right)}{2}$$

These inequalities suggest that more convergence in productivity across cities results in higher high-skill wage and low-skill wage. Figure 3.1 and Figure 3.2 illustrate the concave association between wages and urban productivity after controlling for impacts of urban amenities. An even productivity distribution increases total wages and welfare of both skill groups, and this is one opposing component in our urban accounting to D-RH (2013). Note that the relationship between the rate of labor loss and city size is still convex. If city size distribution is more converged, total frictions tend to become smaller and less welfare is lost.

In equilibrium, suppose there are I cities in a city system, $i = 1, 2, ..., I$, each worker is indifferent in any location, $\bar{U}_1 = U_{1i}$ *and* $\bar{U}_2 = U_{2i}$ *for* $\forall$ *city i.* The indirect utility $\bar{U}_1$ is determined when low-skill labor market clears,

$$\sum_{i=1}^{I} L_{1i} = \bar{L}_1 \tag{3.15}$$

And the indirect utility $\bar{U}_2$ is determined when high-skill labor market clears,

$$\sum_{i=1}^{I} L_{2i} = \bar{L}_2 \tag{3.16}$$

Total population is $\bar{L}_1 + \bar{L}_2 = N$.

In equilibrium, welfare is individually defined by

$$\overline{U}_1 = \log\left(\alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} h_1\left(1+\left(\frac{h_2}{h_1}\right)^{1-\frac{1}{\varepsilon}}\left(\frac{L_2}{L_1}\right)^{1-\frac{1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} e^{-\hat{g}_i\kappa(\frac{N_i}{\pi})^{\frac{1}{2}}}\right)+\log\rho_1 \tag{3.17}$$

$$\overline{U}_2 = \log\left(\alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} h_2^{1-\frac{1}{\varepsilon}} h_1^{\frac{1}{\varepsilon}} L_1^{\frac{1}{\varepsilon}} L_2^{-\frac{1}{\varepsilon}}\left(1+\left(\frac{h_2}{h_1}\right)^{1-\frac{1}{\varepsilon}}\left(\frac{L_2}{L_1}\right)^{1-\frac{1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} e^{-\hat{g}_i\kappa(\frac{N_i}{\pi})^{\frac{1}{2}}}\right)+\log\rho_2 \tag{3.18}$$

3.2 The calibration of parameters

We calibrate values for the parameters in the model both from literature and estimation. human capital input share in the production function is α=0.66, interest rate for renting physical capital is r=0.02, labor losses per mile when commuting and is estimated by 2005 CPS-MORG files data and κ=0.001, the elasticity of substitution adopts estimation from Fu and Hao (2013) and ε=1.6, low-skill quality of human capital is proxied for by the median of schooling years of the low-skill workers across cities and we assume the median is they attend one year of high-school but not finish it, $h_1 = 10$.

Urban success is not only explained by location fundamentals but also by itself. The concentration of population and its skill composition determines productivity, enhancing better urban performance. We call this type of externality as "productivity externality", and by following what has been examined in Fu and Hao (2014), we could derive location advantage shifter, i.e. fundamental productivity, $A = a^{\eta_1}$ from estimation of below equation using a Two-step GMM method,

$$h_{2i} = f\left(a_i, N_i, S_i\right) = a_i^{\eta_1} N_i^{\eta} S_i^{\beta} \tag{3.23}$$

where high-skill share $S_i = L_{2i}/L_{1i}$ and urban employment size $N_i = L_{2i} + L_{1i}$. Utilizing U.S. metropolitan area data in 2005, we get the estimations η=0.07 and β=1.34. Transformation as discussed in Fu and Hao (2014) gives the elasticity of productivity to city size at η=0.05, and skill-share elasticity of productivity is β=0.6. Ahlfeldt, Redding, Sturm, and Wolf (2014) estimate the elasticity of productivity with respect to local density of workplace employment at 0.07 for Germany. Behrens et al. (2010) report a calibration of urban size elasticity at 0.05.

From the transformations of (3.17) and (3.18), log of skill-specified amenities are

$$\log \rho_{1i} = \overline{U}_1 - \log\left(\alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} h_1 \left(1 + \left(\frac{h_{2i}}{h_1}\right)^{1-\frac{1}{\varepsilon}} \left(\frac{L_{2i}}{L_{1i}}\right)^{1-\frac{1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} e^{-\hat{g}_i \kappa (\frac{N_i}{\pi})^{\frac{1}{2}}} \right) \quad (3.19)$$

$$\log \rho_{2i} = \overline{U}_2 - \log\left(\alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} h_{2i}^{1-\frac{1}{\varepsilon}} h_1^{\frac{1}{\varepsilon}} L_{1i}^{\frac{1}{\varepsilon}} L_{2i}^{-\frac{1}{\varepsilon}} \left(1 + \left(\frac{h_{2i}}{h_1}\right)^{1-\frac{1}{\varepsilon}} \left(\frac{L_{2i}}{L_{1i}}\right)^{1-\frac{1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} e^{-\hat{g}_i \kappa (\frac{N_i}{\pi})^{\frac{1}{2}}} \right) \quad (3.20)$$

Normalizations of reservation utility $\overline{U}_1$ and $\overline{U}_2$ allows backing out low-skill preferred amenities ρ_1 and high-skill preferred amenities ρ_2 according to (3.19) and (3.20), for instance, $\overline{U}_1 = 10$ and $\overline{U}_2 = 11$, to correspond to log of annual mean wages.

To estimate "amenity externality", low-skill preferred urban amenities is set as

$$\rho_{1i} = \overline{\rho}_i N_i^{\zeta}$$

and high-skill preferred ones as

$$\rho_{2i} = \overline{\rho}_i^{\omega_2} N_i^{\omega_1 \zeta}$$

where $\overline{\rho}$ is the fundamental urban amenities, high-skill premium over urban amenities is positive if $\omega_2 - 1 > 0$; ζ is the "amenity externality", i.e. urban size elasticity of amenities.

A small strand of studies emphasize that skill composition determines urban amenities since higher income means higher consumption capability, more high-skill concentration thus predicts more urban amenities offered (Handbury, 2012; Diamond, 2012). However, this means more presence of low-skill workers implies worse amenities. Even though more grocery stores and café shops are amenable, these services are provided by the low-skill workers. Some might think of the correlation between higher crime rates and more low-skill workers, yet crime rate is just one dimension of disamenities. Take declined Detroit for example, high crime rate these days do not mean it is all about worse amenities, good public transport system and other existent amenities are not rotten by low-skill workers. Therefore, we admit the contribution from both high-skill and low-skill workers to endogenous amenities, and use urban employment as the driver of amenity externality[6]. By doing so, we place equal weight for the two skill groups' individual role[7]. Glaeser and Saiz (2004) clearly show skills predict productivity growth but not an increase in amenity levels; at the utmost, skills increase amenities at the very local level instead of at the metropolitan area level. That is to say, skilled neighborhoods could be amenities.

We estimate (3.21) and (3.22) to calibrate amenity-related parameters, ω_2, ω_1, ζ,

$$\log \rho_{1i} = constant + \zeta \log(N_i) + error_{1i} \tag{3.21}$$

where the exogenous fundamental is $\log \overline{\rho}_i = error_{1i}$. Due to the effects of amenities on urban size is orthogonal to the impacts of housing supply to population size, geographic housing supply constraints by Saiz (2010) are adopted as instruments, such as share of land that is not ocean within 50 km Radius, share of land with slope < 15% within 50 km Radius, etc.

According to the identification of high-skill preferred amenity,

$$\log \rho_{2i} = constant + \omega_1 \zeta \log(N_i) + \omega_2 \log \overline{\rho}_i + error_{2i} \tag{3.22}$$

so the relationship between the WTP for urban amenities by high-skill and low-skill workers is $\rho_2 = \rho_1^{\omega_2} N^{(\omega_1 - \omega_2)\zeta}$. Instruments for urban size are the same as those for estimation of equation (3.21). The estimations using 2005 data give ω_1= 1.18 and ω_2= 1.18, ζ = 0.2. Consumption externality of amenities is an important agglomeration source. Ahlfeldt, Redding, Sturm, and Wolf (2014) find that elasticity of amenities with respect to urban employment is 0.14 for Germany cities.

Based upon the fact that ω_1= ω_2= 1.18, we use ω= 1.18 in the numerical exercises below to express high-skill amenity, $\rho_2 = \rho_1^{\omega} e^{delta}$, take log and estimate

$$\log \rho_{2i} = constant + \omega \log \rho_{1i} + delta_i$$

where e^{delta_i} represents the idiosyncratic preference of high-skill workers to low-skill preferred

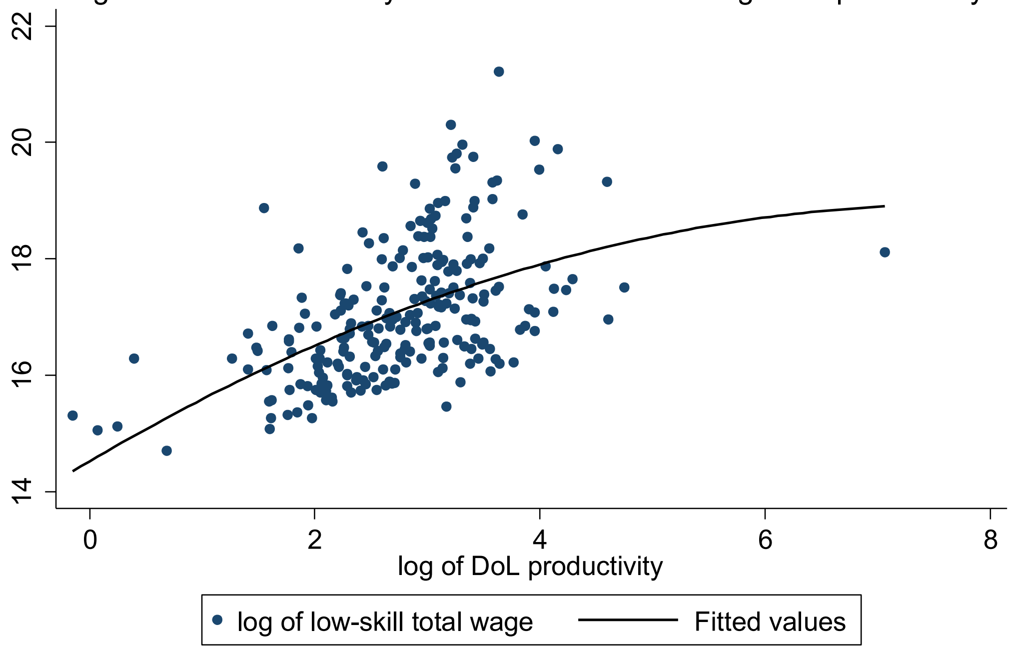
6 Communities within a MSA do have a varied distribution: high-income and low-income communities are distant to each others. This segregation of communities within a MSA might be outstanding, and high-skill community may enjoy better amenities than low-skill community do, but this paper does not accomplish this point of view.

7 Future work could give a bigger weight for the presence of high-skill workers. It has been demonstrated later that amenity externality driven by urban size induces more skill concentration, although adding skill-share as amenity externality driver introduces even more concentration of high-skill workers.

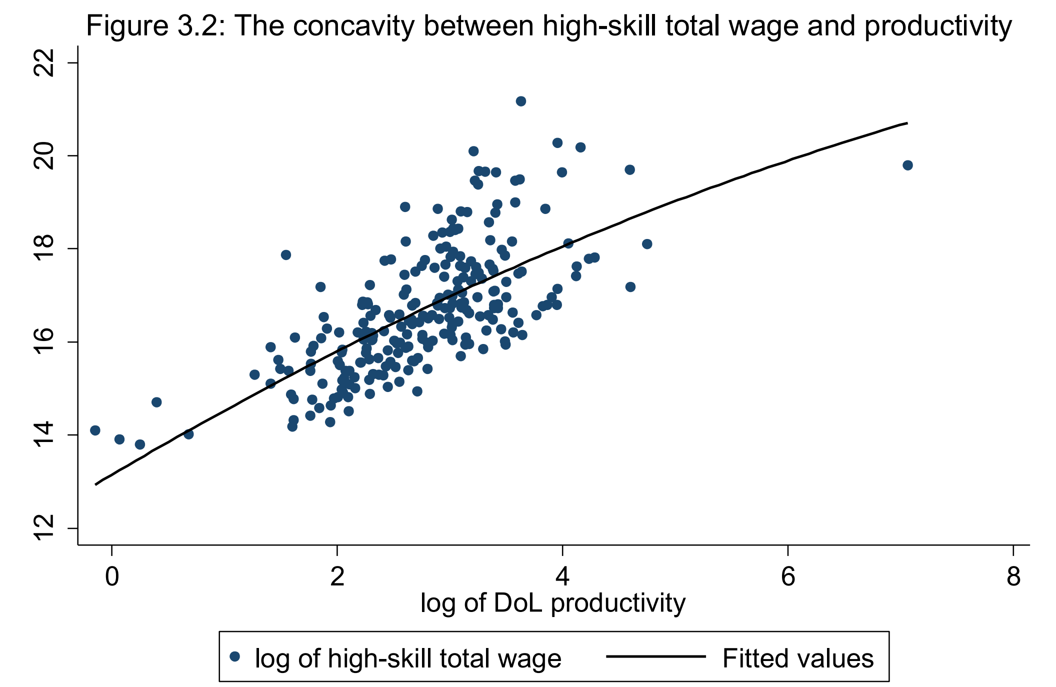
amenity. In the case of equalizing fundamental amenity $\bar{\rho}$, we are able to equalize both ρ_2 and ρ_1 with respect to their exogenous parts; in the case of equalizing full amenities ρ_1 in the case of no externalities, we are able to equalize ρ_2 at the same time.

Association between wage and urban productivity are document the in Figure 3.1 and Figure 3.2, after the pinning down of high-skill preferred amenity and low-skill preferred amenity and calibrations of related parameters, according to equation (3.13) and (3.14). In order to control the impacts of urban amenities, the relationship between log of low-skill wage and log of urban productivity is displayed with low-skill preferred amenity at its mean; due to the log-linear relationship between high-skill preferred amenity and low-skill preferred amenity, Figure 3.2 also demonstrates the association between log of high-skill wage and log of urban productivity when low-skill preferred amenity is averaged. As can be seen from both Figure 3.1 and Figure 3.2, relationships between total skilled wage of individual skill group and productivity from 241 cities of U.S. metropolitan areas in 2005 are respectively concave; these results suggest that equalized urban efficiency enhances welfare of both skill cohorts. Moreover, log of high-skill total wage embodies a steeper slope with respect to log of urban productivity, implying that as urban productivity is improved, it might enlarge welfare gap between the two groups.

Figure 3.1: The concavity between low-skill total wage and productivity



Source: 2005 Bureau of Economic Analysis and 2005 CPS-MORG Files, 241 cities.



Source: 2005 Bureau of Economic Analysis and 2005 CPS-MORG Files, 241 cities.

3.3 The spatial equilibrium

In order to demonstrate how location fundamentals and externalities account for skill concentration and welfare inequality, we transform equations (3.15)-(3.18). Assume *proportion of low-skill in certain city* with respect to aggregate low-skill supply is

$$l_{1i} = \frac{L_{1i}}{L_1} \quad , \text{i=1, 2,…, I}$$

and

$$\sum_{i=1}^{I} l_{1i} = 1 \qquad (3.24)$$

Analogously, for high-skill workers,

$$l_{2i} = \frac{L_{2i}}{L_2} \quad , \text{i=1, 2,…, I}$$

and

$$\sum_{i=1}^{I} l_{2i} = 1 \qquad (3.25)$$

We also assume the *relative skill proportion* in a city is[8]

[8] Note here s_j is relative skill proportion and different from what we define the skill share in Fu and Hao

$$s_i = \frac{l_{2i}}{l_{1i}}$$

Aggregate skill-share is

$$\overline{S} = \frac{\overline{L}_2}{\overline{L}_1}$$

Then skill-share in city i is,

$$\frac{L_{2i}}{L_{1i}} = s_i * \overline{S} \tag{3.26}$$

Then equilibrium condition (3.25) is

$$\sum_{i=1}^{I} l_{1i} s_i = 1 \tag{3.27}$$

Suppose log of wage premium $\delta w_i = \log w_{2i} - \log w_{1i}$ and welfare inequality gap as $\delta U = \overline{U}_2 - \overline{U}_1$, from equation (3.18),

$$\overline{U}_2 = \log\left(\alpha \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} h_{2i}^{1-\frac{1}{\varepsilon}} h_1^{\frac{1}{\varepsilon}} \left(\overline{S} s_i \right)^{-\frac{1}{\varepsilon}} \left(1 + \left(\frac{h_{2i}}{h_1} \right)^{1-\frac{1}{\varepsilon}} \left(\overline{S} s_i \right)^{1-\frac{1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} e^{-\hat{g}_i \kappa \left(\frac{\left(1+\overline{S} s_i\right) l_{1i} \overline{L}_1}{\pi} \right)^{\frac{1}{2}}} \right) + \log \rho_{2i} \tag{3.28}$$

From equations (3.3) and (3.4),

$$\delta\overline{U} = \delta w_i + \left(\log \rho_{2i} - \log \rho_{1i} \right) \tag{3.29}$$

This suggests that for each city, as long as high-skill premium for urban amenities is positive, nominal wage gap is narrower than welfare gap and it is consistent with Diamond (2012).
From equation (3.12),

$$\delta w_i = \frac{\varepsilon - 1}{\varepsilon} \log\left(\frac{h_{2i}}{h_1} \right) - \frac{1}{\varepsilon} \log s_i - \frac{1}{\varepsilon} \log \overline{S} \tag{3.30}$$

Substitute (3.30) to (3.29), we get

$$\delta\overline{U} = \frac{\varepsilon - 1}{\varepsilon} \log\left(\frac{h_{2i}}{h_1} \right) - \frac{1}{\varepsilon} \log s_i - \frac{1}{\varepsilon} \log \overline{S} + \left(\log \rho_{2i} - \log \rho_{1i} \right) \tag{3.31}$$

(2014); s_j here enriches its meaning by taking into account skill/pop endowments.

Proposition 1 (Spatial equilibrium)

Location fundamentals $\left\{A_i,\overline{\rho}_i,g_i\right\}$, together with parameters $\{\alpha,\mathrm{r},\kappa,\varepsilon,\omega,\eta,\beta,\zeta\}$ and constants $\left\{h_1,\overline{L}_1,\overline{S}\right\}$ define the spatial equilibrium such that urban attributes and urban performances $\left\{h_{2i},\rho_{1i},\rho_{2i},\hat{\tau}_i,w_{2i},\delta w_i,s_i,l_{1i},\overline{U}_2,\delta\overline{U}\right\}$ are determined, when equations (3.24), (3.27), (3.28) and (3.31) are all satisfied.

Proposition 1 says there exists one-to-one mappings in counterfactual simulations. We solve the counterfactual equilibrium using initial values of observations in real equilibrium, supposing that the equilibrium follows the selection rule of search for the closest-to-reality. Although there is a probability of multiple equilibria in the case of externalities, the agenda of counterfactual exercises is mainly on predicting roles of location fundamentals and externalities.

3.4 The case of two cities

3.4.1 Identical cities

We subsequently adopt a 2-city case to demonstrate how skill concentration, wage gap, and welfare inequality are jointly determined when the two cities are heterogeneous in productivity, amenities, and excessive frictions, and when agglomeration economies, elasticity of substitution, skilled population endowments are respectively altered.

In an urban system with two cities, city a and city b ($i=a,b$), Figure 3.2-Figure 3.4 respectively depicts the relationships among equations (3.24), (3.27) and (3.30), helping to understand when productivity and amenities are differentiated, how wage gap, skill-share, and welfare inequality are jointly determined. When the two cities are identical with respect to any urban attribute, relative skill proportion $s_a=s_b$=1. In this case, skill-share and nominal wage gap are indifferent between the two cities. Welfare inequality gap is the same as nominal wage gap. If aggregate skill share $\overline{S}$ is also 1, skill-share $L_{2a}/L_{1a}=L_{2b}/L_{1b}$ in the two cities are 1.

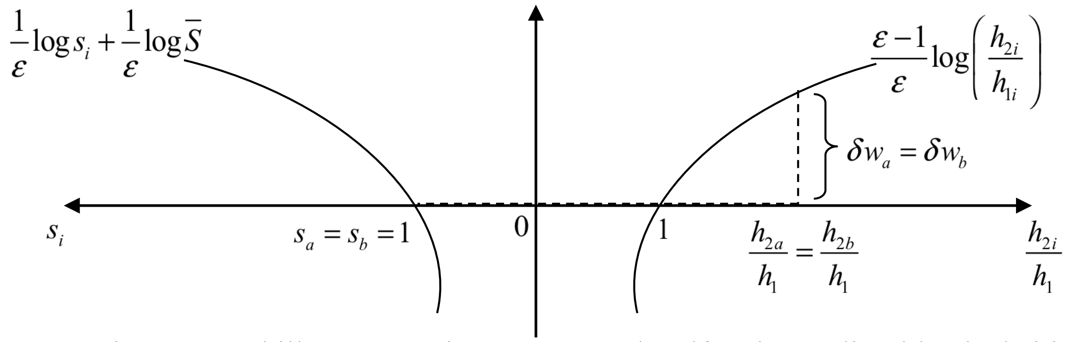


Figure 3.2: Skill concentration, wages, and welfare inequality, identical cities

But we should note that this symmetric equilibrium is unstable, as one worker either of high-skill or low-skill group moves from city b to city a, city a would be more attractive to both types of workers, and it would have a larger urban size.

By setting magnitudes of parameters as $\{\alpha = 0.66, \mathrm{r} = 0.02, \kappa = 0.0015, \varepsilon = 1.6, \omega = 1.16, \eta = 0.07, \quad \beta = 1.34, \zeta = 0.25\}$, constants as $\{h_1 = 1, \overline{L}_1 = 20, \overline{S} = 0.5\}$, and urban performances as $\{w_1 = 10, w_2 = 15, L_1 = 10, L_2 = 5, \overline{U}_1 = 10, \overline{U}_2 = 11, C = 100\}$ for both two cities, it is allowed to pin down location attributes according to equations (3.2), (3.28), (3.30) and (3.31), as $\{h_2 = 0.9287, \rho_1 = 12402, \rho_2 = 22475, g = 259.8118\}$. Nominal wage gap $\delta w = 0.4055$, relative skill proportion $s_a = s_b = 1$, proportion of low-skill to aggregate low-skill supply $l_{1a} = 0.5$ in city a, welfare inequality gap $\delta U = 1$.

3.4.2 Only heterogeneous productivity

We now allow productivity varies between the two cities. Figure 3.3 displays the case when city a is more productive. Since amenities are the same, nominal wage gap should be equivalent $\delta w_a = \delta w_b$.

However, due to the fact that curvature matters, the relative skill proportions in the two cities are not going to be the same, i.e. $s_a \neq s_b$. In addition, if city a is more productive, ceteris paribus, relative skill proportion of city a is bigger than that of city b. According to equation (3.31), city a has a larger concentration of high-skill workers. The asymmetry between s_a and s_b could cause equilibrium condition (3.27) not to be held; moreover, if the LHS of equation (3.27) is bigger than 1, equivalent δw_a and δw_b should both rise in order to adjust to keep the equilibrium condition (3.27) holds. Welfare inequality thus is expanded.

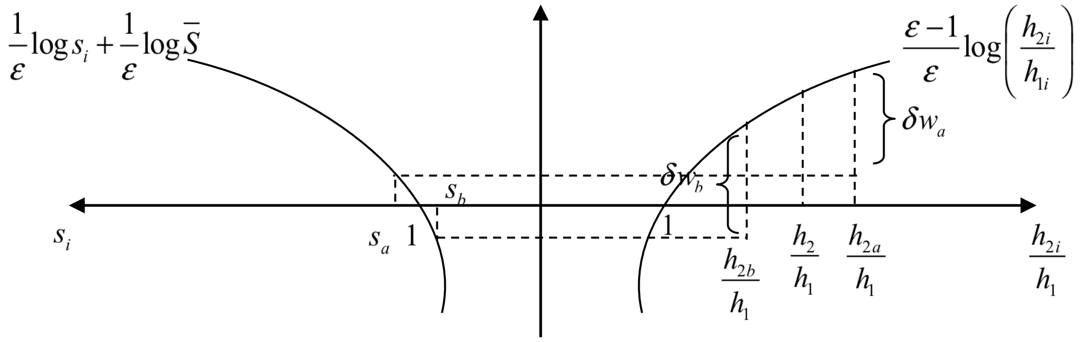


Figure 3.3: Skill concentration, wages, and welfare inequality, $h_{2a} > h_{2b}$

Proposition 2: When two cities are only heterogeneous in productivity, the more productive one has more skill concentration. Welfare inequality gap is wider if $l_{1a}s_a + (1 - l_{1a})s_b > 1$, and narrower if $l_{1a}s_a + (1 - l_{1a})s_b < 1$.

In addition, the relative skill proportion and welfare inequality respond to alternative substitute elasticity, labor loss per mile, the presence of externalities, and aggregate skilled population endowments respectively. Their influences are reported in Table 3.1. We use nominal wage gap, relative skill proportion, proportion of low-skill to aggregate low-skill supply in city a, and welfare inequality gap calculated in identical city case as benchmark, instead we use heterogeneous productivity and alternations of some parameters to demonstrate how they influence skill concentration and welfare inequality.

Compared with the identical city benchmark, proportion of low-skill workers in city a is bigger than the benchmark, relative skill proportion in city a is also bigger than the benchmark, nominal wage gap is narrower and welfare inequality is reduced. These results suggest, in an urban system that cities are equalized with respect to efficiency, there always exists Pareto improvement, from which welfare inequality between skill groups can be curtailed. More productive city captivates not only the high-skill workers but also the low-skill workers, the reason is imperfect substitution between these two skills generates higher demand of the low-skill whose density is relatively low in the skill concentration location; therefore, proportion of low-skill in the more productive city, l_{1a}, has a bigger magnitude. That welfare inequality in the benchmark case when there is no agglomeration economies, 0.9912, versus 1 in equivalent productivity case, corresponds to the reality welfare inequality, 1, versus 1.1208 in equalized productivity case in Table 4.2 of U.S. metropolitan areas in 2005. In the case of identical amenity here, both the two cities have the same nominal wage gap; moreover, uneven productivity makes the gap fall, since low-skill workers are more productive by working near the high-skill ones.

When two cities are indistinguishable, there is no skill concentration in both locations. Skill concentration exists when any one of the three urban attributes are dispersedly distributed across cities. Relative skill proportion s_a in city a is larger than city b, implying that the extent of skill concentration is higher in more productive city.

With the presence of productivity externality due to urban employment, $\eta = 0.07$, welfare inequality is expanded compared with the benchmark case of heterogeneous efficiency. This is due to the compounded effect of the externality attracts more high-skill workers come into the more efficient city, in spite of more low-skill workers as well. Recall that wage is a concave function of urban productivity with steeper slope for the high-skill group, enhanced urban productivity by the externality expands nominal wage and welfare gap. So it is with the productivity externality by skill-share, $\beta = 1.34$. As for the amenity externality, even though more low-skill workers present in city a because of more concentration of high-skill workers, high-skill workers relish more welfare since high-skill premium for amenities is positive, so welfare inequality is severer.

When the elasticity of substitution increases from 1.6 to 1.8, low-skill workers demand curve shifts leftward, they are not demanded more in the more efficient city and high-skill workers come to city a. Due to more supply of high-skill workers into city a to replace low-skill jobs, their labor supply curve shifts rightward, wage of the high-skill group drops, therefore welfare inequality narrows.

Table 3.1: Only heterogeneous productivity in a 2-city case

Cases / Outcomes	Identical	$h_{2a} > h_{2b}$: $h_{2a} = 1, h_{2b} = 0.8$						
Urban outcomes	B.m.k	B.m.k	$\eta = 0.07$	$\beta = 1.34$	$\zeta = 0.26$	$\varepsilon = 1.8$	$\kappa = 0.003$	$\bar{S} = 0.6$ $\bar{L}_1 = 14$ $\bar{L}_2 = 24$
l_{1a}	0.5	0.5461	0.5498	0.7154	0.6283	0.5317	0.5173	0.5283
s_a	1	1.0603	1.0644	1.1640	1.0822	1.0829	1.0644	1.0628
s_b	1	0.9274	0.9214	0.5879	0.8701	0.9059	0.9310	0.9296
$\delta w_a = \delta w_b$	0.4055	0.3966	0.3973	0.4146	0.4083	0.3408	0.3942	0.2988
δU	1	0.9912	0.9918	1.0092	1.0029	0.9354	0.9888	0.8933

Notes: 1. B.m.k represents benchmark. 2. Except cases with the existence of externality, other scenarios of heterogeneous productivity has no externalities.

Severer friction due to doubling of labor loss per mile on road, $\kappa = 0.003$, as well reduce welfare inequality in the only heterogeneous efficiency case. More low-skill workers are crowded out of the productive city and their original jobs are taken place by high-skill workers. Total friction in a large city which is the cluster of high-skill workers is greater, therefore welfare inequality between the two skill groups is reduced.

Relatively large high-skill supply aggravates the scarcity of low-skill workers, especially in those high-skill brain hubs. Therefore less skill concentration is concurrent with scaled down welfare inequality as low-skill demand curve shifts rightward. s_a is a function of skill-share and aggregate skill-share in city a, therefore skill-share of the benchmark case of heterogeneous productivity in city a is larger than of the counterfactual aggregate skill supply case, implying that skill concentration is weakened with more supply of high-skill workers.

3.4.3 Only heterogeneous amenities

Unlike low-skill workers, high-skill ones have a larger WTP for them. So city *a* and city *b* possibly is accounted as heterogeneous in amenities when any of the four cases happen: 1) $\delta\rho_a = \delta\rho_b$ & $\rho_{2a} > \rho_{2b}$ (i.e. $\rho_{1a} > \rho_{1b}$), 2) $\delta\rho_a > \delta\rho_b$ & $\rho_{2a} = \rho_{2b}$ (i.e. $\rho_{1a} < \rho_{1b}$), 3) $\delta\rho_a > \delta\rho_b$ &

$\rho_{2a} > \rho_{2b}$ & $\rho_{1a} = \rho_{1b}$, and 4) $\delta\rho_a > \delta\rho_b$ & $\rho_{2a} > \rho_{2b}$ & $\rho_{1a} > \rho_{1b}$, in which skill-differential WTP is $\delta\rho_i = \log\rho_{2i} - \log\rho_{1i}$.

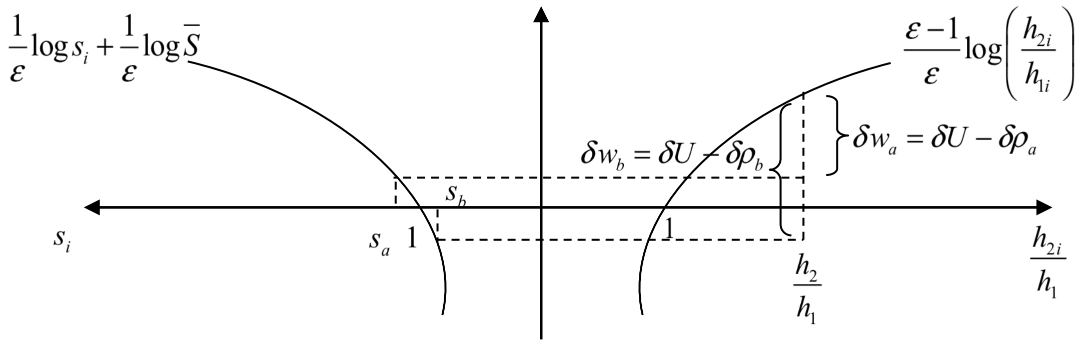


Figure 3.4: Skill concentration, wages, and welfare inequality, $\delta\rho_a > \delta\rho_b$

Figure 3.4 shows how skill concentration and welfare are determined when $\delta\rho_a > \delta\rho_b$. City a of bigger skill-differential WTP has a smaller nominal wage gap since welfare inequality across cities is indifferent, ceteris paribus. The unequal relative skill proportions s_a and s_b challenge equilibrium condition (3.25). In order to keep its validity, nominal wage gaps in two cities should be wider if $l_{1a}s_a + (1-l_{1a})s_b > 1$.

Proposition 3: When two cities are only heterogeneous in amenities, the one with larger skill-differential WTP has more skill concentration. Welfare inequality gap is wider if $l_{1a}s_a + (1-l_{1a})s_b > 1$, and narrower if $l_{1a}s_a + (1-l_{1a})s_b < 1$.

Impacts of amenity, imperfect substitute elasticity, labor loss per mile, externalities, and skilled population endowment respectively on skill concentration and welfare inequality are reported in Table 3.2 and Table 3.3. Table 3.2 exhibits only one scenario of heterogeneous amenity in which skill-differential WTP is wider in city a, low-skill preferred amenity is lower in city a, but high-skill preferred amenity is equivalent. Table 3.3 subsequently provides impacts of amenity on skill concentration and welfare inequality in other three scenarios.

In Table 3.2, it is found that high-skill workers concentrate in city a but larger amount of low-skill workers locate in city *b*. The difference of skill-differential WTP between city *a* and *b* is the difference of nominal wage gap between city *b* and *a*. City *a* has a lower wage inequality in this case. Welfare inequality across the two cities is narrower since more low-skill workers live in city b where their preferred amenity is higher. That welfare inequality in the benchmark case when there is no agglomeration economies, 0.8762, versus 1 which represents equivalent amenity case, corresponds to the reality welfare inequality, 1, versus 1.0279 which speaks for equalized amenity case in Table 4.2 of U.S. metropolitan areas in 2005.

Table 3.2: Only heterogeneous amenities in a 2-city case

Cases / Outcomes	Identical	1. $\delta\rho_a > \delta\rho_b$, $\rho_{2a} = \rho_{2b}$ (i.e. $\rho_{1a} < \rho_{1b}$)						
Urban outcomes	B.m.k	B.m.k	$\eta = 0.07$	$\beta = 1.34$	$\zeta = 0.26$	$\varepsilon = 1.8$	$\kappa = 0.003$	$\bar{S} = 0.6$ $\bar{L}_1 = 14$ $\bar{L}_2 = 24$
l_{1a}	0.5	0.3150	0.3058	0.7308	0.3144	0.3628	0.3951	0.3107
s_a	1	1.2191	1.1994	1.2693	1.0745	1.2266	1.1887	1.2208
s_b	1	0.8993	0.9121	0.2687	0.9556	0.8710	0.8768	0.9005
δw_a	0.4055	0.2817	0.2806	0.3763	0.3605	0.2387	0.2975	0.1845
δw_b	0.4055	0.4718	0.4708	0.5665	0.4339	0.4289	0.4877	0.3746
δU	1	0.8762	0.8751	0.9708	0.8940	0.8333	0.8920	0.7790

Notes: 1. B.m.k represents benchmark. 2. Except cases with the existence of externality, other scenarios of heterogeneous amenity has no externalities.

Although a little bit drop of low-skill workers in city a when productivity amenity is generated by urban employment size, low-skill workers get more welfare since amenity tasted by high-skill ones is not higher in city a . In the case of productivity externality driven by high-skill share, almost double amount of low-skill workers are demanded in city a where productivity is high, even though city b offers better amenities for them. When urban size accounts for amenities, city b is the larger one, although city a has a slightly more concentration of high-skill workers. The more preferred amenities bring about more welfare for high-skill workers and expand the gap of welfare between the two skill groups.

As the elasticity of substitution increases, 36.28 percent of low-skill workers choose to stay in the city where high-skill workers concentrate, which is larger than the 31.5 percent in the benchmark case. Wage gap is much wider in city b because low-skill wage is compensated by desirable amenities there. Since scarcity means less important to the low-skill productivity in this case, a larger proportion of low-skill in abundant low-skill preferred amenity city results in narrower welfare gap.

Rise in the labor loss per mile deters high-skill workers to concentrate in city *a*, so some of them choose to live in the less frictional city *b* when productivity is the same. Larger presence of high-skill workers in city *b* replace some of the jobs previously done by low-skill workers whom are squeezed out to city *a* with a bigger gap of welfare in equilibrium.

Table 3.3: Only heterogeneous amenities, other 3 cases

Cases / outcomes	2. $\delta\rho_a > \delta\rho_b$ $\rho_{2a} > \rho_{2b}$, $\rho_{1a} > \rho_{1b}$	3. $\delta\rho_a > \delta\rho_b$ $\rho_{2a} > \rho_{2b}$, $\rho_{1a} = \rho_{1b}$	4. $\delta\rho_a = \delta\rho_b$ $\rho_{2a} > \rho_{2b}$ (i.e. $\rho_{1a} > \rho_{1b}$)
l_{1a}	0.6669	0.4750	0.9398
s_a	1.1543	1.4148	0.9709
s_b	0.6911	0.6248	1.4544
δw_a	0.3158	0.1886	0.4239
δw_b	0.6364	0.6994	0.1714
δU	1.0089	1.0720	0.9348

Notes: 1. Above three case of heterogeneous amenity has no externalities.

Relatively large supply of high-skill workers than their counterparts again shifts their labor supply curve rightward, resulting in lower wage and narrower welfare gap. Calculations of skill-share from s_a show that skill-share in city *a* is smaller in the case of counterfactual aggregate skill supply, compared with the benchmark case of heterogeneous amenity, so more supply of high-skill workers undermines skill concentration.

As can be seen from Table 3.3, skill concentration and welfare inequality are dependent on how amenities are heterogeneous. High-skill concentration takes place in the city where skill-differential WTP is wider. Shrunken welfare gap comes out of the equilibrium in case 4, of which most of the high-skill workers concentrate in city *b* and most of the low-skill in city *a*.

Ascribing to city *a* is more amenable, welfare gap is narrowed. Case 2 and Case 3 are in line with Lee (2010), since wage premium falls in the city with more high-skill preferred amenities when they would like to consume more of them.

So far we have analyzed how heterogeneity in productivity only and in amenity only affects skill concentration and welfare inequality, simulations in which both productivity and amenities vary at the same time is more intriguing. In reality some cities may be both productive and amenable, while some other cities may be either productive or amenable. These considerations are tackled with in below.

3.4.4 Positive relation between productivity and amenities

We take into account the heterogeneous productivity case and heterogeneous amenity case together in Table 3.4 that shows city *a* is more productive, and skill-differential amenity is larger. Compared with the benchmark case of Table 3.3 where only amenity is heterogeneous, welfare inequality here is even narrower. Because of the higher productivity, city *a* attracts more high-skill workers as well low-skill workers. When amenity and efficiency are positively correlated (enhancing each other), switching off the differences in one of them makes high-skill cohort less concentrate in the more efficient place, of which the welfare inequality widens because equalizing productivity across cities hurts more low-skill workers.

If productivity externality by urban employment size exists, high-skill workers benefit from both higher productivity and agglomeration economies; however, fewer of them would choose to stay in city *a* because amenity is not desirable there. Meanwhile for the low-skill workers, better-off amenity for them reduces welfare inequality. The existence of productivity externality by high-skill concentration lets city *a* be larger and productive, attracting both more high-skill and low-skill workers. More quantity of low-skill workers choose higher productivity over desirable amenity but this shifts rightward labor supply curve of them, nominal wage gap is quite high in city *a*, and in the end welfare gap is wider. Skill concentration in city *a* slightly declines because of amenity externality, so is low-skill population. Although city *b* is more amenable for low-skill workers, deprived higher productivity when leaving city *a* hurts and enlarges welfare inequality.

Table 3.4: Heterogeneous productivity and amenities, positive relation

Cases / Outcomes	$\delta\rho_a > \delta\rho_b$, $\rho_{2a} = \rho_{2b}$ (i.e. $\rho_{1a} < \rho_{1b}$)	1. $h_{2a} > h_{2b} : h_{2a} = 1, h_{2b} = 0.8$; $\delta\rho_a > \delta\rho_b$, $\rho_{2a} = \rho_{2b}$ (i.e. $\rho_{1a} < \rho_{1b}$)						
Urban outcomes	B.m.k	B.m.k	$\eta = 0.07$	$\beta = 1.34$	$\zeta = 0.26$	$\varepsilon = 1.8$	$\kappa = 0.003$	$\bar{S} = 0.6$ $\bar{L}_1 = 14$ $\bar{L}_2 = 24$
l_{1a}	0.3150	0.3552	0.3489	0.8442	0.3410	0.3910	0.4114	0.3323
s_a	1.2191	1.2966	1.2861	1.1617	1.2010	1.3284	1.2640	1.3104
s_b	0.8993	0.8366	0.8467	0.1242	0.8807	0.7891	0.8155	0.8455
δw_a	0.2817	0.2709	0.2689	0.4149	0.3187	0.2273	0.2868	0.1679
δw_b	0.4718	0.4611	0.4591	0.6051	0.4289	0.4175	0.4770	0.3581
δU	0.8762	0.8654	0.8634	1.0094	0.8718	0.8219	0.8814	0.7625

Notes: 1. B.m.k represents benchmark. 2. Benchmark case of heterogeneous productivity and amenity has no externalities.

Elasticity of substitution is crucial for low-skill by determining the importance of scarcity effects on their labor demand. Larger elasticity means less important role of scarcity on low-skill labor demand. Again for low-skill workers, rising substitution elasticity implies a weaker role of scarcity to their productivity, but a larger presence of them in more amenable city b helps reduce welfare inequality.

An interesting finding both seen from Table 3.2 and Table 3.4 is, when labor is more lost on road per mile, low-skill workers locate more in city a . Welfare gap is enlarged due to resemble reason discussed from Table 3.2, although city a has advantage in productivity. And analogously more aggregate supply of high-skill workers slightly reduces skill concentration and welfare inequality.

Table 3.5: Heterogeneous productivity and amenities, positive relation, other 3 cases

Cases / Outcomes	2. $h_{2a} > h_{2b}$, $\delta\rho_a > \delta\rho_b$ $\rho_{2a} > \rho_{2b}$, $\rho_{1a} > \rho_{1b}$	3. $h_{2a} > h_{2b}$, $\delta\rho_a > \delta\rho_b$ $\rho_{2a} > \rho_{2b}$, $\rho_{1a} = \rho_{1b}$	4. $h_{2a} > h_{2b}$, $\delta\rho_a = \delta\rho_b$ $\rho_{2a} > \rho_{2b}$ (i.e. $\rho_{1a} > \rho_{1b}$)
l_{1a}	0.7425	0.5723	0.9822
s_a	1.1398	1.3559	1.0212
s_b	0.5969	0.5238	1.5575
δw_a	0.3514	0.2429	0.4201
δw_b	0.6721	0.7537	0.0726
δU	1.0446	1.1262	0.8835

Notes: 1. B.m.k represents benchmark. 2. Benchmark case of heterogeneous productivity and amenity has no externalities.

Other three scenarios representing positive relation between productivity and amenity are displayed in Table 3.5, from which simulation Case 2 and 3 with both $\delta\rho_a > \delta\rho_b$ and $\rho_{2a} > \rho_{2b}$ drives welfare inequality up compared with the case of identical city. When urban productivity and high-skill preferred amenity is desirable in city a , high-skill and low-skill workers are both appealed to reallocate here. More exposure of high-skill workers to higher preferred amenity and more WTP for it result in wider welfare inequality gap. Equivalent low-skill favored amenity in Case 3 upsurges the inequality relative to Case 2.

3.4.5 Negative relation between productivity and amenities

Table 3.6 and Table 3.7 address the four scenarios that heterogeneous productivity is negatively associated with heterogeneous amenity. Compared with solo heterogeneous amenity case, case of two heterogeneous factors is less intensified in welfare inequality, albeit more distortion occurs due to heterogeneous efficiency to the labor market. Here in the case of Table 3.6, city a loses both two skilled population, and even more low-skill group to city b where their favored amenity is abundant. When amenity and efficiency are negatively correlated (offsetting each other), switching off the differences in one of them makes high-skill cohort more concentrate in the less efficient place, of which the welfare inequality widens because low-skill preferred amenity is less desirable in the less efficient city compared with high-skill preferred amenity to high-skill workers.

When productivity externality by city size explains urban outcomes, fewer high-skill workers concentrate in city *a* despite skill-differential amenity is higher there, so is the low-skill, i.e. this externality contaminate the labor reallocation in the benchmark case relative to only heterogeneous amenity case. High-skill workers are able to reallocate to where their productivity is enhanced by agglomeration economies, which is ampler profit than that to low-skill workers; after all, city *a* has a larger concentration of high-skill workers, low-skill workers in city *b* cannot be closer to large skill concentration in city *a* to be better-off. If skill concentration interprets productivity, low-skill workers together with high-skill ones largely move to the less productive city *a*. However due to that high-skill workers get more benefits from skill concentration, welfare gap is wider than no externality scenario. With the presence of amenity externality, both two skill types migrate to the more productive as well as low-skill preferred city *b* where urban employment size is larger too. More benefits go to the high-skill group thus welfare inequality is raised.

Table 3.6: Heterogeneous productivity and amenities, negative relation

Cases / Outcomes	$\delta\rho_a > \delta\rho_b$, $\rho_{2a} = \rho_{2b}$ (i.e. $\rho_{1a} < \rho_{1b}$)	1. $h_{2a} < h_{2b}$: $h_{2a} = 0.8$, $h_{2b} = 1$; $\delta\rho_a > \delta\rho_b$, $\rho_{2a} = \rho_{2b}$ (i.e. $\rho_{1a} < \rho_{1b}$)						
Urban outcomes	B.m.k	B.m.k	$\eta = 0.07$	$\beta = 1.34$	$\zeta = 0.26$	$\varepsilon = 1.8$	$\kappa = 0.003$	$\bar{S} = 0.6$ $\bar{L}_1 = 14$ $\bar{L}_2 = 24$
l_{1a}	0.3150	0.2773	0.2662	0.5094	0.3309	0.3356	0.3796	0.2895
s_a	1.2191	1.1277	1.0980	1.3985	0.9618	1.1116	1.1077	1.1253
s_b	0.8993	0.9510	0.9644	0.5862	1.0233	0.9436	0.9341	0.9490
δw_a	0.2817	0.2744	0.2754	0.3085	0.3739	0.2271	0.2856	0.1794
δw_b	0.4718	0.4646	0.4656	0.4986	0.4188	0.4713	0.4758	0.3696
δU	0.8762	0.8690	0.8700	0.9030	0.8932	0.8217	0.8802	0.7740

Notes: 1. B.m.k represents benchmark. 2. Benchmark case of heterogeneous productivity and amenity has no externalities.

Although scarcity of the low-skill means less significant when elasticity substitution rises, it does good to them and make them better-off when more low-skill worker migrate closer to the high-skill workers where the high-skill group concentrates, therefore welfare gap reduces. Analogous explanation to Table 3.2 and Table 3.4 could be applied to the case of larger labor loss per mile in Table 3.6, which is even more high-skill workers choose to escape from the traffic in city a and displace some of the low-skill jobs, some low-skill workers have to move away from the more productive and more amenable city b, so welfare gap is enlarged. Massive high-skill supply always lowers both skill concentration and welfare inequality in spite of the relation between productivity and amenity.

Table 3.7: Heterogeneous productivity and amenities, negative relation, other 3 cases

Cases / Outcomes	2. $h_{2a} < h_{2b}$, $\delta\rho_a > \delta\rho_b$ $\rho_{2a} > \rho_{2b}$, $\rho_{1a} > \rho_{1b}$	3. $h_{2a} < h_{2b}$, $\delta\rho_a > \delta\rho_b$ $\rho_{2a} > \rho_{2b}$, $\rho_{1a} = \rho_{1b}$	4. $h_{2a} < h_{2b}$, $\delta\rho_a = \delta\rho_b$ $\rho_{2a} > \rho_{2b}$ (i.e. $\rho_{1a} > \rho_{1b}$)
l_{1a}	0.6048	0.4093	0.9277
s_a	1.1425	1.4134	0.9510
s_b	0.7820	0.7136	1.6287
δw_a	0.2663	0.1333	0.3809
δw_b	0.5869	0.6441	0.1284
δU	0.9594	1.0166	0.8918

Notes: 1. B.m.k represents benchmark. 2. Benchmark case of heterogeneous productivity and amenity has no externalities.

Other three benchmark cases embodying negative relationship between productivity and amenity are shown in Table 3.7. Only when skill-differential amenity and high-skill preferred amenity is more abundant in city a without variation in low-skill preferred amenity, can welfare inequality wider than that of the identical-city benchmark case.

4. *Counterfactuals of Urban Characters and Externalities*

4.1 Data

Previous section documents impacts on skill concentration and welfare inequality by all the possibilities of heterogeneous efficiency and amenities. We now adopt wage and skill data of metropolitan areas in 2005 from NBER version Current Population Survey (CPS) as well for the heterogeneous study of skills. This dataset is of fine quality to report earnings per week in workers' 4th and 8th interview from CPS. CPS is a joint effort conducted by U.S. Census Bureau and the U.S. Bureau of Labor Statistics (BLS). Besides, the foundation is paved by the fact that our homogeneous urban accounting counterfactuals using this dataset share the same results and analyses with D-RH (2013). Geographic measuring units are classified by 2003 Office of Management and Budget (OMB) standard of metropolitan areas (Core Based Statistical Areas). A cutoff of skilled population to determine whether a city exists is defined as the total labor multiplied by the share of that skill group[9]. The threshold of total labor is 2600, which is calculated using 3000 used in D-RH (2013) multiplied by the mean of cities' labor force share[10].

4.2 The results without agglomeration economies

In order to decompose skill concentration and welfare inequality, we simulate various counterfactual situations and demonstrate impacts of location fundamentals and externalities by comparisons among reality and counterfactual outcomes.

Skill concentration refers to large amount of high-skill workers agglomerate in a small number of cities with large high-skill ratio. Skill concentration is calibrated as the mean difference of skill-share between the top 50 percentile cities and the bottom 50 ones, and shrunken or expanded distributions capturing whether high-skill workers relatively cluster are displayed.

Table 4.1 reports the mean difference and the mean and standard deviation of skill-share for both year 2005 and counterfactual cases by alternative equalized urban attributes and externalities. Table 4.2 reports log nominal wage gaps and welfare inequality in year 2005, and in various

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[9] In the following counterfactual Figures and Tables, cities that are too small to be existed are not taken into account.

[10] Mean of labor force share = 1 – mean of old people share. 2600 = 3000 * (1 - mean of old people share in 2005).

counterfactual scenarios. Column (1) in both Table 1 and Table 2 represent the case of equalizing individually full productivity (including location advantage and productivity externality), full amenity (including fundamental amenity and amenity externality), and excess friction. Therefore comparison between year 2005 actual and each of the counterfactual case in Column (1) allows illustrating the decomposition of skill concentration into full productivity, full amenity and excess friction. Column (3) in both Table 1 and Table 2 record the results when skill concentration and welfare inequality are disintegrated into location advantage, productivity externality, full amenity and excess friction. Column (4) in both Table 1 and Table 2 state the outcomes when skill concentration and welfare inequality are accounted for by location advantage, productivity externality, fundamental amenity, amenity externality and excess friction. Comparison between Column (2) and Column (3) allows inspecting the situation if urban size does not do good to urban productivity.

Table 4.1: Skill concentration in counterfactual agglomeration economies scenarios

Scenarios / mean (std. dev)	Weighted Averaged urban characters	2005 Actual	(1) $\beta = 0$ $\eta = 0$ $\zeta = 0$	(2) $\beta = 1.34$ $\eta = 0$ $\zeta = 0$	(3) $\beta = 1.34$ $\eta = 0.07$ $\zeta = 0$	(4) $\beta = 1.34$ $\eta = 0.07$ $\zeta = 0.2$
Top 50% vs. Bottom 50%	Productivity	0.3914	0.0771	0.0302	0.0110	0.0385
	Amenities		0.0591	0.0424	0.0065	0.0823
	Excessive frictions		0.0958	0.0681	0.0599	0.2056
Skill-share (S=L2/L1)	Productivity	0.70 (0.26)	0.74 (0.05)	0.75 (0.02)	0.75 (0.01)	0.74 (0.06)
	Amenities		0.74 (0.04)	0.74 (0.03)	0.75 (0.01)	0.73 (0.07)
	Excessive frictions		0.70 (0.06)	0.73 (0.04)	0.74 (0.05)	0.62 (0.14)

Note: Cities with population < 2600 are dropped.

We discuss firstly the impacts of full productivity on skill concentration and welfare inequality. In Column (1) of Table 4.1, counterfactual exercises are performed by shutting down heterogeneity in full productivity, full amenity and excess friction. Equalization of full productivity results in less skill concentration as the differential mean is smaller than the actual of it in year 2005, 0.0771 vs. 0.3914. The mean of skill-share becomes bigger while the standard deviation tends to be smaller than the actual one. Figure 4.1 and Figure 4.2 respectively depicts high-skill population distribution, and skill-share distribution. As can be seen from Figure 4.1, large cities are more efficient, eliminating their productivity advantage causes high-skill workers migrate to other cities where urban amenities are more desirable and frictions are less severe. So it is with low-skill workers[11]. There are 51.3 percent of high-skill workers moving to other cities than their originated cities, the number for low-skill workers is 43.0 percent. Figure 4.2 shows that skill-share distribution contracts, relative to the actual one in 2005, and it implies that switching off heterogeneous productivity strikes the more educated workers intensively by letting them largely escape, then distribution of high-skill across cities becomes more even.

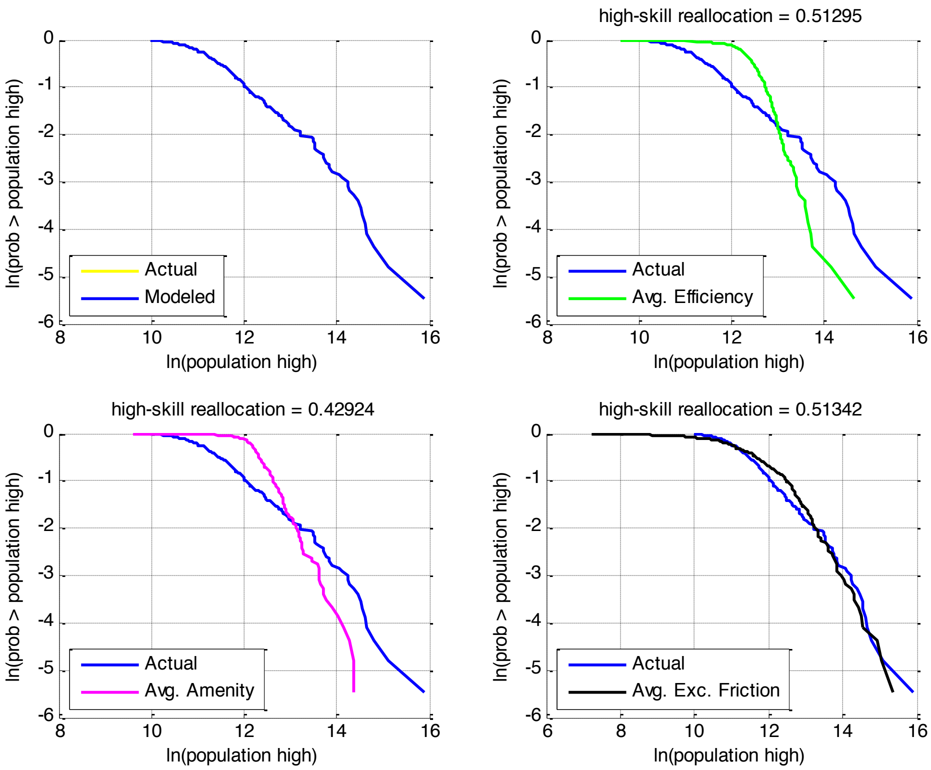

Figure 4.1: Counterfactuals of High-skill Population without One Shock, $\eta = 0$, $\beta = 0$, $\zeta = 0$

Decreased skill-share do not create more low-skill jobs such that log nominal wage gap recorded in Column (1) of Table 2, 0.54, is larger than the actual log nominal wage gap in 2005, but it is opposite as for standard deviation. Figure 4.3 demonstrates that lower wage premium cities are more productive, since equalized efficiency pushes low-skill earnings down to a larger

[11] High-skill population distribution and low-skill population distribution share very similar shape so city size distribution is resembled to high-skill distribution in Figure 4.1.

extent than the high-skill, in order to be comparable with those very low low-skill earnings in the less productive and wider wage gap cities, henceforth wage premium rises. Utility of high-skill and low-skill workers both are raised respectively with a fairly massive amount since distortion of spatial heterogeneous efficiency is removed; yet inequality between their utility levels is escalated too. Productive cities lose efficiency advantage, in which high-skill worker productivity suffers directly, and low-skill worker productivity determined by local skill mix due to the imperfect substitution as well is depleted by less concentration of high-skill workers. Both of the two skill groups migrate to where they are indifferent among all locations. Nonetheless, more even distribution of human capital is unable to benefit as much as in the reality to low-skill workers, their welfare increments cannot outperform those of high-skill workers. Due to the existence of imperfect substitution association between the two skill groups, equalizing productivity leads to less skill concentration but more welfare inequality.

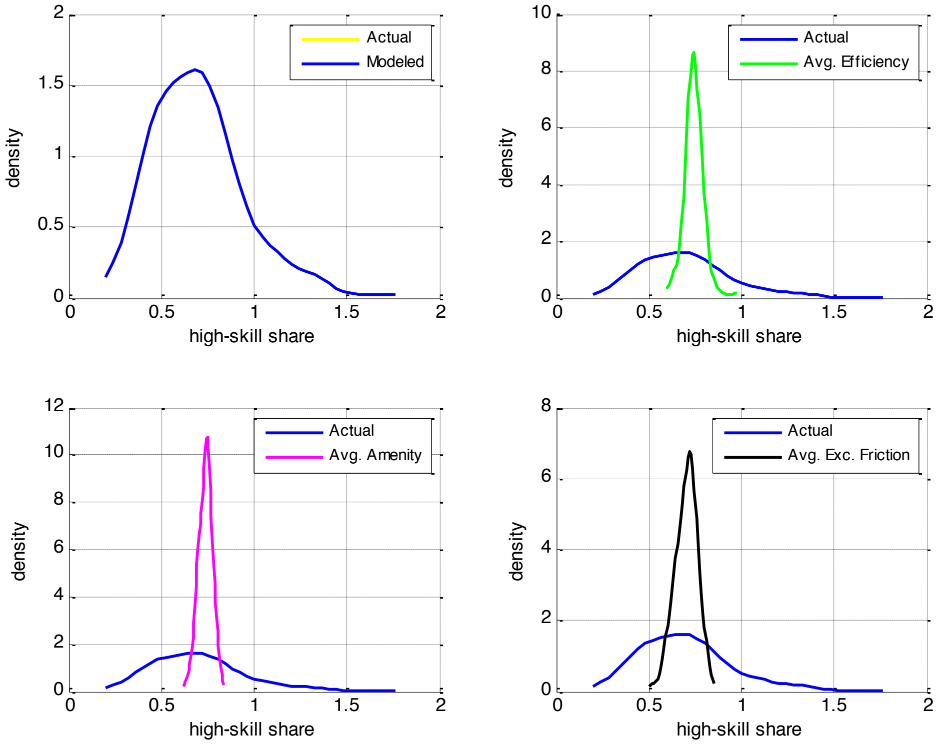

Figure 4.2: Counterfactuals of City Education Level without One Shock, $\eta = 0$, $\beta = 0$, $\zeta = 0$

Conforming to homogeneous skill urban accounting in D-RH (2013) and our improved heterogeneous skill accounting model, proportional reallocations are relatively massive while utility increments are fairly modest although concavity of total wage to population generates more welfare gains if urban size distribution is less dispersed, and convexity of aggregate friction to population lowers utility losses.

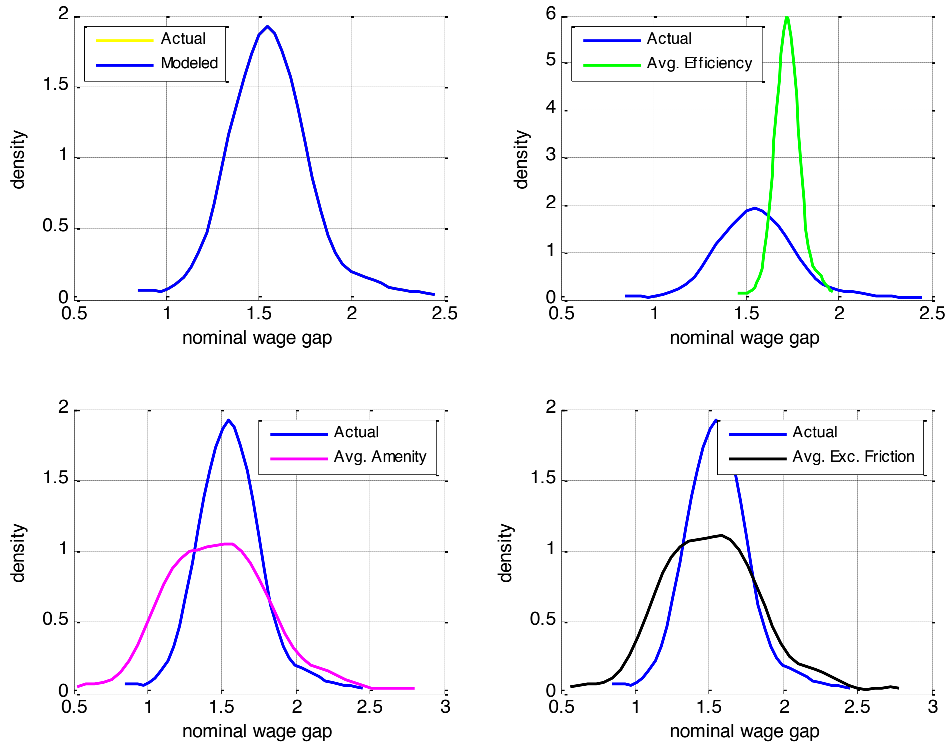

Figure 4.3: Counterfactuals of Nominal Wage Gap without One Shock, $\eta = 0$, $\beta = 0$, $\zeta = 0$

Counterfactual amenity exercise is performed with two steps. First, equalize low-skill preferred amenity by its weighted average; second, calculate equalized high-skill preferred amenity via its relationship with low-skill preferred one governed by ω, residuals and the weighted average low-skill preferred amenity. Absence of heterogeneity in full amenity causes less skill concentration and welfare inequality as found in Column (1) of Table 1 and of Table 2. Fewer reallocations happen than equalizing productivity; there are 42.9 percent of high-skill workers and 42.8 percent of low-skill workers. Different from homogeneous-skill urban accounting, it is concluded from Figure 4.1 that urban size is more evenly distributed, suggesting that larger cities have more desirable full amenities at least to their weighted average. Figure 4.2 tells that losing amenity advantage means loss of population, specifically, more displacement of high-skill workers than low-skill ones of these previously amenable locations to other places. Figure 4.3 demonstrates that lower wage premium cities are more desirable[12], because high-skill

[12] Nominal wage gap is determined by amenity and skill-differential WTP based on Fu and Hao (2014); moreover, we should be aware that amenity is determined by fundamental amenity and amenity externality generated by larger city size, which is determined by productivity, amenity and friction after all, therefore wage gap is also determined by location attributes and externalities.

and low-skill workers get away from their previous locations in absence of amenity heterogeneity, so low-skill wage could have been raised as compensation when low-skill preferred amenities are worse-off in those destination cities, then welfare gap between the two skill types is narrower. One outstanding result from Table 4.2 is that utility levels of the two skill groups are lower than in actual. Removing distortion of spatial heterogeneity in amenity does not attain higher utilities. This is on account of the fact that

Table 4.2: Log nominal wage gap and welfare inequality in counterfactual agglomeration economies scenarios

Welfare \ Scenarios	Weighted Averaged urban characters	2005 Actual	$\beta = 0$, $\eta = 0$, $\zeta = 0$	$\beta = 1.34$, $\eta = 0$, $\zeta = 0$	$\beta = 1.34$, $\eta = 0.07$, $\zeta = 0$	$\beta = 1.34$, $\eta = 0.07$, $\zeta = 0.2$
log nominal wage gap	Productivity	0.43 (0.15)	0.54 (0.04)	0.49 (0.00)	0.46 (0.00)	0.46 (0.00)
	Amenities		0.36 (0.26)	0.35 (0.26)	0.35 (0.29)	0.36 (0.31)
	Excessive frictions		0.39 (0.24)	0.37 (0.25)	0.36 (0.29)	0.48 (0.27)
High-skill utility	Productivity	11	11.3641	11.0744	10.9709	11.0825
	Amenities		10.7413	10.8403	10.8947	10.6527
	Excessive frictions		11.0420	11.0661	11.1268	10.7055
Low-skill utility	Productivity	10	10.262	10.0278	9.9483	10.0365
	Amenities		9.8229	9.8674	9.9052	9.6832
	Excessive frictions		10.0883	10.0874	10.1352	9.7188
Welfare inequality	Productivity	1	1.1021	1.0466	1.0226	1.0460
	Amenities		0.9184	0.9729	0.9895	0.9695
	Excessive frictions		0.9537	0.9787	0.9916	0.9867

Note: Cities with population < 2600 are dropped.

productivity has been changed when size and skill mix of each city change when altering amenities: smaller urban and less skill concentration drive down urban productivity and utility levels.

Urban frictions keep people from locating into the same city, or else people would all locate into the one single place where productivity is the highest and amenity is the most desirable. When equalizing excess friction, a large quantity of population migrate but it is hard to conclude that large cities are less frictional from urban size distribution in Figure 4.1: 51.3 percent of high-skill workers and 53.1 percent of low-skill workers reallocate to other cities. Figure 4.2 shows the probability that large cities are more frictional because lower excess friction creates the chance for low-skill workers coming to large cities and skill-share distribution is more contracted. Figure 4.3 describes that lower wage premium cities have higher excess frictions. When excess friction is equivalent across cities, high-skill workers reallocate to these lower wage premium cities, generating more labor demand for low-skill workers, wage premiums of these cities shrink, and this is why we observe in Table 4.2 that average wage premium is reduced, and welfare inequality between these two groups declines by getting closer to high-skill workers.

The three urban characters are important to high-skill and low-skill workers, since eliminating any of them results in significant changes in skill concentration and welfare inequality. In reality, heterogeneous full productivity significantly contributes to reducing welfare inequality, while full amenity and excess friction respectively expands welfare gap. Spatial variation in each of the three urban factors, efficiency, amenity and excess friction, facilitates geographic cluster of high-skill workers. The results are in line with the two-city example that heterogeneity in either productivity or amenities leads to more skill concentration, but cannot reach the conclusion on welfare inequality. In the case without externalities, full amenity (in terms of fundamental amenity and amenity externality explained by urban size) devotes the most to enhancing skill concentration.

4.3 The results with all agglomeration economies

In this subsection, counterfactual scenarios of location fundamentals with all agglomeration economies are discussed, including productivity externality and amenity externality. Location fundamentals are location advantage, fundamental amenity and excess friction.

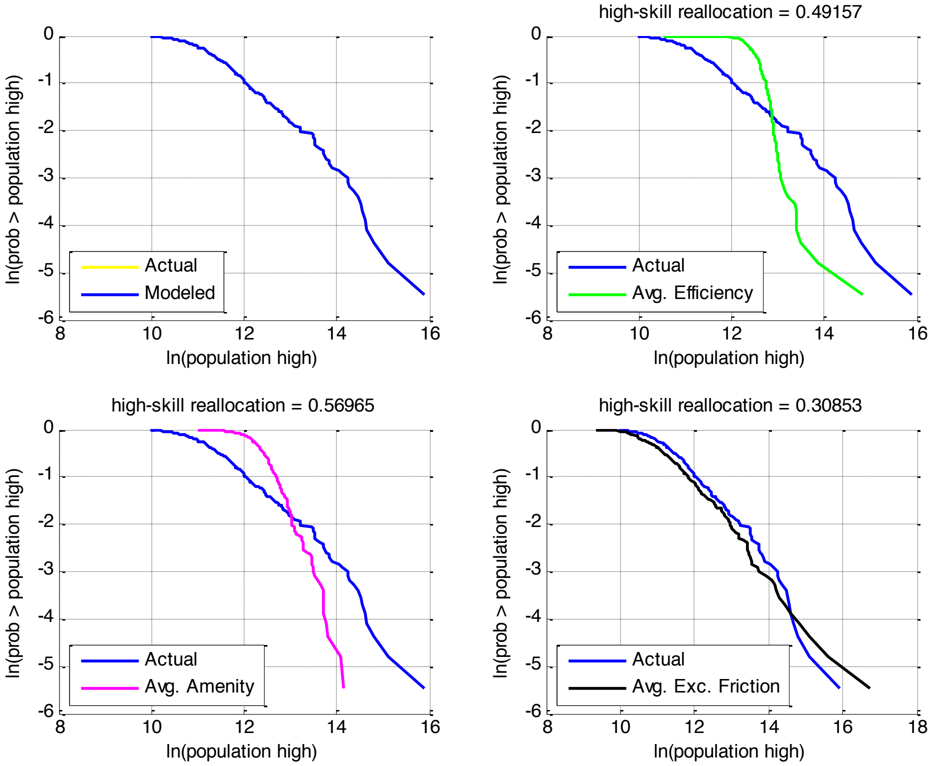

Figure 4.4: Counterfactuals of High-skill Population without One Shock, η = 0.07, β = 1.34, ζ = 0.2

When skill concentration and welfare inequality are decomposed by heterogeneous location fundamentals and two types of externalities, homogeneous location advantage across cities by its weighted average enormously weakens skill concentration, as found from Column (4) in Table 4.1 that difference of average skill-share between the top 50 percentile cities and the bottom 50 percentile ones are all smaller than in reality. Besides, Column (4) in Table 4.2 reveals that welfare inequality gap is wider. Worker reallocations emerge, with 49.2 percent of high-skill workers and 44.7 percent of low-skill workers migrating to cities other than their originations. Out-migrations from large cities imply that large cities own location advantages. Absent heterogeneity in location advantage directly leads high-skill workers to reallocate, but low-skill workers are indirectly affected by altered location advantage through its impacts on skill-composition. High-skill workers in these previously productive cities directly receive large shocks because of both effect of productivity externality and effect of amenity externality compound. To gain productivity, low-skill workers reallocate in order to follow high-skill workers, to where location advantage is counterfactually abundant and high-skill preferred amenity is desirable. Low-skill workers, as the followers, might be unable to voluminously consume their favored amenities; besides, amenity externality reinforces the larger amenity benefits to high-skill workers. Skill-share distribution becomes more even, low-skill wage might fall due to fewer of them close to high-skill workers after the migration, wage premium in general is increased. Figure 4.6 exhibits a more even distribution of wage premium with equalized location advantage, which is more likely to generate wider welfare gap in the case of larger average wage premium as can be seen from Column (4) in Table 4.2. Distinct from the homogenous-skill urban accounting exercises, no city exits with urban size smaller than 2600 using weighted average location advantage. This means each city has its some virtues of being existent, one of which being deprived does not endanger the city to secede.

Comparing Column (1) and (4) in both Table 4.1 and Table 4.2, we find that the compounded effects of the two types of externalities cause even less skill concentration and even lower welfare inequality in the case of equivalent location advantage across cities. The reason is that less skill concentration is worse-off to low-skill workers due to no-utilization of imperfect substitution, less skill concentration and smaller urban size as well hinder high-skill workers to exploit productivity externality in the case with presence of agglomeration economies. Therefore we also find that high-skill utility level is lower with externalities relative to the case without externalities. Total externality plays the role of influencing skill concentration and welfare gap in the same direction: reducing both skill concentration and welfare gap.

A more even counterfactual urban size distribution with equalized fundamental amenity is found in Figure 4.4, with large cities losing population while small cities gaining population. There are 57.0 percent of high-skill workers and 50.0 percent of low-skill workers choose different cities other than their originations. Consistent with equalized full amenity case in above subsection, indifferent fundamental amenity across cities let large and fundamentally amenable cities lose population. Ascribing to the compounded effect from amenity externality, percentages of reallocation are larger than the case without externality. Once again lost amenity advantage let high-skill workers reallocate, then skill concentration is weakened. Low-skill workers follow high-skill migration pattern to get benefits by imperfect substitution, low-skill workers' co-locating behavior is proved by even skill-share distribution in Figure 4.5. Workers in large cities can only migrate to destinations where location advantage in productivity is lower, since large cities are productive and amenable. In this case, utility levels of both high-skill and low-skill workers decline, relative to those in actual.

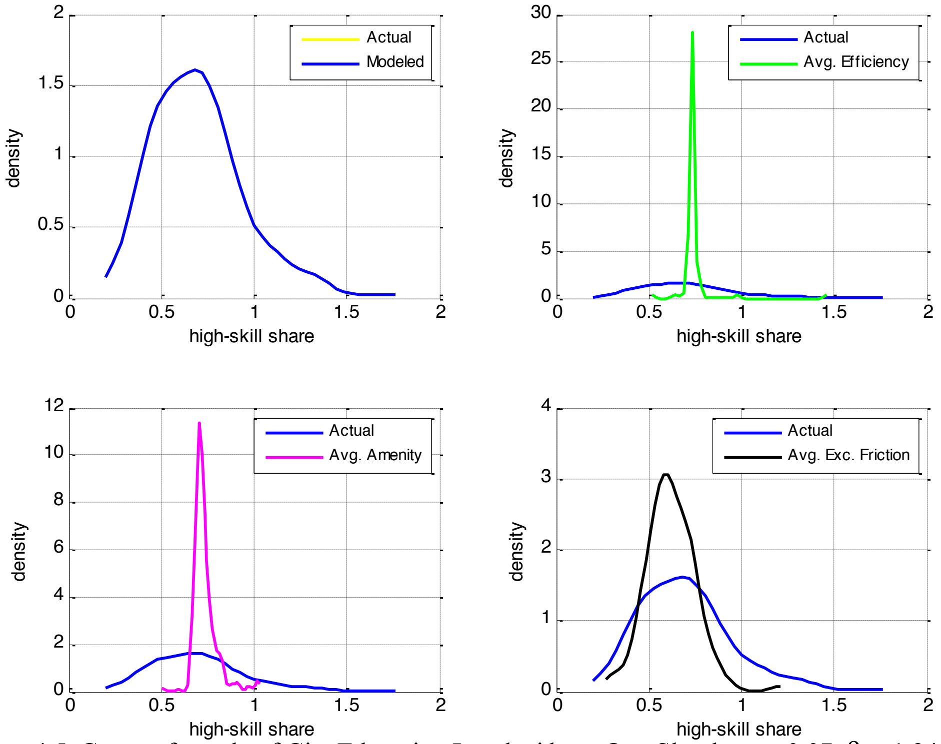

Figure 4.5: Counterfactuals of City Education Level without One Shock, η = 0.07, β = 1.34, ζ = 0.2

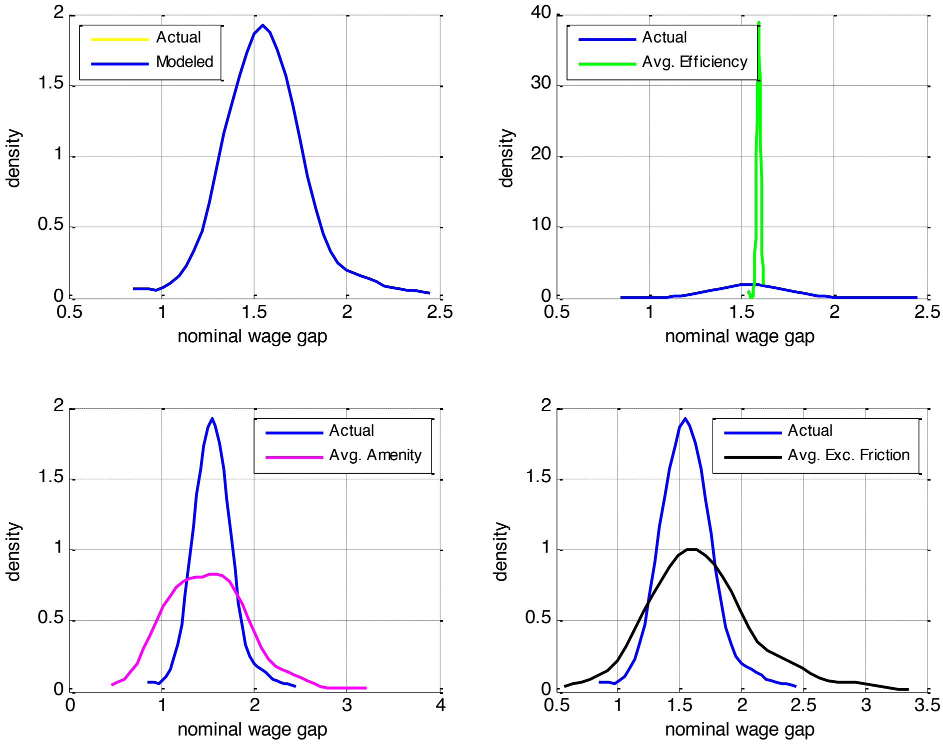

Figure 4.6: Counterfactuals of Nominal Wage Gap without One Shock, η = 0.07, β = 1.34, ζ = 0.2

Compared with Column (1), total externality affects both skill concentration and welfare inequality in the same direction. Total externality engenders intensified skill concentration and welfare inequality. That high-skill workers cluster means they are able to bask higher productivity and desirable amenity, which substantially cause widened welfare gap with the presence of total externality.

When excess friction is identical by its weighted average across cities with the existence of two types of externalities, 51.3 percent of high-skill workers and 19.0 percent of low-skill workers choose to reallocate, and this is divergent from the case without externalities in which similar percentages of high-skill and low-skill workers migrate, this is also against what Diamond (2012) claims that low-skill workers are more sensitive to frictions. This suggests that existence of the two types of externalities is of great importance to accounting for skill concentration and welfare inequality. As can be seen from Figure 4.2, urban size distribution is more dispersed when switching off excess friction heterogeneity. This implies that large cities are more excessively frictional, opposed to D-RH (2013). Lower excess friction attracts even more in-migration into large cities, and this results in the probability that some newly generated low-skill jobs (by imperfect substitution) should have been done by low-skill workers are replaced by in-migrated high-skill ones. Therefore we observe fewer low-skill labor reallocation, and more skill concentration than the cases of equalizing location advantage and fundamental amenity. Once again crowdedness hurts more high-skill workers than low-skill ones, so welfare gap is narrower than in reality.

Comparing Column (1) and (4) in respectively in Table 4.1 and Table 4.2, skill concentration and welfare inequality are both intensified in the case of indifferent excess friction. Reallocation of high skilled workers to large, fundamentally productive and amenable cities generates substantial externalities, amplifying the inequality in welfare and degree of skill concentration. Note also that in Column (4) utility levels for both two skill groups are lower than in reality when excess friction heterogeneity is eliminated. Equivalent excess friction results in more dispersed the productivity distribution by changing urban size distribution, hence concavity between total wages and productivity undermines welfare gains and convexity between total friction and urban size intensifies welfare losses. Utility will be lower as long as losses outweigh gains.

By and large, total externality (including productivity externality and amenity externality) affect skill concentration and welfare inequality in the same direction. It undermines welfare inequality and skill concentration in the case of equalizing location advantage, but magnifies skill concentration and welfare inequality in the counterfactual fundamental amenity and excess friction cases.

Comparisons among all the four columns allow investigating roles played by productivity externality, and by amenity externality. Presence of productivity externality driven by skill mix from Column (1) to Column (2) in any of the three counterfactual urban attributes cases, weakens skill concentration; in addition, emergence of productivity externality driven by urban size from Column (2) and Column (3) in each of the three equalized urban attributes cases, further crippling skill concentration. Based upon that each equalized location attribute results in less skill concentration, productivity externality reinforces impact of location attributes and leads to even less skill concentration. However impacts on welfare inequality are divergent. In the case of equivalent efficiency, productivity externality impelled by skill-composition lowers welfare inequality, so does the one impelled by urban size. In either case of equivalent amenity or indifferent excess friction, productivity externality propelled by skill-share enlarges welfare inequality, so does the one propelled by urban size. The distinguished impacts of productivity externality center on that less skill concentration generates insufficient low-skill demand, then leads to wider welfare gap in the counterfactual amenity and excess friction cases. Productivity externality effect is lessened when urban size is smaller and skill-share is lower; as there is no spatial heterogeneity in efficiency, less skill concentration directly harms high-skill workers so much that their utility level is decreased in the presence of productivity externality, and hence welfare gap is lowered.

Comparing Column (3) and Column (4) respectively in Table 4.1 and Table 4.2 allows investigating role of amenity externality. Unlike productivity externality, amenity externality enhances skill concentration in all the three counterfactual cases, undermines welfare inequality in the cases of counterfactual amenity and excess friction, and enlarges welfare inequality when equalizing location advantage. The effect of total externality is produced by counteractive productivity externality and amenity externality. Amenity externality enhancing skill concentration is because high-skill workers prefer more urban amenities than low-skill ones do. As for welfare inequality, imperfect substitution benefits low-skill workers sufficiently in the case of equalizing fundamental amenity and excess friction so that it shuts down a little of welfare gap. With enhanced skill concentration resulted from the case of equivalent location advantage, more high-skill workers are able to reallocate to where their preferred urban amenities are more desirable, this increases welfare inequality.

4.4 The results with alternative substitution elasticity and labor loss per mile

As the elasticity of substitution augments, high-skill demand curve turns to be less downward sloping, meaning that the relative output price between the two skill groups rises and fewer low-skill jobs are created. Column (1) in Table 4.3 and Table 4.4 demonstrate less skill concentration and narrower welfare inequality when elasticity of substitution is raised from 1.6 to 1.8. Column (2) in which both two types of externalities exist suggests that total externality contributes to enhancing skill concentration and welfare inequality. A larger elasticity of substitution is a direct bummer for low-skill workers and indirectly dreadful for high-skill workers, so utility levels decline relative to their actual magnitudes.

High-skill population distribution, skill-share distribution and nominal wage gap distribution across cities are respectively displayed in Figure 4.7, Figure 4.8 and Figure 4.9. There are 42.1 percent of high-skill workers and 42.3 percent of low-skill workers reallocate. Urban size distribution becomes more even due to raised elasticity of substitution with or without agglomeration economies, i.e. larger cities lose population and smaller cities gain population. There are 40.6 percent of high-skill workers and 31.4 percent of low-skill workers migrate when total externalities are present. The results emphasize the significance of the elasticity of substation which documents the imperfect substitution relationship between the two skill groups.

Table 4.3: Skilled concentration and nominal wage gap in counterfactual scenarios of substitution elasticity and labor loss of commuting

Scenarios / mean (std. dev)	2005 Actual	(1) ε=1.8 β = 0 η = 0 ζ = 0	(2) ε=1.8 β = 0.07 η = 1.34 ζ = 0.2	(3) κ=0.002 β = 0 η = 0 ζ = 0	(4) κ=0.002 β = 0.07 η = 1.34 ζ = 0.2
Top 50% vs. Bottom 50%	0.3914	0.0725	0.2001	0.1730	0.2450
Skill share	0.70 (0.26)	0.73 (0.05)	0.79 (0.17)	0.68 (0.11)	0.73 (0.16)

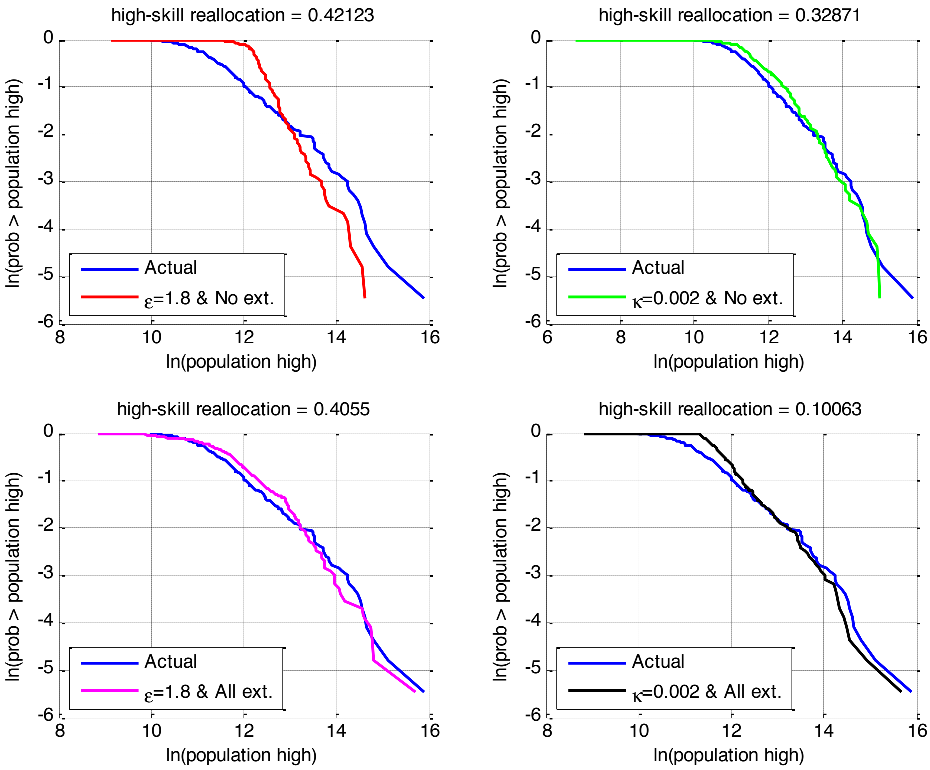

Figure 4.7: Counterfactuals of High-skill Population with Alternative ε, κ, and externalities

As can be seen from Figure 4.7, large cities lose population and small cities gain. This suggests that imperfect substitution is one important source of shaping large cities. Both high-skill and low-skill workers are affected due to stronger imperfect substitution between skill cohorts, which directly impairs low-skill workers because their demand generated by lower elasticity of substitution declines, so does their productivity. An indirect harm is toward high-skill workers, since less extensive division of labor by stronger imperfect substitution accentuates the human capital service provided by high-skill workers, so their productivity is lower. As a result, skill-share distributions in both cases (with and without total externality) are more contracted as found in Figure 4.8. Wage premium gaps become more expanded with lower means of wage premium. Recall that higher skill-share cities are more likely to have lower wage gaps from subsections 4.2 and 4.3, loss of benefits to low-skill workers by imperfect substitution in large cities tends to enlarge wage premium, while shrink wage premium in small cities since high-skill workers could have reallocated there and get close to low-skill workers despite benefits by imperfect substitution is lower as elasticity of substitution is larger. Welfare inequality thus is narrower.

In the presence of total externality, it intensifies both skill concentration and welfare inequality, because high-skill worker productivity is sustained by productivity externality, the existence of amenity externality also keeps high-skill workers in large cities, therefore it is observed that urban size distribution is relatively less even relative to the case without agglomeration economies. But as for low-skill workers, larger agglomeration economy of productivity is an indirect benefit, therefore welfare gap is widened.

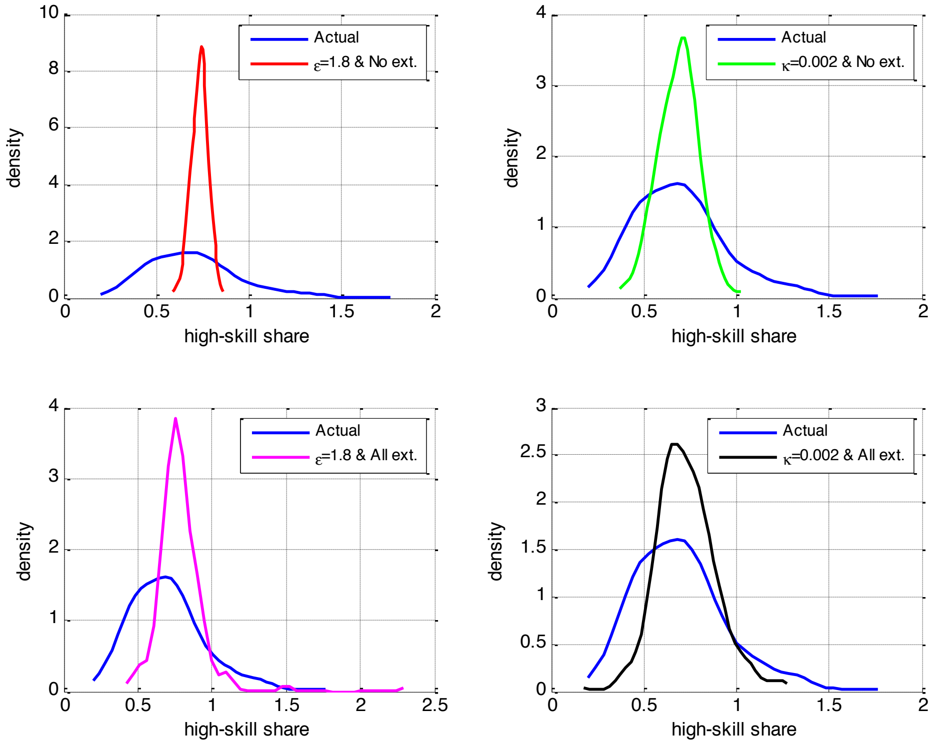

Figure 4.8: Counterfactuals of City Education Level with Alternative ε, κ, and externalities

Counterfactual cases of increased labor loss per mile with and without total externalities have similar results as the alternation of substitution elasticity. Workers in large cities respond more sensitively to the increased labor loss per mile. 32.9 percent of high-skill workers move to other cities than originations and 34.6 percent of low-skill workers do so without externalities; while 10.1 percent of high-skill workers move to other cities than originations and 8.4 percent of low-skill workers migrate with total externality. On account of that labor loss per mile takes effect on everyone in the cities, augmentation of it leads to losses of both high-skill utility and low-skill utility.

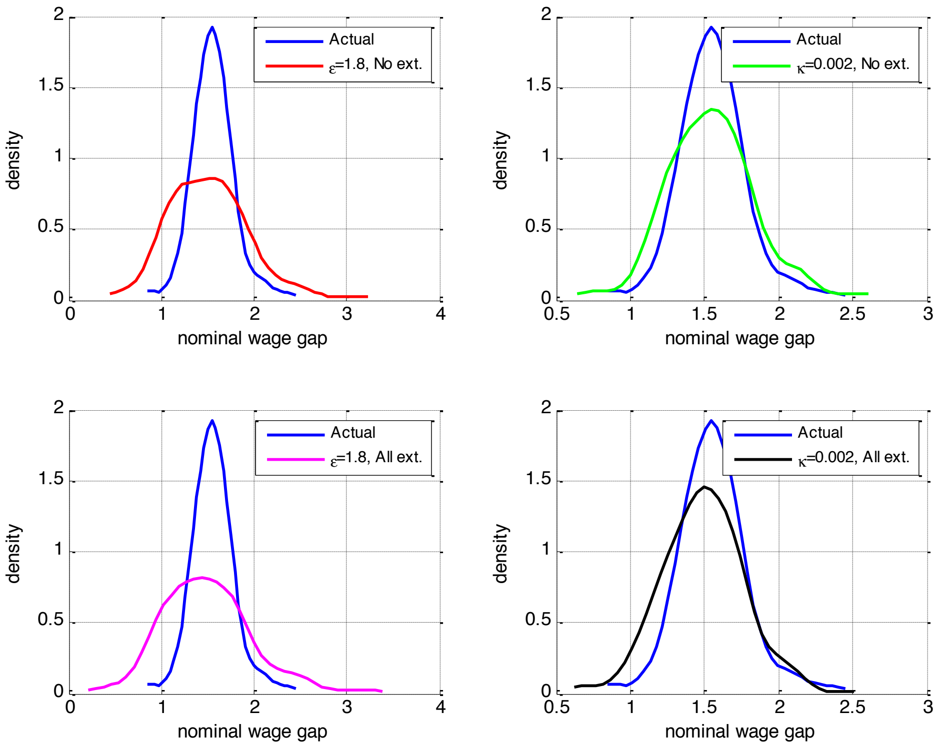

Figure 4.9: Counterfactuals of Nominal Wage Gap With Alternative ε, κ, and externalities

Table 4.4: Skilled utility in counterfactual scenarios of substitution elasticity and labor loss of commuting

Scenarios / Welfare	2005 Actual	(1) ε=1.8 $\beta=0$ $\eta=0$ $\zeta=0$	(2) ε=1.8 $\beta=0.07$ $\eta=1.34$ $\zeta=0.2$	(3) κ=0.002 $\beta=0$ $\eta=0$ $\zeta=0$	(4) κ=0.002 $\beta=0.07$ $\eta=1.34$ $\zeta=0.2$
log nominal wage gap	0.43 (0.15)	0.37 (0.31)	0.34 (0.36)	0.42 (0.20)	0.38 (0.20)
High-skill utility	11	10.3324	10.5710	10.4193	10.6001
Low-skill utility	10	9.399	9.5711	9.4446	9.6037
Welfare inequality	1	0.9334	0.9999	0.9747	0.9964

Large cities should be more sensitive to the increased costs on road since they have already been congested, so urban size distribution turns to be more even. High-skill workers reallocate from large and highly educated cities, leading to more even distribution of skill mix as seen in

Figure 4.8; meanwhile, low-skill workers productivity is lowered due to getting close to insufficient quantity of high-skill workers, and wage premium is expanded. Comparison between Column (3) and (4) suggests that total externality enhances both skill concentration and welfare inequality, which conforms to the comparison between Column (1) and (2). These two consistent results imply that in reality that all three urban attributes are heterogeneous, total externalities reinforce both welfare inequality and skill concentration.

In addition, utility levels of counterfactual substitution elasticity are respectively lower than those of counterfactual labor loss per mile, because reallocations in counterfactual substitution elasticity are larger (eg. 42.1% vs. 32.9%), bringing about more even urban size distribution which adjusts lower utility levels.

5. *Conclusions*

Welfare inequality, skill mix, and nominal wage gap are essentially the mapping outcomes of spatial fundamentals and externalities together with parameters. This paper aims to decompose welfare gap between high-skill and low-skill workers, geographic concentration of high-skill and nominal wage gap distribution into urban attributes such as productivity, amenities and excess friction, together with productivity externality and amenity externality. It also examines skill concentration and welfare inequality in cases that imperfect substitution and labor loss per mile individually alters. Through counterfactual exercise in each counterfactual situation, we simulate the new spatial equilibrium, and expound role of each source via comparisons among scenarios.

We first modify the theoretical urban accounting model that D-RH (2013) has built. Since labor income losses instead of labor losses should have been accounted for, working hours then are invariant over cities. We find analogous outcomes with analogous intuitions to explain these outcomes, even though counterfactual distribution patterns are much clearer after modification. Utilizing Jones (2014) productivity accounting framework, we extend D-RH (2013) to skill accounting model. High-skill workers are productive because of lower coordination costs which are generated by location advantage, skill mix and urban size, while low-skill workers are productive by local skill mix due to imperfect substitution between the two skill groups.

Both urban attributes and two types of externalities are found important to determine skill concentration and welfare inequality. Impacts of location fundamentals are examined. Heterogeneity in any one of location advantage, fundamental amenity and excess friction enhances skill concentration; moreover, either heterogeneous fundamental amenity or excess friction across cities enlarges welfare inequality, but due to the imperfect substitution engendering benefit to low-skill workers, distinct location advantage across cities curtails welfare inequality gap. The presence of both productivity externality and amenity externality, further weakens skill concentration and welfare inequality in the case of equalized location advantage in productivity, but further enhances skill concentration and welfare inequality in either the case of indifferent fundamental amenity or equivalent excess friction case, albeit in reality that all three urban attributes are heterogeneous across cities, total externality intensifies both welfare inequality and hinders skill concentration. Unlike Diamond (2012), this study does not incorporate the amenity externality driven by skill mix, is not only because only a small strand of literature discusses about it, but also due to that amenity externality by urban size has already been able to facilitate skill concentration.

This paper validates Jones (2013) that high-skill and low-skill workers are imperfectly substitutable into urban accounting framework, brings about comprehensive understandings on geographic skill concentration and welfare inequality. As more and more higher educated workers concentrate in large, productive, amenable as well as frictional cities, this study offers one view of investigating the contemporary phenomenon. It also links closely to explanations for change of welfare inequality over the last three decades in U.S. cities. Our 2-city example illustrates that relatively large supply of high-skill workers endowment narrows welfare gap. As U.S. workers are becoming more and more educated, this endowment change over the past years could have contributed to weaken welfare inequality. More research on the influences of place-based policy on skill concentration welfare inequality in future could also benefit from this study.

Appendix: Data

A Core Based Statistical Area (CBSA) is a U.S. geographic area defined by the Office of Management and Budget (OMB) based around an urban center of at least 10,000 people and adjacent areas that are socioeconomically tied to the urban center by commuting; areas defined on the basis of these standards applied to Census 2000 data were announced by OMB in June 2003. There are 942 CBSAs, of which 336 Metropolitan Statistical Areas (MSAs) are populated with at least 50,000 people, and the rest of which are Micropolitan Statistical Areas (μSA). Geographic measuring units in this paper are MSAs.

Consumption:

Aggregate consumption is the consumption net of housing and transportation costs by goods and services. Aggregate consumption of all cities C = Market-based PCE – Motor vehicles and parts – Gasoline and other energy goods – Transportation services. Market-based PCE (Personal Consumption Expenditure) is a supplemental measure that is based on household expenditures for which there are observable price measures in the addenda from National Income and Product Accounts (NIPA).

Following D-RH (2013), we also proxy private consumption net of housing and transportation in each city, C_i, by its retail earnings:

$$C_i = \frac{retail\ earnings\ in\ MSA_i}{retail\ earnings\ in\ U.S.} * private\ consumption\ net\ of\ housing\ and\ travel\ in\ U.S.$$
$$= \frac{retail\ earnings\ in\ MSA_i}{\sum_i retail\ earnings\ in\ MSA_i} * C$$

Retail earnings are defined as personal earnings from retail trade in the Regional Economic Accounts (REA) of Bureau of Economic Analysis (BEA).

The correlation between total private consumption in this paper and in D-RH(2013) is as high as 0.9993 for year 2005.

Consumption per capita: $c_i = C_i / (total\ population)$.

Total population:

We adopt data of population (Census Bureau mid-year population estimates, July 1) in 2005 from Regional Economic Accounts, Bureau of Economic Analysis.

Total population excluding retirees:

Total population excluding retired people, N_i, is obtained by

$$N_i = total\,population_i * (1 - oldshare05)$$

in which people aged 65 and above are counted as old and retired. Old population share in 2005 cannot be directly retrieved so we use the proxy by its 2000 and 2010 levels and assume old population share increases within the decade exponentially:

$$oldshare05 = oldshare2000 * \left[\left(\frac{oldshare2010}{oldshare2000} \right)^{\frac{1}{10}} \right]^5$$

Old people share in 2000 and in 2010 respectively comes from census 2000 and census 2010. We also calculate old people share in 2005 using America Community Survey (ACS 2005 1-year estimate) to check its correlation with our computation and find very large positive association. Reason of not adopting the direct measurement of old people share in 2005 from ACS is that ACS sample has too many missing MSAs and unidentified households.

Aggregate personal income in homogeneous-skill urban accounting:

Aggregate personal income, W_i, is from the REA of BEA.

Average personal income of labourforce in homogeneous skill case:

$w_{it} = \frac{W_i}{N_i}$. Dividing by population excluding retirees is because some amounts of these wage supports a family, so N_i=employed workers + dependent children.

Gross Metropolitan Product (GMP):

Gross Metropolitan Product, Y_{it}, is from REA of BEA.

Average GMP of laborforce:

Average GMP of laborforce, $y_{it} = \frac{Y_i}{N_i}$.

Annual average wage and skilled labor in heterogeneous skill case:

Annual average wages of high-skill and low-skill groups w_{1i}, w_{2i}, and Skilled population L_{1i}, L_{2i}, are computed by data retrieved from extracts of Current Population Survey, i.e. Merged

Outgoing Rotation Groups files, REA of BEA and 2005 Census. In MORG Files, usual weekly hours/earnings are asked in the 4th and 8th interviews, so one fourth of the households are in the outgoing rotation each month. Weekly earnings offers a more accurate picture regarding the information quality of survey data, more details about this dataset could be found in Eeckhout, J., Pinheiro, R., & Schmidheiny, K. (2010). w_{1i} and w_{2i} are calculated by aggregate personal income multiplied by their respective share; L_{1i} and L_{2i} are calculated by total population excluding retirees multiplied by their individual share.

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